Symbolic Search in Planning and General Game Playing

Peter Kissmann

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Gutachter: Prof. Dr. Stefan Edelkamp
Universität Bremen

Prof. Dr. Malte Helmert
Universität Basel

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Zusammenfassung

Diese Arbeit behandelt drei Themenkomplexe. Der erste davon, symbolische Suche unter Nutzung von binären Entscheidungsdiagrammen (engl. *binary decision diagrams*, kurz BDDs) stellt dabei auch direkt das Verknüpfungselement mit den anderen beiden Themen, Handlungsplanung und Allgemeines Spiel dar.

Suche ist in vielen Anwendung der künstlichen Intelligenz (KI) vonnöten. Sei es, dass Graphen durchsucht, kürzeste Wege ermittelt oder möglichst gute Züge in einem Spiel gefunden werden sollen (um nur ein paar Themengebiete anzuschneiden). In all diesen Fällen ist Suche von höchster Priorität.

Viele dieser Suchprobleme bringen eine Schwierigkeit mit sich, nämlich das sogenannte Suchraumexplosionsproblem. Dabei handelt es sich um das Problem, dass auch für vermeintlich kleine Probleme die Menge an Zuständen, die durchsucht werden muss, exponentielle Ausmaße annehmen kann. Das führt in der Praxis zum einen zu langen Laufzeiten, zum anderen aber auch dazu, dass die Suche nicht vollständig durchgeführt werden kann, da der zur Verfügung stehende Arbeitsspeicher nicht ausreicht. Aufgrund des exponentiellen Wachstums der Suchräume besteht auch keine Hoffnung, in absehbarer Zukunft eine Rechenleistung und Speicherkapazität zu erreichen, die mit diesem Problem umgehen kann, da es zumeist nur geringe Verbesserungen von einer Rechnergeneration zur nächsten gibt.

Um das Problem des geringen Speichers zu behandlen kann man auf verschiedene Verfahren zurückgreifen. Eine Möglichkeit wäre die Nutzung externer Algorithmen. Diese führen die Suche weitestgehend auf der Festplatte durch, da diese typischerweise über erheblich mehr Platz verfügt als der interne RAM. In dieser Arbeit beschäftigen wir uns hingegen mit der sogenannten symbolischen Suche.

Bei der symbolischen Suche werden BDDs eingesetzt, um ganze Zustandsmengen zu speichern. Gegenüber sogenannter expliziter Suche bringen BDDs den Vorteil, dass sie auch große Zustandsmengen sehr effektiv speichern können und dabei oftmals erheblich weniger Speicherplatz benötigen als die expliziten Verfahren. Jedoch bringt der Schritt von Einzelzuständen hin zu Zustandsmengen das Problem mit sich, dass wir oftmals völlig neue Algorithmen entwerfen müssen, die für dieses Konzept ausgelegt sind.

Im Rahmen dieser Arbeit untersuchen wir BDDs auf ihre Komplexität für einzelne Suchprobleme. So ist es beispielsweise ein bekanntes Ergebnis, dass BDDs für Permutationsspiele wie etwa das Schiebepuzzle (15-Puzzle) eine exponentielle Anzahl an internen Knoten benötigen, um alle erreichbaren Zustände abzuspeichern. Wir untersuchen hier einige weitere Probleme und stellen teils untere exponentielle Schranken, teils obere polynomielle Schranken für die BDD-Größen auf.

Der zweite Teil dieser Arbeit beschäftigt sich mit der Handlungsplanung. In der Handlungsplanung sind eine Menge von Variablen, mit denen sich ein Zustand beschreiben lässt, eine Menge von Aktionen, ein Initialzustand und eine Menge von Zielzuständen gegeben. Basierend auf dieser Eingabe wird nun ein sogenannter Plan gesucht, der eine Sequenz von Aktionen darstellt, die, wenn sie in dieser Reihenfolge auf den Initialzustand angewendet werden, letztlich zu einem Zielzustand führen. Wir beschäftigen uns hier mit optimalem Planen, das heißt wir suchen nach Plänen minimaler Länge.

Häufig werden den einzelnen Aktionen auch unterschiedliche Kosten zugeordnet. In diesem Fall geht es darum einen Plan zu finden, dessen Kosten (als Summe der Kosten aller Aktionen in diesem Plan) minimal ist.

Hier stellen wir symbolische Suchalgorithmen vor, die eben solche Pläne finden können. Einige setzen auf blinde Suche, wie etwa Breitensuche oder Dijkstras Single-Source Shortest-Paths Algorithmus, andere auf heuristische Suche wie sie in A* eingesetzt wird. Ferner haben wir einen optimalen Handlungsplaner implementiert, der bereits sehr erfolgreich an internationalen Wettbewerben teilgenommen hat und diesen

im Jahr 2008 gewann. Im Rahmen der Vorstellung unseres Planers gehen wir weiter auf einige Verbesserungen an den zugrundeliegenden Algorithmen ein.

Neben diesen relativ einfach strukturierten Handlungsplanungsproblemen widmen wir uns ferner einer etwas komplexeren Domäne, dem sogenannten net-benefit Planen. Darin sind neben den normalen Zielzuständen weitere Ziele deklariert, die jedoch nicht zwingend erreicht werden müssen, typischerweise jedoch in einem höheren Gewinn resultieren. Entsprechend geht es in dieser Domäne nicht mehr nur darum, einen möglichst kostengünstigen Plan zu finden, sondern einen, der einen möglichst hohen Gewinn (durch das Erfüllen der zusätzlichen Ziele) erzielt und dabei möglichst wenig kostet (in Hinblick auf die Summe der Aktionskosten).

Für diese zweite Planungsdomäne erarbeiten wir schrittweise einen symbolischen Algorithmus, den wir ebenfalls implementiert haben und den resultierenden Planer äußerst erfolgreich in einem internationalen Wettbewerb eingesetzt haben: Auch dieser konnte den Wettbewerb im Jahre 2008 gewinnen.

Das Spannende an der Handlungsplanung ist die Tatsache, dass wir als Programmierer der Planer nicht wissen, welche Planungsprobleme letztlich tatsächlich zu lösen sind. Von einfachen Turmbau- oder Logistikproblemen bis zu erheblich komplexeren Bereichen wie einem automatisierten Gewächshaus ist vieles möglich. Der Planer muss einzig auf Basis der Eingabe intelligente Entscheidungen treffen, um in möglichst kurzer Zeit einen möglichst guten Plan zu finden.

Ein sehr ähnliches Konzept findet sich auch im Allgemeinen Spiel wieder. Hier müssen wir einen Spieler implementieren, der mit einer breiten Auswahl an Spielen konfrontiert werden kann und sie alle möglichst sinnvoll spielen soll. Wie auch in der Handlungsplanung wissen wir als Programmierer vorher nicht, welche Spiele gespielt werden. Entsprechend könnte man das Allgemeine Spiel als eine Erweiterung der Handlungsplanung ansehen. Während in der klassischen Handlungsplanung nur Einpersonenspiele möglich sind, können wir im Allgemeinen Spiel mit einer Menge von Gegen- oder Mitspielern konfrontiert werden. In der Praxis stimmt dieser erste Eindruck jedoch nicht immer vollständig, unter anderem da es im Allgemeinen Spiel keine Aktionskosten gibt und die Lösungslänge völlig belanglos ist, solange man ein möglichst gutes Ziel erreicht. Auf die entsprechenden Gemeinsamkeiten und Unterschiede beider Formalismen werden wir im Rahmen dieser Arbeit ebenfalls eingehen.

Während viele Forscher im Zusammenhang mit dem Allgemeinen Spiel an möglichst guten Spielern interessiert sind, liegt unser Hauptaugenmerkt auf dem Lösen von Spielen. Im Falle von Einpersonenspielen möchten wir also eine Art von Plan finden, der uns sicher vom Startzustand zu einem möglichst guten Zielzustand führt. Im Falle von Zweipersonenspielen hingegen ist eher eine Strategie gefragt. Diese sagt uns für jeden erreichbaren Zustand, welchen Zug wir wählen sollten, um den größtmöglichen Gewinn zu erreichen, auch unter Berücksichtigung der Züge der Gegner.

In der Vergangenheit wurde schon eine ganze Reihe von Spielen gelöst. Dafür wurden jedoch zumeist sehr spezialisierte Verfahren eingesetzt. Einige dieser Verfahren und wie sie zur Lösungsfindung genutzt wurden werden wir in dieser Arbeit in Erinnerung rufen. Anschließend widmen wir uns einigen Algorithmen, die Allgemeine Spiele lösen können. Für diese haben wir erneut auf symbolische Suche gesetzt.

Da viele Spiele einen immens großen Zustandsraum aufweisen, erweist es sich als Vorteil, diese effizient durch die BDDs verarbeiten zu können. Für BDDs ist es wichtig, eine instanziierte (das heißt variablenfreie) Problembeschreibung zu haben. Im Falle der Handlungsplanung gibt es bereits eine Reihe etablierter Instanziierer, die die Eingabe in ein solches Format überführen können. Im Allgemeinen Spiel gab es solche Instanziierer bisher noch nicht. Im Rahmen dieser Arbeit wurde ein solcher Instanziierer entwickelt und implementiert.

Auch wenn unser Hauptaugenmerk ganz klar auf der symbolischen Suche liegt, so haben wir uns doch auch mit der Programmierung eines Allgemeinen Spielers beschäftigt. Wir werden zunächst einige etablierte Algorithmen zum Spielen allgemeiner Spiele, etwa Minimax Suche und UCT, vorstellen. Anschließend gehen wir näher auf unseren eigenen Spieler ein. Dieser setzt auf einen hybriden Ansatz, indem wir auf der einen Seite versuchen, das Spiel zu lösen, und auf der anderen Seite zwei klassischere Allgemeine Spieler nutzen. Beide Spieler setzen auf UCT, wobei einer eine Schnittstelle zu Prolog nutzt, um die Eingabe verarbeiten zu können, während der andere die Ausgabe unseres Instanziierers verarbeitet und damit erheblich schneller den Spielbaum durchsuchen kann. Der Vorteil des hybriden Ansatzes gegenüber einem reinen Spieler ist ganz klar, dass wir optimal spielen können, wenn eine Lösung gefunden wurde und damit letztlich besser abschneiden können als ein normaler Spieler allein.

Chapter 1

Introduction

> The man in black fled across the desert, and the gunslinger followed.
>
> Stephen King, *The Gunslinger*
> (*The Dark Tower* Series, Book 1)

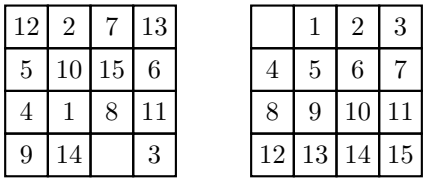
1.1 Motivation

In this thesis we are mainly concerned with two sub-disciplines of artificial intelligence, namely *action planning* and *general game playing*. In both we are confronted with large sets of states (also called *state spaces*), in which we must search for certain properties. Some examples for these properties are the following.

- In single-player games or puzzles we want to find a way to establish a goal state, given a state from which we start.
- In multi-player games we want to find a way to reach a goal state where we achieve a reward as high as possible, even if the other players play against us.
- In action planning we want to find a plan transforming an initial state into a goal state, and often wish the to contain the minimal number of actions or the sum of the costs of the actions within the plan to be minimal.
- In non-deterministic planning we want to find a policy that specifies for each state an action to take, so that a goal state can be reached independent of any non-determinism.

It is also easy to see some more practical applications of these two sub-disciplines.

- Results from game theory, especially the famous equilibria according to Nash, Jr. (1951), are often used in economics and similar disciplines.
- For entertainment we often play games. When playing against a computer we want it act reasonably, because playing against a stupid opponent often is just boring.
- When designing a mobile robot, we want it to be able to reach a certain position, so that it has to find a way to it, given noisy observations of its surroundings.
- In navigation software we want to find a route from a start location to a destination given an entire map of a country. The desired route can be the shortest, the fastest, the most energy-efficient etc.



12	2	7	13
5	10	15	6
4	1	8	11
9	14		3

(a) A possible starting state.

	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

(b) The desired goal state.

Figure 1.1: The 15-PUZZLE.

In all cases we are confronted with state spaces that can be extremely large, a fact that is often referred to as the *state space explosion problem*. Take the simple $n^2 - 1$-PUZZLE as an example (Figure 1.1 shows an instance of the well-known 15-PUZZLE). There we have a square board having n^2 positions to place tiles, and $n^2 - 1$ tiles. Given an initial state described by the positions of the tiles on the board, we want to find a way to establish a certain goal state, where all tiles are placed in specified positions. We can move the tiles only horizontally or vertically onto the one empty position, the blank. At first glance it might appear that all possible $n^2!$ permutations can be reached, but actually Johnson and Story (1879) proved that due to the special way the tiles are moved only $n^2!/2$ can really be reached. A typical instance of this puzzle is the 15-PUZZLE, in which we can reach $10{,}461{,}394{,}944{,}000$, i. e., roughly 1×10^{13} different states. Increasing the width and height of the board by only two—resulting in the 35-PUZZLE—the number of reachable states increases to $185{,}996{,}663{,}394{,}950{,}608{,}733{,}999{,}724{,}075{,}417{,}600{,}000{,}000$, which is more than 1.8×10^{41}. Finding a solution, i. e., a way to move the pieces in order to reach the goal state, requires us to search large parts of this state space.

Handling such large state spaces requires immense amounts of memory, a resource that is still comparably scarce and expensive. In the 32 bit era it was only possible to use up to 4 GB RAM. Since the change to 64 bit machines in theory it is possible to use up to 16 EB (exa bytes, i. e., $\left(2^{10}\right)^6$ bytes), which is roughly 16.8 million TB, though in practice in early 2012 personal computers seldom have more than eight or 16 GB, and recent mainboards support at most 64 GB. On server machines the physical limit is higher and mainboards supporting up to 4 TB can be found. Apart from the hardware limits, not all operating systems support so much RAM, e. g.,, the current versions of Microsoft Windows support only up to 192 GB (Windows 7), resp. up to 2 TB (Windows Server 2008).[1] Furthermore, the price for RAM is rather high. Now, early 2012, 8×8 GB RAM can be bought for roundabout 350 Euros.

If the internal memory does not suffice, another way to handle these immense search spaces would be to use external memory algorithms, which make use of hard disks or solid state disks. Nowadays, classical magnetic hard disks have a capacity of up to 4 TB, while the newer solid state disks, which use flash memory instead of magnetic disks and thus have a much better access time, currently have a capacity of up to 3 TB. Few desktop mainboards contain up to ten SATA ports to handle the disks, which would allow us to access up to 40 TB in hard disk space. If we compare the prices for RAM with the prices for hard disks (a 4 TB magnetic disk is available for a bit more than 250 Euros; a 3 TB solid state disk is available for more than 12,500 Euros) we see that using classical RAM is much more expensive than external memory.

Nevertheless, external memory algorithms also bear certain disadvantages. For example, they must be written in such a way that the random accesses to the disks are minimized, as the disk operations typically dominate the entire runtime. In other words, by switching from internal to external memory we can switch from the memory problem (too few memory to handle all the states) to a runtime problem (searching the states on the disk takes a lot longer than in internal memory). Thus, instead of using external memory algorithms we use only internal memory but utilize a memory efficient datastructure, namely *binary decision diagrams* (BDDs) Bryant (1986).

In classical explicit search each state is handled on its own, i. e., for each state we must check, if it is a goal state or which actions can be applied in it, resulting in which successor states. Using BDDs and what is then called *symbolic search* this is not a good idea, as their operations are typically slower than in explicit search and there is no advantage in memory usage in case of single state exploration. Rather, BDDs are

[1] `http://msdn.microsoft.com/en-us/library/windows/desktop/aa366778%28v=vs.85%29.aspx`

made to represent large sets of states, which is where they typically safe huge amounts of memory compared to explicit search, and there are operators allowing us to generate all the successors of all stored states in a single step.

In this thesis we handle two domains, namely action planning and general game playing. In both we are confronted with a similar setting. We have to implement an *agent* (called a *planner* or a *player*, respectively) that is able to find plans in case of action planning or play or solve the games that it is provided in general game playing. In contrast to specialized players or solvers we as the programmer do not know what kind of planning problem or game the agent will be confronted with, we only know the description languages that are used to model them. Thus, we cannot insert any domain specific knowledge to improve the agent, but rather it has to come up with efficient ways to search within the state spaces of the specific problems on its own.

This kind of behavior can be seen as much more intelligent than classical approaches. So far, the best game players such as DEEP BLUE (Campbell et al., 2002) for CHESS or CHINOOK (Schaeffer et al., 1992) for AMERICAN CHECKERS were able to dominate the best human players in their respective games, but cannot play any other games, as they simply do not know the rules. Also, they were mainly able to beat the humans due to good heuristic functions and intelligent search mechanisms. But here, the intelligence does not come from the program but rather from the programmer that inserted all the expert knowledge, e. g., the relative importance of the various types of figures in the games, which moves to prefer in general, how to evaluate a certain situation in the game an so on. In an automated action planner or a general game player the intelligence inserted is much less prominent, as no such heuristic functions are general enough to cover the entire set of possibilities. Rather, the agents must come up with such functions themselves and thus act in a much more problem independent manner, which we can perceive as more intelligent behavior.

In the remainder of this introductory chapter we will briefly introduce state space search (Section 1.2), BDDs (Section 1.3), action planning (Section 1.4), and general game playing (Section 1.5), and present a small example which will be used throughout the thesis (Section 1.6). Furthermore, the following sections give a more detailed overview over the three parts of this thesis. Part I is concerned with BDDs, Part II with action planning and Part III with general game playing. At the end of this thesis, we draw some conclusions and point out possibilities for future research in these areas in Chapter 12.

1.2 State Space Search

In many areas of artificial intelligence we are concerned with state space search (Russell and Norvig, 2010; Edelkamp and Schrödl, 2012). This means that we need to search for certain properties within a set of states with transitions between them. Examples for this are finding shortest paths from one state to another, reaching some predefined goal state given a starting state, finding the optimal outcome for each state, if some of these states are terminal and an outcome is defined for those. An outcome of a non-terminal state can be the best (according to some metric) outcome reachable if we are not confronted with some opponent or the outcome that will be reached if all agents act optimally (according to some optimality criterion).

In this work, we are confronted with logic based input, which defines certain binary atoms or fluents that specify a state.

Definition 1.1 (Fluent)**.** *A* fluent *is a binary atom that can change over time.*

Definition 1.2 (State)**.** *Given a set of fluents $\mathcal{F}$ a* state *is the set of all fluents that are currently true. Thus, all fluents not in this set are false.*

There are two representations of state spaces, explicit and implicit.

Definition 1.3 (Explicit State Space)**.** *An* explicit state space *corresponds to a given set of states $\mathcal{S}$ and a set of transitions $\mathcal{TR} : \mathcal{S} \mapsto \mathcal{S}$ between these states. Applying the transition relation to a state results in a successor of that state. Additionally we typically need an initial state $\mathcal{I} \in \mathcal{S}$, which determines where the search starts, and some terminal states $\mathcal{T} \subseteq \mathcal{S}$, which typically do not have any successors.*

Such a set of transitions is often given in form of an adjacency list or adjacency matrix. While the former is linear in the number of transitions (which themselves can be quadratic in the number of states), the latter has a fixed size of $|\mathcal{S}|^2$.

In this thesis the state spaces are not provided explicitly beforehand, but only implicitly, so that they must be calculated at runtime.

Definition 1.4 (Implicit State Space)**.** *An* implicit state space *is given by a tuple $\mathcal{P} = \langle \mathcal{F}, \mathcal{O}, \mathcal{I}, \mathcal{T} \rangle$ with $\mathcal{F}$ being the set of fluents constructing states, $\mathcal{O}$ a set operators transforming states to successors, $\mathcal{I}$ an initial state, and $\mathcal{T}$ a set of terminal states.*

Given such an implicit state space declaration we can calculate an explicit one in the following way. The initial state $\mathcal{I}$ and the terminal states $\mathcal{T}$ are already given. Starting at the initial state $\mathcal{I}$ we can determine the set of all reachable states $\mathcal{S}$ by repeatedly applying all possible operators to all states found so far until a fix point is reached. Applying operators to a state to determine its successors is also called an *expansion* of the state. The transitions then correspond to the applied operators and can be stored as well.

In many cases we do not need all reachable states but rather some smaller subset of them. Thus, in this case we can already safe some memory because we do not have to represent the entire state space in memory. Also, the set of operators is typically a lot smaller than the full set of transitions, because each operator of an implicit state space declaration can affect several states. An example for this might be the $n^2 - 1$-PUZZLE, where we have only four operators—i. e., moving the blank up, down, left, or right—, while in an explicit declaration the number of transitions is between two and four times the number of states, because in each state two up to four transitions are applicable (two in states where the blank is in a corner, three where it is on the border but not in a corner, four where it is not on the border of the board).

A question that comes to mind is which parts of the state space are necessary, but the answer to this depends on the actual problem at hand, so that there is no general answer.

There is a close relation between explicit state spaces and graphs (see, e. g., Edelkamp and Schrödl, 2012), so that we can actually transform a state space search problem into a graph search problem and use algorithms that are designed for that.

Definition 1.5 (Graph)**.** *A* graph *$G = (V, E)$ consists of a set of nodes or vertices V and a set of edges E connecting certain nodes.*

In case of an undirected *graph the edges are not oriented, but rather an edge connecting nodes u and v is given by a set $\{u, v\}$.*

In case of a directed *graph the edges are oriented and thus an edge from node u to node v is given by an ordered pair (u, v). An edge in the opposite direction must not necessarily exist.*

A rooted *graph is a graph with a special node v_0, called the root.*

Corollary 1.6 (Equivalence of explicit state spaces and rooted directed graphs)**.** *There is a one-to-one mapping between explicit state spaces and rooted directed graphs with a set of distinguished terminal nodes.*

Proof Idea. Every node of a graph corresponds to a state in an explicit state space. The root of the graph corresponds to the initial state, the terminal nodes to the terminal states. An edge $(u, v) \in E$ corresponds to a transition from the state represented by node u to the state represented by node v. □

In implicit state space search we might say that we generate the graph at runtime and extend it whenever we expand some state.

1.3 Binary Decision Diagrams and Symbolic Search

While explicit search handles single states, in case of symbolic search (see, e. g., Burch et al., 1992; McMillan, 1993) we are confronted with sets of states, which are all expanded in a single step. For this we use binary decision diagrams (BDDs) (Bryant, 1986), a datastructure used to represent Boolean formulas. With our definition of states as sets of fluents that are currently true, we can represent a state as a Boolean formula as well. Each fluent is represented by one binary variable, and a state is the conjunction of all the variables representing the fluents holding in this state with the negation of those variables representing the fluents not holding in it, i. e., each state can be represented by a *minterm*. A set of states can thus be modeled as a

disjunction of the minterms representing the states. We will give a more detailed introduction to BDDs in Chapter 2.

Set-based search calculates all the successors of a set of states in a single step. This is called an *image*. Because of this behavior it is easy to perform breadth-first search (BFS) using symbolic search. In case of BFS we start with the initial state, expand it, continue with the first successor and expand it, continue with the second successor of the initial state and expand it and so on, until all immediate successors are expanded. Then we continue with the first successor of the first successor of the initial state and so on. In other words, we expand the search space in *level-order*. Using BDDs, we start with the initial state and call the image. The resulting states are all those of the first BFS level. For the BDD representing that level we again call the image and receive the second BFS level and so on. Further details on this will be given in Chapter 3.

For BDDs as we use them today there are two reduction rules. Due to these rules they can efficiently represent large sets of states without using too much memory; in good cases they can save an exponential amount of memory compared to an explicit representation. It is hard to predict beforehand whether the BDDs representing certain sets of states are exponentially smaller or not. For some domains it is already known that they safe exponential amounts of memory (e. g., symmetric functions (see, e. g., Shi et al., 2003)), while for others they are still of exponential size (e. g., for representing the set of all permutations of a set of variables (Hung, 1997)). We provide the results for some domains from planning and general game playing we have analyzed in more detail in Chapter 4, which is based on the following publications.

- Stefan Edelkamp and Peter Kissmann. *Limits and Possibilities of BDDs in State Space Search*. In 23rd AAAI Conference on Artificial Intelligence (AAAI) by Fox and Gomes (eds.). Pages 1452–1453, AAAI Press, 2008.
- Stefan Edelkamp and Peter Kissmann. *On the Complexity of BDDs for State Space Search: A Case Study in Connect Four*. In 25th AAAI Conference on Artificial Intelligence (AAAI) by Burgard and Roth (eds.). Pages 18–23, AAAI Press, 2011.

Especially will we determine a polynomial upper bound for the BDD sizes to represent all reachable states in GRIPPER, and the problem of CONNECT FOUR is two-fold. On the one hand we can only calculate the set of reachable states, continuing after reaching a terminal state. In that case we will prove that we can find a variable ordering for which the BDDs remain polynomial. On the other hand we have the termination criterion, for which we will prove that even a drastically reduced version of it requires the BDD to have exponential size.

1.4 Action Planning

One of the domains we investigate in this thesis is automated action planning. It is interesting because we as a programmer do not know what kind of problem the agent, i. e., the planner, will have to handle, so that we must implement a general approach that can work for any problem that can be described.

The description languages used today are quite strong, so that a lot of problems can be described in a concise way. While in the beginning especially simple problems such as the BLOCKSWORLD problem, where several blocks have to be unstacked or stacked to achieve one or more stacks of pre-defined order, have been extensively studied, it is actually possible to model some scenarios closer to real-world problems, such as LOGISTICS problems, where the planner is supposed to find a way to bring some packages from their starting locations to their destinations with a number of trucks and airplanes, or some smart greenhouses (the SCANALYZER domain (Helmert and Lasinger, 2010)), where plants are to be moved around conveyors, scanned, and finally returned to their starting positions. Of course, often a lot of abstraction has to be done in order to come up with a problem that can still be handled by a general approach. A rather general introduction to action planning is provided in Chapter 5, where we also introduce the planning domain definition language PDDL (McDermott, 1998), which is used to model planning problems.

There are several sub-domains in the general action planning domain, such as classical planning, non-deterministic planning, metric planning, or net-benefit planning. We will also briefly introduce these concepts in Chapter 5. In this thesis we are concerned with only two of these sub-domains, namely classical and net-benefit planning

In Chapter 6 we investigate classical planning. There we are given an initial state, some actions to calculate successor states, a set of goal states, and possibly action costs. The goal for the planner is to find a plan, i. e., a sequence of actions, that transforms the initial state into one of the goal states, and that is supposed to be minimal (in the number of actions, or in the sum of the action costs, if we are not only confronted with unit costs). Our contributions are also proposed in that chapter and are based on the following publications.

- Stefan Edelkamp and Peter Kissmann. *Partial Symbolic Pattern Databases for Optimal Sequential Planning*. In 31st Annual German Conference on Artificial Intelligence (KI) by Dengel, Berns, Breuel, Bomarius, and Berghofer (eds.). Pages 193–200 of volume 5243 of Lecture Notes in Computer Science (LNCS), Springer, 2008.
- Stefan Edelkamp and Peter Kissmann. *Optimal Planning with Action Costs and Preferences*. In 21st International Joint Conference on Artificial Intelligence (IJCAI) by Boutilier (ed.). Pages 1690–1695, 2009.
- Peter Kissmann and Stefan Edelkamp. *Improving Cost-Optimal Domain-Independent Symbolic Planning*. In 25th AAAI Conference on Artificial Intelligence (AAAI) by Burgard and Roth (eds.). Pages 992–997, AAAI Press, 2011.

The main contributions are partial pattern databases in the symbolic setting, while they before were only defined for explicit search, and the implementation of a classical planner. For the planner we also propose a number of improvements that greatly enhance its power to find more solutions.

In Chapter 7 we are concerned with net-benefit planning. In this sub-domain we are given an initial state, actions to calculate successor states, a set of goal states one of which must be reached by a plan and an additional set of preferences, which correspond to properties of the goal states that are to be preferred. The actions can have certain costs, while satisfying properties gives us certain rewards. The goal of a planner is to find a plan that reaches a goal and achieves the maximal net-benefit, which is the total reward for satisfying the desired properties minus the total cost of the actions needed to satisfy them. The approach we propose in that chapter is based on the following publication.

- Stefan Edelkamp and Peter Kissmann. *Optimal Planning with Action Costs and Preferences*. In 21st International Joint Conference on Artificial Intelligence (IJCAI) by Boutilier (ed.). Pages 1690–1695, 2009.

The algorithm is based on a symbolic variant of branch-and-bound search and allows us to find optimal plans in this setting. We have implemented this algorithm and included it into a full-fledged net-benefit planner.

1.5 General Game Playing

General game playing (GGP) goes in a similar direction as automated action planning, but it can be seen to be even more general. In general game playing the agent, i. e., the player, has to handle any game that can be described by the used description language without intervention of a human, and also without the programmer knowing what game will be played. Instead it has to come up with a good strategy on its own.

Nowadays the most used description mechanism is the game description language GDL (Love et al., 2006). In its original form it allows the description of simultaneous-move multi-player games[2] that are discrete, deterministic, finite, and where all players have full information of the current state. A more recent extension, GDL-II (for GDL for games with incomplete information) (Thielscher, 2010) lifts the two most constricting conditions, i. e., it allows to model games that can have some element of chance, as it is present in most dice games, and also provides a concept to conceal parts of the states from the players in order to model incomplete information games, as it is needed for most card games.

In this work we are only concerned with the concept of deterministic games with full information for all players, as even in this domain research still seems to be quite at the beginning. We will start by describing

[2]Multi-player here means games for one or more players.

the idea of general game playing in more detail in Chapter 8. There we will also introduce all the elements of GDL.

One problem of GDL, similar to PDDL, is that it comes with a lot of variables, while for BDDs we are dependent on grounded input. In the beginning we modeled the games in a slightly extended PDDL derivative by translating the GDL input by hand. We did this in order to use existing grounders from the planning domain, as so far there were none present in general game playing. Later we decided to implement our own grounding utility, or instantiator, so that we can use our symbolic algorithms in a fully automated general game player as well. Two approaches we implemented to instantiate general games are described in Chapter 9, which is based on the following publications.

- Peter Kissmann and Stefan Edelkamp. *Instantiating General Games*. In 1st IJCAI-Workshop on General Intelligence in Game-Playing Agents (GIGA). Pages 43–50, 2009.
- Peter Kissmann and Stefan Edelkamp. *Instantiating General Games using Prolog or Dependency Graphs*. In 33rd Annual German Conference on Artificial Intelligence (KI) by Dillmann, Beyerer, Schultz, and Hanebeck (eds.). Pages 255–262 in volume 6359 of Lecture Notes in Computer Science (LNCS), Springer, 2010.

For several games optimal solutions are already known. In this context we distinguish three different types of solutions (Allis, 1994), from ultra-weak, where we only know whether the starting player wins the game but no actual strategy is known, over weak, where we know what move to choose in any state that can be reached, no matter what the opponent does, up to strong, where we know the optimal move to take for every reachable state, i. e., even for states that are only reachable if we chose a sub-optimal move ourselves.

We implemented a symbolic solver that can find strong solutions for single-player games as well as non-simultaneous two-player games. After a discussion of specialized approaches that yield solutions for some games we will present our general symbolic approach in Chapter 10, which is based on the following publications.

- Stefan Edelkamp and Peter Kissmann. *Symbolic Explorations for General Game Playing in PDDL.* In 1st ICAPS-Workshop on Planning in Games. 2007.
- Stefan Edelkamp and Peter Kissmann. *Symbolic Classification of General Two-Player Games*. In 31st Annual German Conference on Artificial Intelligence (KI) by Dengel, Berns, Breuesl, Bomarius, and Berghofer (eds.). Pages 185–192 of volume 5243 of Lecture Notes in Computer Science (LNCS), Springer, 2008.
- Peter Kissmann and Stefan Edelkamp. *Layer-Abstraction for Symbolically Solving General Two-Player Games*. In 3rd International Symposium on Combinatorial Search (SoCS). Pages 63–70, 2010.

Normally the overall goal in general game playing is to implement a player that can play general games, and play them as good as possible. In practice it must decide which move to take in a very short time. Symbolic search with BDDs is often quite time-consuming, so that it is not too well suited for this task. Instead we decided to implement a hybrid player. One part is a rather traditional player that is based on explicit search using a simulation-based approach called *upper confidence bounds applied to trees* (UCT) (Kocsis and Szepesvári, 2006), which has been the most successful in recent years. We actually implemented two of these players, one making use of Prolog in order to handle the original input, and another one capable to handle the grounded input we generate for the BDD based solver, which is a lot faster than the Prolog based player. The other part of the hybrid player is our BDD based solver, so that games that are solved can be played optimally. Our main interest lies with symbolic search, which means that the explicit search players are not yet high-performance, even though they can play reasonably well in several games. The UCT approach along with several extensions and ways for parallelizing it as well as our player are presented in Chapter 11, which is based on the following publications.

- Stefan Edelkamp, Peter Kissmann, Damian Sulewski, and Hartmut Messerschmidt. *Finding the Needle in the Haystack with Heuristically Guided Swarm Tree Search*. In Multikonferenz Wirtschaftsinformatik – 24th Workshop on Planung / Scheduling und Konfigurieren / Entwerfen (PuK) by Schumann, Kolbe, Breitner and Frerichs (eds.). Pages 2295–2308, Universitätsverlag Göttingen, 2010.

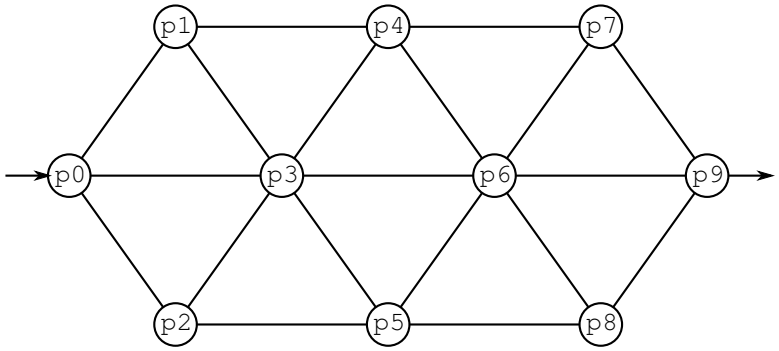


Figure 1.2: The running example.

- Peter Kissmann and Stefan Edelkamp. *Gamer, a General Game Playing Agent*. In volume 25 of KI—Zeitschrift Künstliche Intelligenz (Special Issue on General Game Playing). Pages 49–52, 2011.

1.6 Running Example

Throughout this work we will use a simple example to explain things. This example is slightly inspired by the opening words of Stephen King's *The Dark Tower* series, i. e., "The man in black fled across the desert, and the gunslinger followed". We model the desert as a graph consisting of ten locations `p0` to `p9`, which are connected by undirected edges, so that the protagonists can move from one location to a connected one (cf. Figure 1.2). Location `p0` serves as the entrance to the desert, while it can be left only from location `p9`.

We take this graph as the basis whenever we refer to our running example, though it will be refined in several places. For example, we must specify locations for the man in black and the gunslinger to start. Also, we must specify what the goal of the planner respectively the players might be. If we allow actions to have different costs we can assume that some paths are harder to travel on than others, maybe due to less shade or more and steeper dunes, which we can model by using different weights for the edges. Additionally we might assume that it is necessary to find some water to drink in order not to die of dehydration. Furthermore it might be that the two protagonists move with different speeds. Further details will be given when they are needed in the corresponding places in the remainder of this thesis.

Part I

Binary Decision Diagrams

Chapter 2

Introduction to Binary Decision Diagrams

Logic is the beginning, not the end, of Wisdom.

Captain Spock in *Star Trek VI*
(The Undiscovered Country)

Binary decision diagrams (BDDs) are a memory-efficient data structure used to represent Boolean functions as well as to perform set-based search.

Definition 2.1 (Binary Decision Diagram)**.** *A* binary decision diagram *(BDD) is a directed acyclic graph with one* root *node and two terminal nodes, i. e., nodes with no outgoing edges, the* 0 sink *and the* 1 sink*. Each non-terminal node corresponds to a binary variable, so that it has two outgoing edges, the* low edge *for the variable being false (or* 0*) and the* high edge *for it being true (or* 1*).*

Definition 2.2 (Satisfying Assignment)**.** *A path in a BDD starting at the root and ending in the* 1 *sink corresponds to a* satisfying assignment *of the variables. The variables touched by the path are assigned the value according to the outgoing edge, all others can take any value.*

Given a BDD, it is fairly easy to check whether an assignment of the variables is satisfying or not. Starting at the root, it is only necessary to follow the outgoing edge corresponding to the value of the variable in the assignment. If finally the 1 sink is reached, the assignment is satisfying; if the 0 sink is reached it is not.

BDDs are often used to store sets of states. According to our definition of a state (cf. Definition 1.2) a state is specified by those binary fluents (or *variables*, as we typically call them in the context of BDDs) that are true in that state, while all other variables must be false. Thus, a state can be seen as a simple conjunction. Let $\mathcal{V}$ be the set of all variables and let $\mathcal{V}^1 \subseteq \mathcal{V}$ and $\mathcal{V}^0 \subseteq \mathcal{V}$ be the set of variables being true or false, respectively, with $\mathcal{V}^1 \cap \mathcal{V}^0 = \emptyset$ and $\mathcal{V}^1 \cup \mathcal{V}^0 = \mathcal{V}$. Then a state s can be described by the Boolean function

$$bf\,(s) = \bigwedge_{v^1 \in \mathcal{V}^1} v^1 \wedge \bigwedge_{v^0 \in \mathcal{V}^0} \neg v^0.$$

A set of states $\mathcal{S}$ can then be described the disjunction of all the Boolean functions describing the states within the set:

$$bf\,(\mathcal{S}) = \bigvee_{s \in \mathcal{S}} bf\,(s)\,.$$

This being a Boolean function it is of course possible to represent the set of states by a BDD. Any satisfying assignment then corresponds to a state that is part of the set represented by the BDD.

The BDD representing the empty set consists of only the 0 sink and is denoted by $\bot$, while the one that consists only of the 1 sink and is denoted by $\top$ represents the set of all states describable by the set of variables, i. e., for n binary variables it corresponds to all possible 2^n states.

In the following sections of this chapter we will give a brief overview of the history of BDDs, followed by a short introduction to the most important efficient algorithms to handle and create BDDs and some notes on the importance of the variable ordering that was introduced to reduce the BDDs' sizes.

Chapter 3 introduces the use of BDDs in state space search, which is then called *Symbolic Search*. Finally, in Chapter 4 we present theoretical results on BDD sizes in some planning and game playing domains.

2.1 The History of BDDs

BDDs go back more than 50 years, when Lee (1959) proposed the use of, what he then called, *binary-decision programs*. They already consisted of some of the basic elements, i. e., a set of binary variables along with the 0 and 1 sink to model Boolean functions.

Akers (1978) came up with the name we still use today and already presented some approaches to automatically generate BDDs to represent Boolean functions using Shannon expansions (Shannon, 1938) or truth tables. Nevertheless, he did not limit the generation of the BDDs in any way, so that variables might appear more than once on certain paths from the root node to one of the sinks. Apart from the name and the similarity in the graphical representation he already saw the possibility to reduce the BDDs in some cases but did not provide an algorithmic description of this process.

Finally, Bryant (1986) proposed the use of a predefined ordering of the binary variables.

Definition 2.3 (Variable Ordering)**.** *Given a set of binary variables $\mathcal{V}$, the* variable ordering π *is a permutation of the variables $\pi : \mathcal{V} \mapsto \{0, \ldots, |\mathcal{V}| - 1\}$.*

Due to this ordering a BDD can be organized in layers, each of which contains nodes representing the same variable. These layers can be addressed by a unique index, which is the position of the corresponding variable within the ordering. Thus, the root node resides in the layer with smallest index 0. All edges are directed from nodes of one layer to those of other layers with greater index, so that any path starting at the root visits the variables in the same predefined order. With the fixed ordering it is thus possible to automatize the process of generating a BDD given any Boolean function. Such a BDD is also called an *Ordered Binary Decision Diagram* (OBDD).

Definition 2.4 (Ordered Binary Decision Diagram)**.** *An* ordered binary decision diagram *(OBDD) is a BDD where the variables represented by the nodes on any path from the root to one of the sinks obey the same fixed variable ordering.*

Due to the fixed variable ordering an OBDD can be reduced by following two simple rules, which are depicted in Figure 2.1.

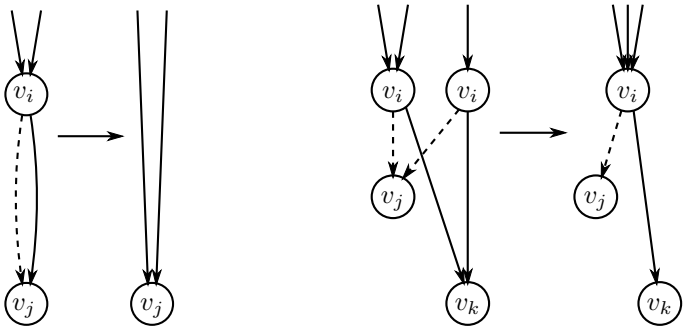
Definition 2.5 (Reduction Rules)**.** *For OBDDs two* reduction rules *are known.*

1. *If the two successors of a node u reached when following the low and the high edge actually are the same node v then u can be removed from the BDD and all ingoing edges are redirected to v. This is depicted in Figure 2.1a.*

2. *If two nodes u_1 and u_2 represent the same variable and the node reached by the low edge of u_1 is the same as the one reached by the low edge of u_2 and the node reached by the high edge of u_1 is the same as the one reached by the high edge of u_2 then u_1 and u_2 can be merged by removing u_2 and redirecting all ingoing edges to u_1. This is depicted in Figure 2.1b.*

The resulting OBDD is also called a *Reduced Ordered Binary Decision Diagram* (ROBDD).

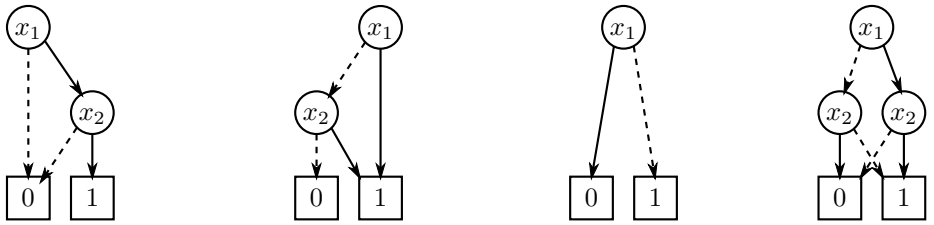
Definition 2.6 (Reduced Ordered Binary Decision Diagram)**.** *A* reduced ordered binary decision diagram *(ROBDD) is an OBDD where the reduction rules are repeatedly applied until none of them can be applied any more.*



(a) Rule 1. Removal of nodes with equal successors.

(b) Rule 2. Merging of nodes with same low and high successors.

Figure 2.1: The two rules for minimizing BDDs for a given variable ordering $\pi = \{v_1, \ldots, v_n\}$ ($i < j$ and $i < k$). Solid lines represent high edges, dashed ones low edges.



(a) Conjunction ($v_1 \wedge v_2$). (b) Disjunction ($v_1 \vee v_2$). (c) Negation ($\neg v_1$). (d) Equivalence ($v_1 \leftrightarrow v_2$).

Figure 2.2: BDDs representing basic Boolean functions. Solid lines represent high edges, dashed ones low edges.

An ROBDD is a canonical representation of the given Boolean function, i. e., two equivalent formulas are represented by the same ROBDD (apart from isomorphisms). Due to the reduction several of the internal nodes can be saved, so that ROBDDs are quite memory-efficient. Throughout this work we are only concerned with ROBDDs, so that whenever we write about BDDs we actually refer to ROBDDs.

For the set of variables $\mathcal{V} = \{v_1, v_2\}$ and the ordering (v_1, v_2) a number of simple ROBDDs representing some basic Boolean functions are depicted in Figure 2.2.

Another extension Bryant (1986) proposed is to use one BDD with several root nodes to represent a number of different Boolean functions instead of a single BDD for each one. This brings the advantage of sharing the same nodes, i. e., for representing formulas whose BDD representation is largely the same we do not need to store the same BDD nodes more than once in main memory, so that these are even more memory-efficient. Such BDDs are called *Shared BDDs* (Minato et al., 1990).

2.2 Efficient Algorithms for BDDs

Along with the idea of using a fixed variable ordering resulting in OBDDs, Bryant (1986) also proposed several algorithms to automatically generate BDDs and to work with them efficiently.

2.2.1 Reduce

One of the most important algorithms is the *Reduce* algorithm, which is an implementation of the reduction rules to create ROBDDs.

Bryant (1986) proposed a version that has a time complexity of $\mathcal{O}(|V| \log |V|)$ with $|V|$ being the number of nodes within the original BDD. It starts at the two sinks and works in a layer-wise manner backwards towards the root. In each layer it checks the applicability of the two rules and deletes those

nodes for which one can be applied. Once the root node is reached the BDD is fully reduced and thus of minimal size.

Later, Sieling and Wegener (1993) have shown that using two phases of bucket sort the time complexity is reduced to linear time $\mathcal{O}(|V|)$.

2.2.2 Apply

Another important algorithm is the *Apply* algorithm. In contrast to other approaches, there is no need for different algorithms for each Boolean operator. Instead, Apply generates a new BDD given two existing ones and the operator for the specified composition. The idea is as follows. The algorithm starts at the root nodes of each BDD. For the current nodes it adds a new node into the resulting BDD. If the current nodes of both BDDs are in the same layer, both low successors as well as both high successors are combined using the Apply algorithm. Otherwise it combines the low and high successor of the node from the layer with smaller index with the other node. The values of the sinks are determined according to the rules for the specified Boolean operator. Finally, the Reduce algorithm is called to make sure that the resulting BDD is reduced as well.

This algorithm would require time exponential in the number of variables, so Bryant (1986) also proposed two extensions. The first one makes use of a table to store the result of applying two nodes. This way, the same operation does not have to be performed more than once. The second one improves the algorithm by adding a sink already when one of the two current nodes is a sink with a controlling value (e. g., 1 for the disjunction or 0 for the conjunction). With these extensions the algorithm generates the combination of two BDDs with $|V_1|$ and $|V_2|$ BDD nodes in time $\mathcal{O}(|V_1||V_2|\log(|V_1|+|V_2|))$.

2.2.3 Restrict, Compose, and Satisfy

Further algorithms Bryant (1986) proposed are *Restrict*, *Compose*, *Satisfy-one*, *Satisfy-all*, and *Satisfy-count*. In Restrict, a variable of the represented formula is assigned a fixed Boolean value and the resulting BDD is returned. This algorithm can be performed in time $\mathcal{O}(|V|\log|V|)$, with $|V|$ being the number of nodes in the original BDD. In Compose, a variable is replaced not just by a constant but by a full Boolean function. This can be done in time $\mathcal{O}\left(|V_1|^2|V_2|\log(|V_1|+|V_2|)\right)$ for two BDDs of sizes $|V_1|$ and $|V_2|$.

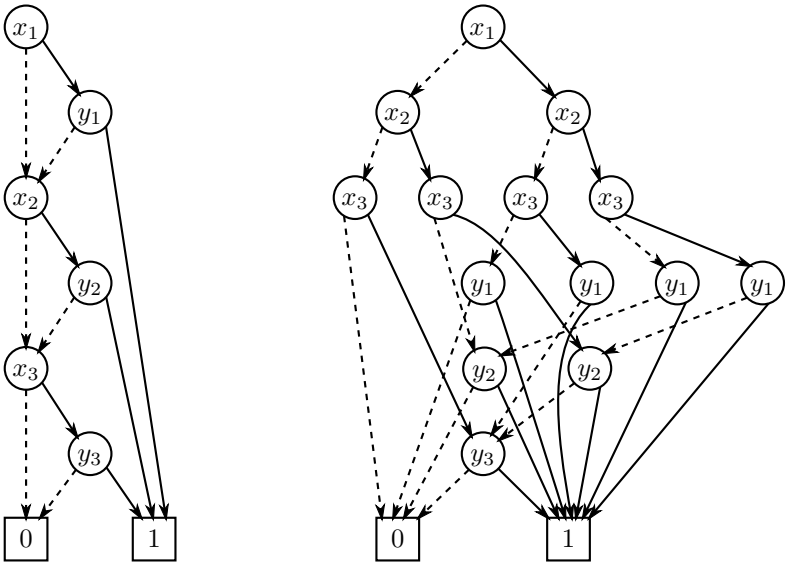
While all of the other algorithms create or reorganize the BDDs, the Satisfy algorithms are concerned with the sets of satisfying assignments. The Satisfy-one algorithm returns exactly one of the satisfying assignments. Given an initial array of Boolean values, it starts at the root of the BDD and operates towards the sinks. For each encountered node it checks if the successor according to the provided Boolean value is the 0 sink. In that case it flips the value in the array and continues with the other successor. Otherwise it does not change the value in the array and continues with the indicated successor. As soon as the 1 sink is reached the array holds a satisfying assignment. Thus, at most $\mathcal{O}(n)$ nodes must be evaluated, with n being the number of variables.

Satisfy-all returns the set of all satisfying assignments. Here, the idea is to start at the root node and to follow all possible paths toward the 1 sink. If a node for the current index is present on the current path, both successors are investigated next. If for some index i no such node is present, i. e., the formula is independent of v_i for the current assignment, both cases ($v_i = 1$ and $v_i = 0$) are investigated and returned in the end. Thus, the algorithm can be performed in time $\mathcal{O}(n|S_f|)$ with n being the number of variables and $|S_f|$ the number of satisfying assignments for the represented Boolean function f.

The Satisfy-count returns the number of satisfying assignments. For this, the algorithm assigns a value α_v to each node v. For the terminal nodes, the value is the corresponding value (0 / 1). For a non-terminal node v, the value is set to

$$\alpha_v = \alpha_{low(v)}2^{index(low(v))-index(v)-1} + \alpha_{high(v)}2^{index(high(v))-index(v)-1}$$

with $low(v)$ and $high(v)$ being the successor of v along the low and high edge, respectively, and $index(v)$ the ordering's index of the variable that the node v represents. This way, the complete BDD is traversed only once in a depth-first manner and the resulting runtime is at most $\mathcal{O}(|V|)$ with $|V|$ being the number of nodes in the BDD.



(a) Good variable ordering. (b) Bad variable ordering.

Figure 2.3: Two possible BDDs representing DQF_3. Solid lines represent high edges, dashed ones low edges.

2.3 The Variable Ordering

For many Boolean functions the corresponding BDD can be of polynomial or even linear size, given a good variable ordering π, although the number of satisfying assignments (i. e., the number of states) might be exponential in the number of variables. One such case is the disjoint quadratic form (DQF).

Definition 2.7 (Disjoint Quadratic Form)**.** *For two sets of binary variables $x_1, \ldots x_n$ and $y_1, \ldots, y_n$ the* disjoint quadratic form *DQF_n is defined as*

$$DQF_n(x_1, \ldots, x_n, y_1, \ldots, y_n) := (x_1 \wedge y_1) \vee (x_2 \wedge y_2) \vee \cdots \vee (x_n \wedge y_n).$$

On the one hand, if the two sets of variables are ordered in an interleaved fashion the corresponding BDD is of linear size.

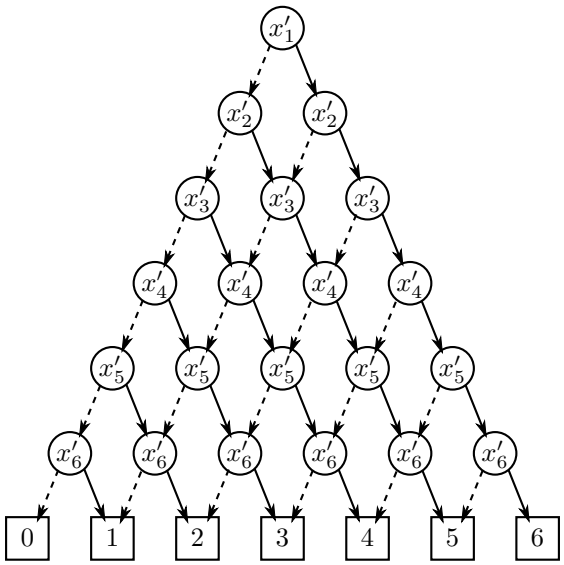
Proposition 2.8 (Linear size of a BDD for DQF)**.** *Given the variable ordering $\pi = (x_1, y_1, \ldots, x_n, y_n)$, the BDD representing the DQF_n contains $2(n+1)$ nodes.*

On the other hand, for the DQF we can also find a bad variable ordering, so that the size of the corresponding BDD is exponential in n.

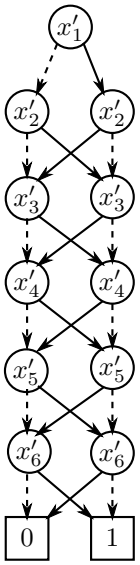
Proposition 2.9 (Exponential size of a BDD for DQF)**.** *The BDD representing the DQF_n contains 2^{n+1} nodes if the variable ordering $\pi = (x_1, \ldots, x_n, y_1, \ldots, y_n)$ is chosen.*

Figure 2.3 illustrates both cases for the DQF_3.

There are also some functions for which any BDD contains an exponential number of nodes, no matter what variable ordering is chosen. One example the integer multiplication. Given two inputs $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$, the binary encoding of two integers a and b with a_1 and b_1 being the least significant bits. Then there are $2n$ outputs, which correspond to the binary encoding of the product of a and b. These can be described by functions $mul_i(a_1, \ldots, a_n, b_1, \ldots, b_n)$ $(1 \leq i \leq 2n)$. For a permutation π of $\{1, \ldots, 2n\}$, let $G(i, \pi)$ be a graph, which represents the function $mul_i\left(x_{\pi(1)}, \ldots, x_{\pi(2n)}\right)$ for inputs $x_1, \ldots, x_{2n}$. Then the following Theorem can be proved (Bryant, 1986).



(a) Not fully reduced BDD. The sinks represent the numbers of high edges on the path from the root. In a real BDD the sinks would be replaced by the 0 or 1 sink, dependent on the actual function.



(b) Reduced BDD for the case that the function represents all cases with an odd number of 1s in the input.

Figure 2.4: BDDs for a symmetric function with six arguments ($f(x_1, \ldots, x_6)$), with $(x'_1, \ldots, x'_6)$ being an arbitrary permutation of $(x_1, \ldots, x_6)$. Solid lines represent high edges, dashed ones low edges.

Theorem 2.10 (Exponential size of any BDD for integer multiplication)**.** *For any variable ordering π there exists an i, $1 \leq i \leq 2n$, so that $G(i, \pi)$ contains at least $2^{n/8}$ nodes.*

Finally, for some functions any variable ordering results in a polynomial sized BDD. Symmetric functions are an example for this behavior.

Definition 2.11 (Symmetric Function)**.** *A function $f : \{0,1\}^n \mapsto \{0,1\}$ is called* symmetric *if the value remains the same, independent of the order of the arguments, i. e., if $f(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n) = f(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$ for all $i, j \in \{1, \ldots, n\}$.*

Thus, for a symmetric function the truth value depends only on the number of 1s in the input. Any BDD representing a symmetric function is of size $\mathcal{O}(n^2)$, no matter what variable ordering is used (see, e. g., Shi, Fey, and Drechsler, 2003).

Theorem 2.12 (Quadratic size of any BDD for symmetric functions)**.** *For any symmetric function $f : \{0,1\}^n \mapsto \{0,1\}$ a BDD contains at most $\mathcal{O}(n^2)$ nodes, no matter what variable ordering is used.*

Proof. As the value of a symmetric function does not depend on the order of the input variables it is clear that any BDD, no matter what ordering is used, is of the same size. It is possible to generate a (not fully reduced) BDD as shown in Figure 2.4a for any such function, which contains $\mathcal{O}(n^2)$ BDD nodes and is thus quadratic in the number of variables. All nodes on a diagonal going from top-right to bottom-left can be reached by the same number of high edges. Due to the fact that swapping two variables does not matter, the low successor of the high successor of some node always equals the high successor of the low successor of the same node. Thus, the BDD contains exactly $n(n+1)/2 + 2$ nodes. Reducing such a BDD cannot increase the size, so that a reduced BDD cannot contain more than $\mathcal{O}(n^2)$ nodes as well (cf. Figure 2.4b for an example). □

For many functions there are good variable orderings and finding these is often crucial, as due to the Apply algorithm operations such as the conjunction or disjunction of two BDDs take time in the order

of the size of the two BDDs. Yet, finding an ordering that minimizes the number of BDD nodes is Co-NP-complete (Bryant, 1986). Furthermore, the decision whether a variable ordering can be found so that the resulting BDD contains at most s nodes, with s being specified beforehand, is NP-complete (Bollig and Wegener, 1996). Finally, Sieling (2002) proved that there is no polynomial time algorithm that can calculate a variable ordering so that the resulting BDD contains at most c times as many nodes as the one with optimal variable ordering for any constant $c > 1$, so that we cannot expect to find a good variable ordering using reasonable resources. This shows that using heuristics to calculate a variable ordering (see, e. g., Fujita et al., 1988; Malik et al., 1988; Butler et al., 1991) is a sensible approach.

Chapter 3

Symbolic Search

Where do I go?

Never had a very real dream before.
Now I got a vision of an open door.
Guiding me home, where I belong,
Dreamland I have come.

Oh where do I go?

Avantasia, *The Tower*
from the Album *The Metal Opera*

State space search is an important topic in many areas of artificial intelligence (Russell and Norvig, 2010; Edelkamp and Schrödl, 2012). No matter if we want to find a shortest path from one location to another, or some plan transforming a given initial state to another state satisfying a goal condition, or a strategy specifying good moves for a game, in all these (and many more) cases search is of high importance.

In explicit state search we need to come up with good data structures to represent the states efficiently or algorithms for fast transformation of states. In case of symbolic search (see, e. g., Burch et al., 1992; McMillan, 1993) with BDDs the data structure to use is already given. As to efficient algorithms, here we need to handle things in a different manner compared to explicit search. While explicit search is concerned with expansion of single states and the calculation of one successor after the other, in symbolic search we need to handle sets of states.

To perform symbolic search we need two sets of binary variables, both ranging over the entire set of variables needed to represent a state. One set, $\mathcal{V}$, is needed to store the current states, while the other set, $\mathcal{V}'$, stores the successor states. Because the variables true in a successor state are often closely related to those true in the current state we decided to store the two sets in an interleaved fashion (Burch et al., 1994). With these sets we can represent a transition relation, which connects the current states with their successors.

As we have mentioned in the introduction, in this work we are only concerned with implicit search spaces, i. e., given an initial state, we can calculate all successor states by applying all possible operators. This we can repeat for the successors, until in the end all states are calculated and the full state space resides in memory. Actually, in practice we are often also given some terminal states and try to reduce the state space to be stored as much as possible, e. g., by using some heuristic to determine whether a successor actually brings us closer to a terminal state. Nevertheless, in the following we will present the most basic algorithms, i. e., pure symbolic breadth-first search (BFS) and its bidirectional extension.

For a given set of operators $\mathcal{O}$, an operator $o \in \mathcal{O}$ consists of a precondition pre_o and some effects eff_o to specify the changes to a state satisfying the precondition, which enables us to calculate the successor state.

Definition 3.1 (Transition Relation)**.** *Given an operator $o = (pre_o, \mathit{eff}_o) \in \mathcal{O}$ the* transition relation $\mathcal{TR}_o$ *is defined as*

$$\mathcal{TR}_o(\mathcal{V}, \mathcal{V}') := pre_o(\mathcal{V}) \wedge \mathit{eff}_o(\mathcal{V}') \wedge frame_o(\mathcal{V}, \mathcal{V}')$$

with $frame_o$ being the frame *for operator o, which specifies what part of the current state does not change when applying o.*

Combining these transition relations results in a single (monolithic) transition relation $\mathcal{TR}$ by calculating the disjunction of the transition relations $\mathcal{TR}_o$ of all operators $o \in \mathcal{O}$.

Definition 3.2 (Monolithic Transition Relation)**.** *Given a set of operators $\mathcal{O}$ and a transition relation $\mathcal{TR}_o$ for each $o \in \mathcal{O}$, the* monolithic transition relation $\mathcal{TR}$ *is defined as*

$$\mathcal{TR}(\mathcal{V}, \mathcal{V}') := \bigvee_{o \in \mathcal{O}} \mathcal{TR}_o(\mathcal{V}, \mathcal{V}').$$

3.1 Forward Search

To calculate the successors of a set of states, the *image* operator (see, e. g., Burch et al., 1992) is used.

Definition 3.3 (Image Operator)**.** *Given a set of states $current$, specified in $\mathcal{V}$, all successor states are calculated by the* image operator

$$image(current) := (\exists \mathcal{V} \,.\, \mathcal{TR}(\mathcal{V}, \mathcal{V}') \wedge current(\mathcal{V}))[\mathcal{V}' \to \mathcal{V}].$$

In this formula $[\mathcal{V}' \to \mathcal{V}]$ denotes the replacement of all variables of $\mathcal{V}'$ by those of $\mathcal{V}$, so that the successor states are represented in $\mathcal{V}$ and we can continue the search.

This combination of conjunction and existential quantification is also called the *relational product*. Though we are able to find efficient variable orderings for many problems we cannot expect to be able to calculate exponential search spaces in polynomial time. This comes from the fact that the calculation of the relational product is in itself NP-complete, which was proved by McMillan (1993) by reduction from the 3-SAT problem.

Note that it is not essential to use a monolithic transition relation. For many problems the BDD for the monolithic transition relation is larger than the sum of the sizes of the transition relations for the operators. Thus, the image calculations take even longer. Also, the creation of the monolithic transition relation takes a long time and it consumes a large amount of memory. Instead, we can manage the transition relations of all operators separately, apply the relational product to one after the other and calculate the disjunction of these smaller sets of successor states afterward (Burch et al., 1991).

Definition 3.4 (Single-Operator Image Operator)**.** *Given a set of states $current$, specified in $\mathcal{V}$, the successor states resulting from the application of the operator $o \in \mathcal{O}$ is calculated by the* single-operator image operator

$$image_o(current) := (\exists \mathcal{V} \,.\, \mathcal{TR}_o(\mathcal{V}, \mathcal{V}') \wedge current(\mathcal{V}))[\mathcal{V}' \to \mathcal{V}].$$

Corollary 3.5 (Image operator with non-monolithic transition relation)**.** *The image operator can be rewritten to use the smaller transition relations $\mathcal{TR}_o$ for each operator $o \in \mathcal{O}$.*

Algorithm 3.1: Symbolic Breadth-First Search

Input: Implicit definition of a state space problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{O}, \mathcal{I} \rangle$.
Output: BDD $reach$ representing the set of all states reachable from $\mathcal{I}$.

```
1 reach ← I                                   // Start at initial state I.
2 newStates ← reach                           // newStates is used for stopping once all states have been generated.
3 while newStates ≠ ⊥ do                      // Repeat until no new states found in last iteration.
4 |  newStates ← image(reach) ∧ ¬reach        // Calculate new successor states.
5 |  reach ← reach ∨ newStates                // Add new states to set of all reachable states.
6 return reach                                // Return set of all reachable states.
```

Proof.

$$\begin{aligned}
image\,(current) &= (\exists \mathcal{V} \,.\, \mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \wedge current\,(\mathcal{V}))\,[\mathcal{V}' \to \mathcal{V}] \\
&\equiv \left(\exists \mathcal{V} \,.\, \left(\bigvee_{o \in \mathcal{O}} \mathcal{TR}_o\,(\mathcal{V}, \mathcal{V}')\right) \wedge current\,(\mathcal{V})\right)[\mathcal{V}' \to \mathcal{V}] \\
&\equiv \left(\exists \mathcal{V} \,.\, \bigvee_{o \in \mathcal{O}} (\mathcal{TR}_o\,(\mathcal{V}, \mathcal{V}') \wedge current\,(\mathcal{V}))\right)[\mathcal{V}' \to \mathcal{V}] \\
&\equiv \bigvee_{o \in \mathcal{O}} (\exists \mathcal{V} \,.\, \mathcal{TR}_o\,(\mathcal{V}, \mathcal{V}') \wedge current\,(\mathcal{V}))\,[\mathcal{V}' \to \mathcal{V}] \\
&\equiv \bigvee_{o \in \mathcal{O}} image_o\,(current)\,[\mathcal{V}' \to \mathcal{V}]
\end{aligned}$$

□

Using the image operator to implement a symbolic BFS is straight-forward (cf. Algorithm 3.1). All we need to do is to repeatedly apply the image operator to the set of states reachable from the initial state $\mathcal{I}$ found so far. The search ends when a fix-point is reached, i. e., when no new successor states can be found. In this version we store the set of all reachable states as one BDD $reach$, so that, due to the structure of a BDD, this does not contain any duplicates of states.

Another possibility for implementing symbolic BFS is to store each layer separately. This often brings the advantage that the BDDs remain smaller, but we must handle the duplicate elimination manually by calculating the conjunction of the new layer with the negation of each of the preceding layers. Not removing duplicates results in a non-terminating algorithm if some paths, i. e., sequences of states and operators, contain loops.

3.2 Backward Search

For search in backward direction we use the pre-image operator.

Definition 3.6 (Pre-Image Operator)**.** *The* pre-image operator *calculates the set of predecessor states given a set of states* $current$*, represented in* $\mathcal{V}'$*.*

$$pre\text{-}image\,(current) := (\exists \mathcal{V}' \,.\, \mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \wedge current\,(\mathcal{V}'))\,[\mathcal{V} \to \mathcal{V}']$$

Similar to the image operator we can define the pre-image operator for a single operator.

Definition 3.7 (Single-Operator Pre-Image Operator)**.** *Given a set of states* $current$*, specified in* $\mathcal{V}'$*, the predecessor states resulting from the application of the operator* $o \in \mathcal{O}$ *is calculated by the* single-operator pre-image operator

$$pre\text{-}image_o\,(current) := (\exists \mathcal{V}' \,.\, \mathcal{TR}_o\,(\mathcal{V}, \mathcal{V}') \wedge current\,(\mathcal{V}'))\,[\mathcal{V} \to \mathcal{V}']\,.$$

For symbolic backward search we need at least a set of operators, which are transformed to a transition relation, and some terminal states $\mathcal{T}$. It is implemented the same way as forward search, only that it starts at the terminal states and uses the pre-image operator instead of the image. Due to the use of BDDs starting at the set of terminal states is no problem, as we can store all in one BDD.

Finally, for some algorithms we need to calculate another set of predecessor states.

Definition 3.8 (Strong Pre-Image Operator)**.** *The* strong pre-image operator *calculates all the predecessors of a given set of states* $current$ *whose successors all reside in the set* $current$*. It is defined as*

$$strong\ pre\text{-}image\,(current) := (\forall \mathcal{V}'\ .\ \mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \rightarrow current\,(\mathcal{V}'))\,[\mathcal{V} \rightarrow \mathcal{V}']\,.$$

In most BDD packages the relational product is not implemented in the most obvious way—first performing the conjunction and then the quantification—but a more efficient implementation is used, where the existential quantification and the conjunctions are intertwined (Burch et al., 1994). Thus, it is important to know that the strong pre-image operator can be transformed to the normal pre-image operator, so that the efficient implementations of the relational product can be used.

Corollary 3.9 (Strong pre-image transformed to pre-image)**.** *The strong pre-image operator can be transformed to the pre-image operator, so that efficient implementations of the relational product can be used.*

Proof.

$$\begin{aligned}
strong\ pre\text{-}image\,(current) &= (\forall \mathcal{V}'\ .\ \mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \rightarrow current\,(\mathcal{V}'))\,[\mathcal{V} \rightarrow \mathcal{V}'] \\
&\equiv \neg\neg\,(\forall \mathcal{V}'\ .\ \neg\mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \vee current\,(\mathcal{V}'))\,[\mathcal{V} \rightarrow \mathcal{V}'] \\
&\equiv \neg\,(\exists \mathcal{V}'\ .\ \neg\,(\neg\mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \vee current\,(\mathcal{V}')))\,[\mathcal{V} \rightarrow \mathcal{V}'] \\
&\equiv \neg\,(\exists \mathcal{V}'\ .\ (\mathcal{TR}\,(\mathcal{V}, \mathcal{V}') \wedge \neg current\,(\mathcal{V}')))\,[\mathcal{V} \rightarrow \mathcal{V}'] \\
&\equiv \neg pre\text{-}image\,(\neg current)
\end{aligned}$$

□

It follows immediately that we can make use of the non-monolithic transition relations in the calculations of the strong pre-image as well.

3.3 Bidirectional Search

For state spaces specified by terms of operators $\mathcal{O}$, initial state $\mathcal{I}$ and terminal states $\mathcal{T}$ we are often interested in finding a shortest path from the initial state $\mathcal{I}$ to a terminal state $t \in \mathcal{T}$. For the unidirectional, i. e., forward or backward, BFS this means that the search can be stopped once a terminal state or the initial state, respectively, has been reached.

Given the image and pre-image operators we can also devise a symbolic bidirectional BFS. For this we perform one search in forward direction (using the image operator) starting at $\mathcal{I}$ and another in backward direction (using the pre-image operator) starting at $\mathcal{T}$. Once the two searches overlap, i. e., the conjunction of the BDDs representing the states reachable in forward and backward direction, respectively, is no longer empty ($\bot$) we can stop both and generate a shortest path.

Chapter 4

Limits and Possibilities of Binary Decision Diagrams in State Space Search

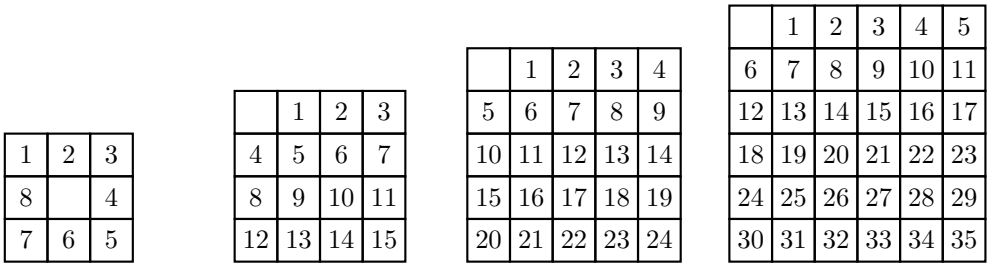
I see shadows of giant machines
cast upon the land
I see a world where kings nor queens
but chips are in command

Ayreon, *Computer Reign (Game Over)*
from the Album *The Final Experiment*

In this chapter we are concerned with finding (exponential) lower or (polynomial) upper bounds for the sizes of BDDs representing the reachable states for a number of benchmark domains from planning or (general) game playing. This is an important indicator whether the use of BDDs brings an advantage or not. Of course, if the reachable states of an exponential search space can be represented by a BDD of polynomial size, we are able to handle much larger search spaces than in case of explicit search. But if any BDD contains an exponential number of nodes the advantage or disadvantage is not immediately clear. It can be expected that in those cases the advantage of using a BDD is rather small, but in some domains we actually are able to generate the complete set of reachable states in case of symbolic search, while we would need much more memory to achieve the same in case of explicit search. Thus, exponential sized BDDs are only an indicator that they might not work well, even though they actually might help a lot.

In the following we will start with some established results, concerning permutation games such as the $n^2 - 1$-PUZZLE (Section 4.1.1) or BLOCKSWORLD (Section 4.1.2). For those it has been shown that no matter what variable ordering is used a BDD contains exponentially many nodes. Then we will continue with two domains from the set of planning benchmarks. The first one, GRIPPER (Section 4.2.1), admits BDDs of polynomial size, while for the second one, SOKOBAN (Section 4.2.2), we can only prove that a superset of the reachable states can be represented by BDDs of polynomial size. Both proofs work only for a special variable ordering; for others the BDDs might still become exponential sized. Finally, we will analyze a domain from general game playing, namely CONNECT FOUR (Section 4.3). For this we can show that the size of the BDDs for a superset of the reachable states is polynomial as long as we omit the removal of terminal states while the BDD for representing the terminal states is exponential. The prove for these two properties leads to results for further k-in-a-row games such as GOMOKU.

Apart from theoretical analyzes of these domains we also provide experimental evaluations. For all these we used the same machine, namely one with an Intel Core $i7$ 920 CPU with 2.67 GHz and 24 GB RAM.



(a) The 8-PUZZLE. (b) The 15-PUZZLE. (c) The 24-PUZZLE. (d) The 35-PUZZLE.

Figure 4.1: Goal positions of typical instances of the $n^2 - 1$-PUZZLE.

4.1 Exponential Lower Bound for Permutation Games

For permutations games on $(0, \ldots, N-1)$ an exponential number of states is reachable. The characteristic function f_N of all permutations on $(0, \ldots, N-1)$ needs $N \lceil \log N \rceil$ binary state variables. It evaluates to true if every block of $\lceil \log N \rceil$ variables corresponds to the binary encoding of an integer and the integers form a permutation on $(0, \ldots, N-1)$.

Concerning the BDD encoding of such games, Hung (1997) has shown the following result.

Lemma 4.1 (Exponential bound for permutation games)**.** *The BDD for f_N needs more than $\left\lfloor \sqrt{2^N} \right\rfloor$ nodes for any variable ordering.*

We show this exponential behavior experimentally in the following two sections for two typical planning benchmarks.

4.1.1 The SLIDING TILES PUZZLE

In general, in the domain of the SLIDING TILES PUZZLES we are concerned with puzzles of (often rectangular) pieces that can be moved around within a certain frame. The goal is to establish a predefined positioning (either of one block, such as in RUSH HOUR, or of the entire board).

In AI especially the $n^2 - 1$-PUZZLE has become a benchmark problem. It consists of a square board with an edge length of n, so that a total of n^2 square pieces fit onto it. One of these pieces is left out resulting in a blank square. The pieces adjacent to this blank can be moved onto its position, leaving the piece's starting position empty. The goal is to achieve a predefined goal position. For of some instances these are depicted in Figure 4.1 (the pieces here are denoted by numbers from 1 to $n^2 - 1$).

The $n^2 - 1$-PUZZLE is a permutation game, so that there is a total of $n^2!$ different states of which only one half is reachable (Johnson and Story, 1879). Using explicit state BFS, Korf and Schultze (2005) have generated the entire state space of the 15-PUZZLE—consisting of 10,461,394,944,000 states—on hard disk. An important result they achieved by this is that the minimal distance of any (reachable) state to the goal state (as depicted in Figure 4.1b) is at most 80 moves.

We used symbolic BFS to generate as many layers of the 15-PUZZLE as possible on our machine. With it we were able to fully generate the first 29 layers before it started swapping due to the limited main memory. At that time 822,416,571 states were generated. The results (cf. Figure 4.2) show that both the number of states and the number of BDD nodes grow exponentially, no matter whether we consider the sizes for each layer or the total sizes, though the growth of the BDD nodes is slightly less steep, so that after 15 steps the total number of states exceeds the number of BDD nodes. Similarly, from layer 20 onward the number of states in the corresponding layer is smaller than the number of BDD nodes needed to represent it.

4.1.2 BLOCKSWORLD

In BLOCKSWORLD we are given a number of blocks that are placed either on the table or on top of each other (cf. Figure 4.3) and a hand to move the blocks around. The goal is to stack them into one tower in a

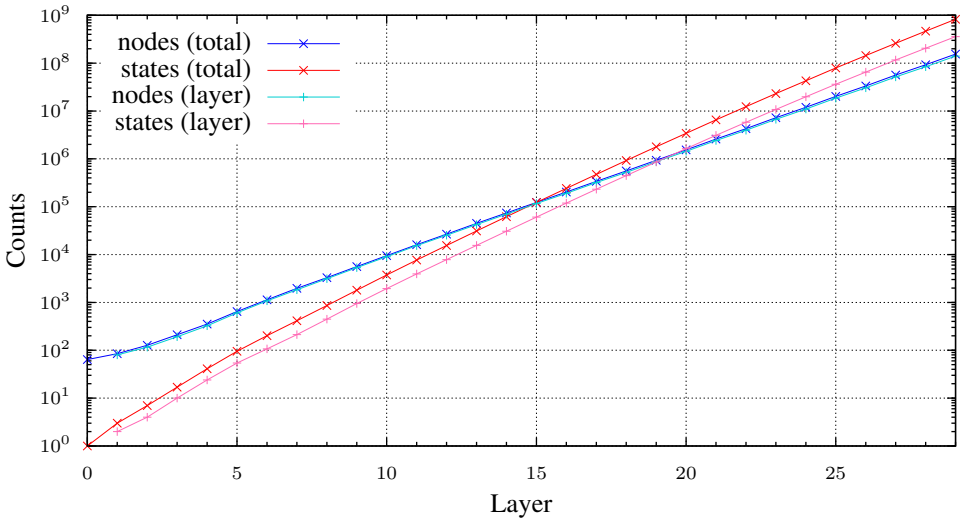


Figure 4.2: Number of states and BDD nodes in the reached layers and in total during symbolic BFS of the 15-PUZZLE.

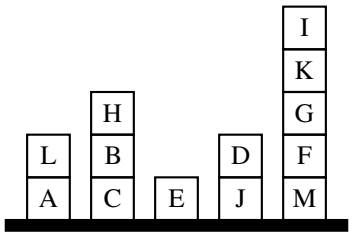


Figure 4.3: The BLOCKSWORLD domain.

certain order. For this the planner can take a block from the table or the top of a tower if the hand is empty. If it is not empty it may only place the held block on top of an existing tower or on the table to start a new one. Apart from the problem of stacking the blocks into one tower, the number of configurations with exactly one tower containing all blocks is clearly exponential, as a tower can be in any permutation of the given blocks. This permutation problem leads us to expect that BDDs will not work too well in this domain.

For BLOCKSWORLD we compare the two approaches of symbolic uni- and bidirectional BFS on the set of default benchmark problems from the second international planning competition in the year 2000[3]. For our tests we used a timeout of 60 minutes. The runtime results for all successful runs are given in Figure 4.4. It clearly shows that in this domain the bidirectional search outperforms unidirectional search given at least seven blocks. Using unidirectional search we can solve all instances of up to nine blocks, while bidirectional search can handle up to 15 blocks on our machine.

For the largest problem we could solve (problem 15–1), we provide the number of BDD nodes and reachable states for each layer of the forward as well as the backward search in Figure 4.5. We can see that the forward search is similar to the one of the 15-PUZZLE. The number of BDD nodes and the number of states grow exponentially with the number of nodes growing less steep, so that from layer 16 onward the number of states exceeds the number of BDD nodes.

In backward search the number of states is a lot larger than the number of BDD nodes. Due to the encoding many states are generated that are not reachable in forward direction. In this case, a state is defined by four predicates

on The two-ary predicate *on* denotes which block resides on which other block.

table The unary predicate *table* holds if the block provided as the argument resides on the table.

[3] http://www.cs.toronto.edu/aips2000

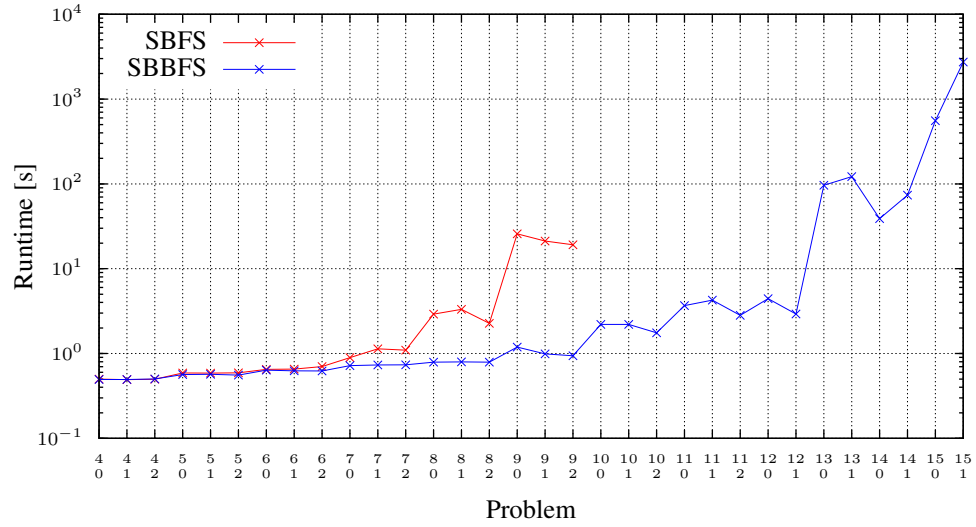


Figure 4.4: Runtime comparison of symbolic uni- (SBFS) and bidirectional (SBBFS) breadth-first search for the BLOCKSWORLD domain, averaged over 20 runs. The problem names consist of the number of blocks and some index to differentiate the various problems with equal number of blocks.

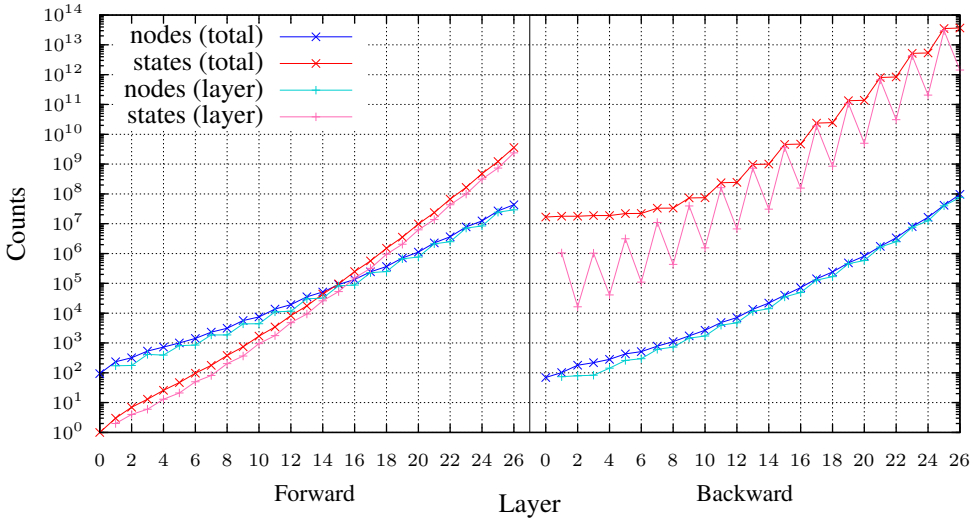


Figure 4.5: Number of states and BDD nodes in the reached layers and in total during symbolic bidirectional BFS for the problem BLOCKSWORLD 15–1.

clear The unary predicate *clear* holds if the specified block is the topmost of a tower, so that no other block resides on it.

handempty The 0-ary predicate *handempty* holds if no block is held in the hand.

Some of these are actually mutually exclusive, but this is not properly detected in all cases. For example, it was detected that for every block x it holds either $on\,(y, x)$ for exactly one other block y, or if it does not hold for any block then $clear\,(x)$ must hold, i. e., either some other block y is stacked on a block x or it is the topmost one of a tower. However, it was not detected that for every block x either $on\,(x, y)$ holds for exactly one other block y, or if it does not hold for any block then $table\,(x)$ must hold, i. e., either a block x is stacked on some other block y or it is on the bottom of a tower and thus resides on the table. In the goal only the tower to be achieved is specified (in terms of the corresponding on predicates). Opposed to the initial state this is not unique as no information concerning the hand or the placement on the table is given. For us humans both are clear immediately (the hand must be empty and the bottom-most block must

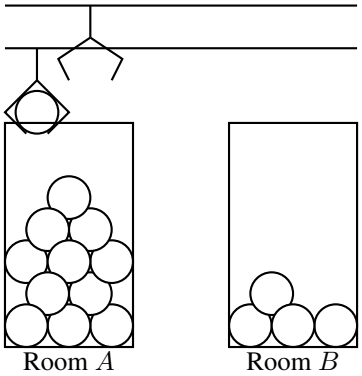


Figure 4.6: The GRIPPER domain.

reside on the table), but this is not found automatically, so that the program must consider all possibilities. If, similar to the initial state, the goal state was fully specified the backward search would very likely look the same as the forward search.

4.2 Polynomial Upper Bounds

Apart from the cases where any variable ordering necessarily results in a BDD of exponential size we can also find some planning benchmarks for which there is a variable ordering so that the size of the BDD representing the reachable states—or at least a superset of those—is polynomial in the number of variables (Edelkamp and Kissmann, 2008b,c). In the following we will show that this holds for the GRIPPER domain (Section 4.2.1) as well as for a superset of the reachable states in the SOKOBAN domain (Section 4.2.2).

4.2.1 GRIPPER

In the GRIPPER domain, which was introduced in the first international planning competition in 1998, we have two rooms, A and B, an even number of $n = 2k$ balls and a robot with two hands, the grippers, which can move between the rooms (cf. Figure 4.6). Initially, all balls are in room A and the goal is to bring them to room B. The possible actions are to take one ball in the left or the right gripper, move between the rooms, and drop the ball from the left or the right gripper.

The search space grows exponentially in the number of balls. As $2^n = \sum_{i=0}^{n} \binom{n}{i} \leq n\binom{n}{k}$ for $k = n/2$, the number of states with k balls in one room is $\binom{n}{k} \geq \frac{2^n}{n}$. More precisely, Helmert and Röger (2008) have shown that the exact number of reachable states S_n is

$$S_n = 2^{n+1} + n2^{n+1} + n\,(n-1)\,2^{n-1},$$

with $S_n^0 := 2^{n+1}$ being the number of states with no ball in a gripper. All states with an even number of balls in both rooms and none in the grippers (except for those with all balls in one room and the robot in the other) are part of an optimal solution. Thus, for larger n heuristic search planners with as little as a constant error of only 1 are doomed to failure.

Each optimal solution contains cycles of six steps, i. e., taking one ball in one gripper, taking another ball in the other gripper, moving from room A to room B, dropping one ball, dropping the other ball, and moving back to room A. Thus, every sixth BFS layer contains those states of an optimal solution with no ball in a gripper. Of these there are still exponentially many, namely S_n^0.

Lemma 4.2 (Polynomial bound for GRIPPER)**.** *There exists a binary state encoding and an associated variable ordering for which the BDD size for the characteristic function of the states in any optimal solution in the BFS exploration of* GRIPPER *is polynomial in n, the number of balls.*

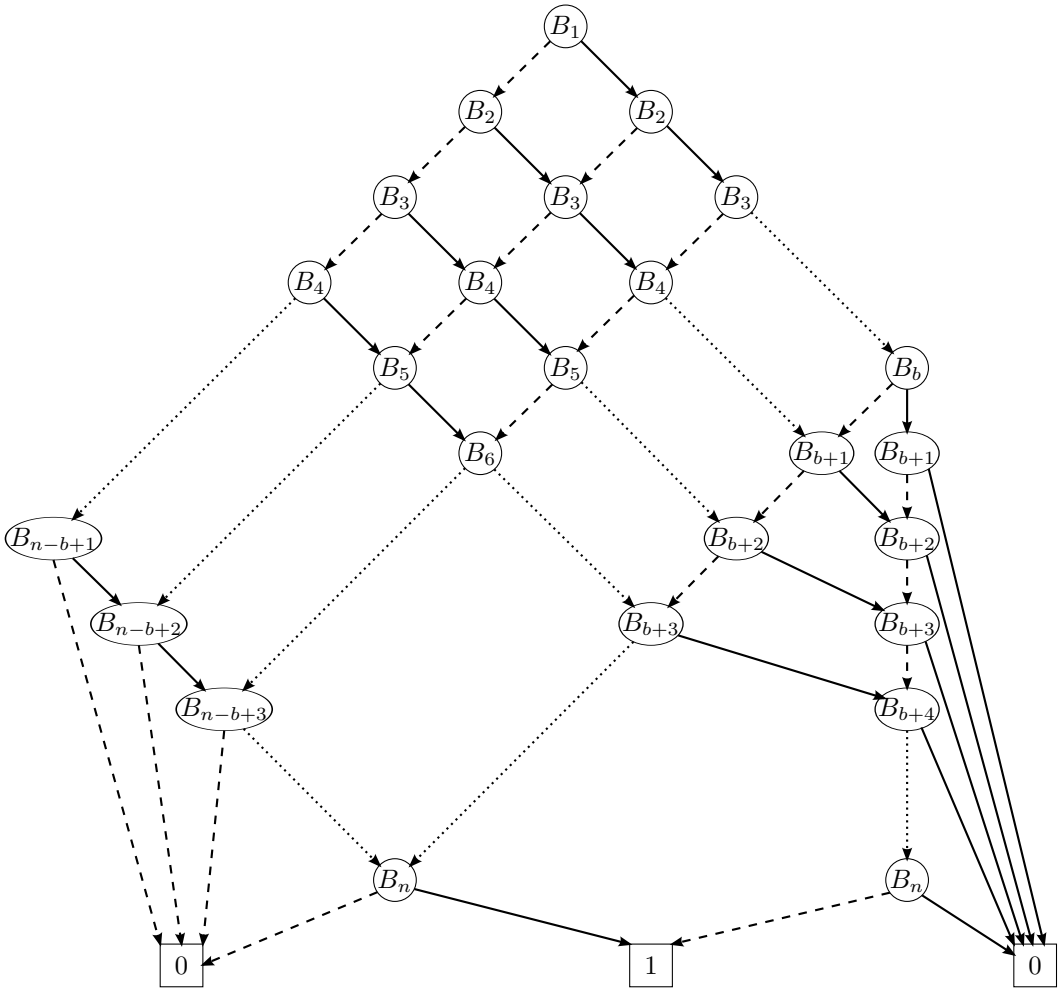


Figure 4.7: Critical part of the BDD for representing b balls in room B in the GRIPPER domain. Dashed edges correspond to the ball being in room A, solid ones to the ball being in room B, dotted edges denote something left out due to unspecified size. We use two 0 sinks here only to keep the diagram more readable.

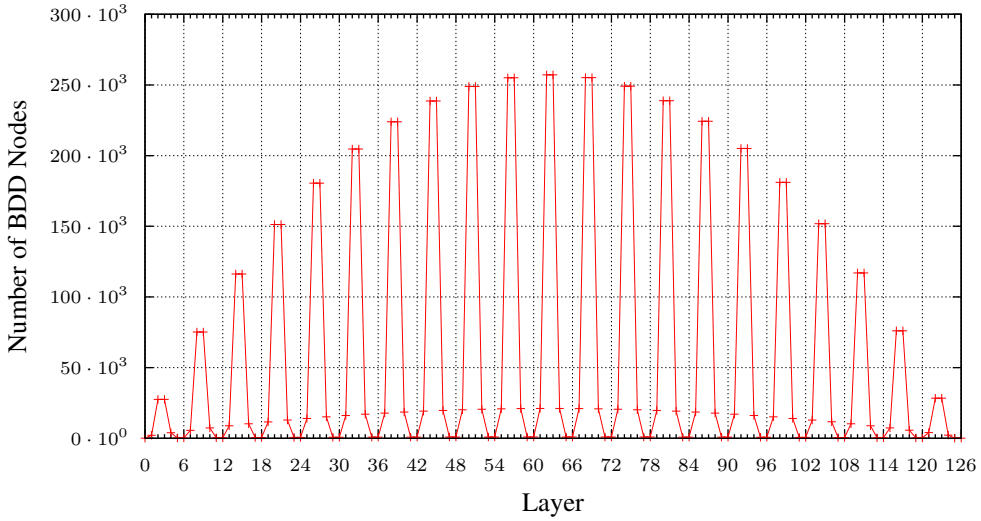
Proof. To encode states in GRIPPER, we need $2n + 2\lceil \log(n+1) \rceil + 1$ bits, $2n$ for the location of the n balls (A / B / none = in one of the grippers), $\lceil \log(n+1) \rceil$ for each of the grippers to denote which ball it currently carries (including none) and one for the location of the robot (A / B).

The states with no balls in the grippers, b balls in room B and $n - b$ balls in room A can be represented using $\mathcal{O}(bn)$ BDD nodes (cf. Figure 4.7), which is at most $\mathcal{O}(n^2)$.

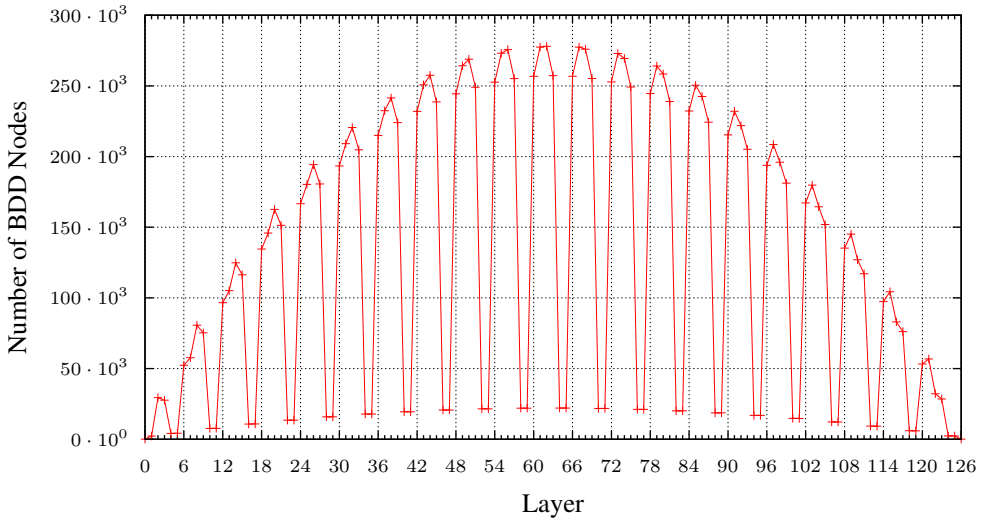
The number of choices with one or two balls in the grippers is bounded by $\mathcal{O}(n^2)$, so that this leads to an at most quadratic growth (after each of these choices, the BDD contains a part that corresponds to the one in Figure 4.7). Thus, each layer contains at most $\mathcal{O}(n^4)$ BDD nodes in total.

Accumulating this over the complete path, whose length is linear in n, results in $\mathcal{O}(n^5)$ BDD nodes needed for the entire exploration, if we store all the $\mathcal{O}(n)$ BFS layers as separate BDDs of size $\mathcal{O}(n^4)$. □

If we allow only states that are part of optimal solutions we achieve an interesting effect. In that case, each sixth layer l contains all those states with no balls in the grippers and the robot in room A. In the previous layer $l - 1$, the robot was in room B and also had no balls in the grippers. In layers $l - 2$ and $l + 1$ it holds one ball and in the remaining layers $l - 3 = l + 3$ and $l + 2$ it holds two balls. Thus, in layers $l - 1$ and l, the BDD size is quadratic in n, in layers $l - 2$ and $l + 1$ it is cubic (for one gripper we have one choice, for the other n) and in layers $l + 2$ and $l + 3$ it is in n^4. This effect can be seen in Figure 4.8a.



(a) Only optimal actions allowed.



(b) All actions allowed.

Figure 4.8: Symbolic BFS for the problem GRIPPER 20.

A suboptimal solution where the robot can take only one ball from room A to room B is also possible. This brings a shifted result. In the first step, the robot takes one ball. In the second it moves to room B or takes a second ball. In the third step it drops the ball in room B or moves there. In the fourth step it moves back to A empty-handed or drops one ball. In the fifth step it takes one ball in room A or drops the second ball in room B. In the sixth step it takes a second ball in room A or moves there. From that time on both processes always take two balls from room A, move between the two rooms and drop both balls in room B. Other seemingly possible procedures are impossible if we remove duplicates.

The result is that we never reach a layer in which on every path both grippers are empty. The possible paths are detailed in Table 4.1, from which we see that for every sixth layer l, in layers $l + 4$ and $l + 5$ the highest number of balls in the grippers is one, while in the other states the highest number is two. For layers l and $l + 3$, the one process contains states with two held balls while the other contains states with no held balls, which results in a slightly smaller BDD than the one for layers $l + 1$ and $l + 2$, where in the one process two balls are held and one in the other. This effect can be seen in Figure 4.8b, where we show the number of BDD nodes for the problem GRIPPER 20, which contains 42 balls.

Table 4.1: Actions and number of balls in the grippers for the two processes in GRIPPER The value l corresponds to every sixth layer, Action_i is the action taken by the ith process and Balls_i the number of balls the robot holds in its grippers afterward.

Layer	Action_1	Balls_1	Action_2	Balls_2
l	grab	2	move to A	0
$l+1$	move to B	2	grab	1
$l+2$	drop	1	grab	2
$l+3$	drop	0	move to B	2
$l+4$	move to A	0	drop	1
$l+5$	grab	1	drop	0

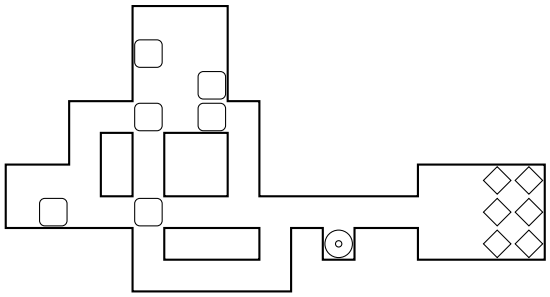
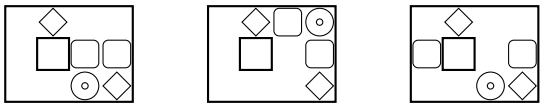


Figure 4.9: The SOKOBAN domain. The boxes are denoted by squares, the player by two concentric circles and the target positions by diamonds.



(a) Initial state. (b) Unreachable state. (c) Another unreachable state.

Figure 4.10: Possible initial state and two unreachable states in the SOKOBAN domain.

4.2.2 SOKOBAN

In the game SOKOBAN we are concerned with one player that is located in a maze of n cells. Additionally, there are b boxes and an equal number of target positions (cf. Figure 4.9). The aim of the player is to move the boxes to the target positions. The player can move only horizontally or vertically and can move the boxes only by pushing them. Pushing more than one box at a time is impossible.

SOKOBAN is a well studied domain of AI (see, e. g., Junghanns, 1999). Similar to GRIPPER we can show that using BDDs we can save an exponential amount of memory. The argument is quite similar. Placing the b boxes in the n cells results in exponentially many states which we can represent by a polynomial number of BDD nodes. Including the player doubles the BDD's size. But it is important to note that actually not all these states are reachable, as illustrated in Figure 4.10, mainly because the player cannot pull the boxes. Thus, BDDs for representing the exact set of reachable states might still be exponential sized.

Lemma 4.3 (Polynomial bound for SOKOBAN)**.** *In a* SOKOBAN *maze consisting of n cells and containing b boxes at most $\binom{n}{b}(n-b)$ states are reachable. For these, there is a binary state encoding in which the BDD size for the characteristic function of all possibly reachable states in* SOKOBAN *is polynomial in the number of cells n.*

Proof. In our encoding we do not distinguish between the different boxes, so that we need $2n$ bits for a state, i. e., two bits for each cell, which can be occupied by a box, the player, or nothing.

Removing the player results in the pattern shown in Figure 4.7 for the GRIPPER domain, with the dashed edge denoting that the cell is empty and the solid one that the cell is occupied by a box. Thus, the BDD for this consists of $\mathcal{O}(nb) = \mathcal{O}(n^2)$ nodes.

Integrating the player results in an additional BDD of size $\mathcal{O}(nb)$ and links from the first to the second. In contrast to the previous case, in the first of these BDDs we need twice as many BDD nodes to distinguish not only between empty cells and cells occupied by boxes as here the player is placed on some cell as well. A first node determines whether a cell is occupied by a box or not. If there is a box on a cell the successor is the same as the one following the solid edge in Figure 4.7. If there is none and the player is not on that cell, the successor node is the same as the one following the dashed edge in Figure 4.7. If the player resides on that cell the successor is the same as the one following the dashed edge in Figure 4.7, but now in the second BDD. The second BDD is the same as the one for GRIPPER, as the player can occupy only one cell at a time.

This results in total in a polynomial number of BDD nodes needed to represent the exponential number of possibly reachable states, which are a superset of the set of states that are actually reachable from the initial state. □

In Figure 4.11a we present the results for symbolic BFS for the example shown in Figure 4.9. An optimal solution has a length of 230 actions, while the maximum layer in which new states can be found is 263. As can be seen, even in this setting, where we cannot reach all possible states, the number of BDD nodes for representing the set of all actually reachable states is smaller by a factor of roughly four orders of magnitude than the total number of states (9,362 vs. 180,144,838).

To get closer to the superset used in Lemma 4.3 we have adapted the rules in such a way that the player can not only push but also pull a box (if the cell behind the player is empty). This way, the two unreachable states from Figure 4.10 are now reachable, though still not all possible states can be reached. As an example, it is never possible to place a box in the cell where the player starts in the maze in Figure 4.9. In this setting, the results are very similar (cf. Figure 4.11b). Here, the last new states can be found in layer 238 and the total number of states is increased to 1,282,783,263. In the end, representing all reachable states takes a BDD consisting of a number of BDD nodes (7,293) that is roughly five orders of magnitude smaller than the number of states .

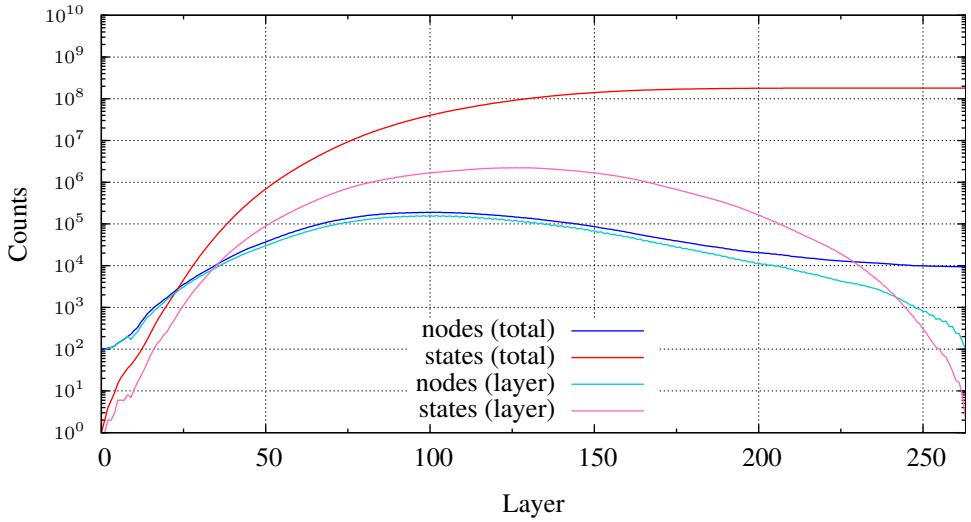
4.3 Polynomial and Exponential Bounds for CONNECT FOUR

The game CONNECT FOUR belongs to the broader class of k-in-a-row games such as TIC-TAC-TOE or GOMOKU. Given a $w \times h$ board (w being the width and h the height) the two players (red and yellow) take turns placing their pieces on the board starting with the red player. In contrast to most other games of this class gravity is important in CONNECT FOUR, so that the pieces fall as far to the bottom as possible. Thus, in each state at most w moves are possible (placing the piece in one of the columns if it is not yet completely filled). The goal is to construct a line of (at least) four pieces of the own color (cf. Figure 4.12).

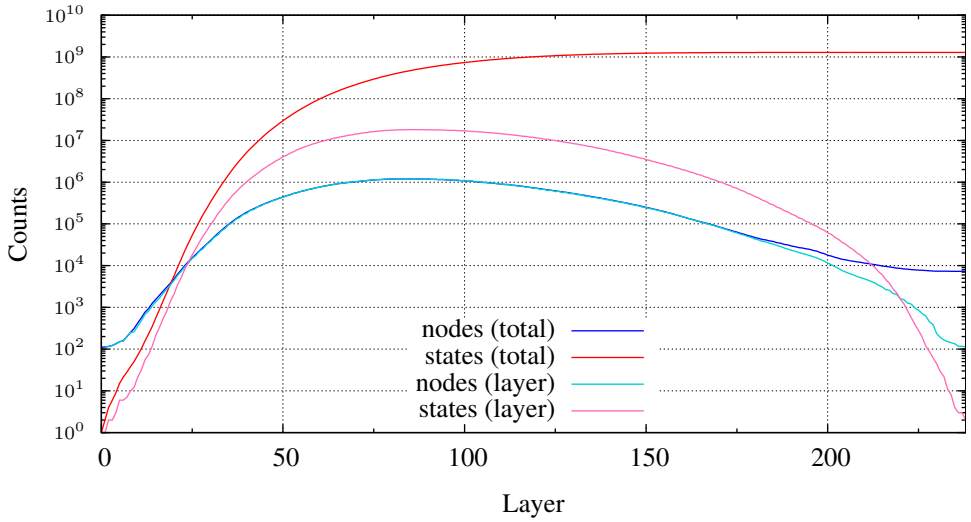
The game CONNECT FOUR has been widely studied in computer sciences. Actually, Allen (1989, 2011)[4] and Allis (1988; Uiterwijk et al. 1989)[5] solved the game for the default instance on a 7×6 board independently of each other only a few days apart in 1988. The optimal outcome in case of perfect play of both players is a victory for the starting player, if that puts the first piece in the middle column. Putting the first piece in one of the two adjacent columns results in a draw while putting it in one of the four outermost columns results in a victory for the second player.

In his Master's Thesis, Allis (1988) provided an estimate of 70,728,639,995,483 for the total number of reachable states, which is too much by a factor of about 15. Using symbolic search we found the precise number of reachable states to be 4,531,985,219,092 (Edelkamp and Kissmann, 2008e). This took about 15 hours of work. The same number was later confirmed by Tromp using explicit search[6], which took about 10,000 hours[7]. Given these differences in runtime it seems likely that BDDs are good for generating

[4] Originally reported in rec.games.programmer on October 1st 1988.
[5] Originally reported in rec.games.programmer on October 16th 1988.
[6] `http://homepages.cwi.nl/~tromp/c4/c4.html`
[7] According to a private communication.



(a) Normal SOKOBAN rules.



(b) SOKOBAN rules with pulling allowed.

Figure 4.11: Number of states and BDD nodes in the reached layers and in total during symbolic BFS for the example SOKOBAN problem.

the set of reachable states. Furthermore, using 85 bits per state (two for each cell to denote the color it is occupied with (red / yellow / none) and one to denote the active player), in explicit search we would need nearly 43.8 TB to store them all at the same time, while with BDDs a machine containing 12 GB RAM is sufficient.

In contrast to the problems studied so far in this chapter, for CONNECT FOUR we must divide the analysis into two parts (Edelkamp and Kissmann, 2011). The first one is concerned with finding a bound on the BDD size for representing the reachable states when ignoring terminal states, the second one with finding a bound for the representation of the termination criterion, which is much more complex than in the other games.

4.3.1 Polynomial Upper Bound for Representing Reachable States

CONNECT FOUR is another game where l pieces are placed on $n = wh$ cells of the game board, so that it has an exponential search space as well. Nevertheless, we can find a variable ordering, so that a superset of

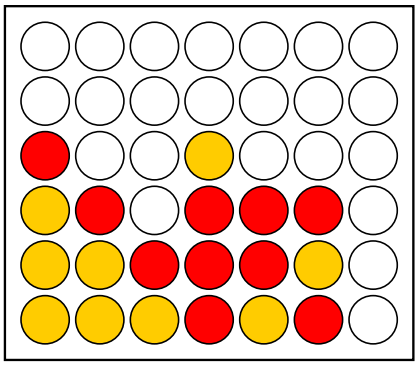


Figure 4.12: The CONNECT FOUR domain. Here, the red player has achieved a line of four pieces and thus won the game.

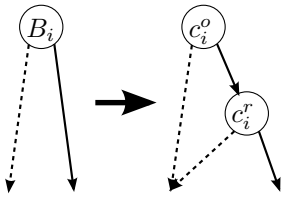


Figure 4.13: Replacement of a node from the BDD for the GRIPPER domain by two nodes denoting that cell i is occupied (c_i^o) and the occupying piece is red (c_i^r) for the CONNECT FOUR domain. Solid arrows represent high edges, dashed arrows represent low edges.

the reachable states can be represented by a number of polynomial sized BDDs (one for each BFS layer) if we continue after a terminal state has been reached, i. e., if we omit the check for termination. In this case, the superset contains all those states that can be reached by placing the pieces somewhere on the board and activating gravity only afterward.

Lemma 4.4 (Polynomial bound for CONNECT FOUR without termination and delayed gravity)**.** *For* CONNECT FOUR *there is a binary state encoding and an associated variable ordering for which the BDD size for representing all the states with l pieces on the board is polynomial in $n = wh$, with w being the width and h the height of the board, if we do not cut off the search after a terminal state and if we use gravity only in layer l, not the previous ones.*

Proof. We use $2n + 1$ bits to encode a state in CONNECT FOUR, two for each cell (red / yellow / none) and one for the active player. If we organize the variables in a column-wise manner, we need to calculate the conjunction of three polynomial sized BDDs.

Two BDDs are similar to the one in Figure 4.7 we used for the GRIPPER domain. The first represents the fact that we have $b = \lceil \frac{l}{2} \rceil$ red pieces on the board, the second one that we have $b = \lfloor \frac{l}{2} \rfloor$ yellow pieces on the board. In this game the cells can also be empty, so that we need to replace each node by two, the first one distinguishing between the cases that a cell is empty or not and the second one between the colors. If the cell is empty, we insert an edge to a node for the next cell; if it is not, we distinguish between the two colors (cf. Figure 4.13 for red pieces; for yellow pieces, the low and high edges of the lower node are swapped). Due to the replacement of each cell by two BDD nodes, each BDD has a size of $\mathcal{O}(2ln) = \mathcal{O}(ln)$.

The third BDD represents the gravity, i. e., the fact that all pieces are located at the bottom of the columns. For this the column-wise variable ordering is important. Starting at the lowest cell of a column, if one is empty, all the following cells of this column are necessarily empty as well. If one is not empty, the others can still be empty or not. Thus, in this BDD it is only relevant if a cell is occupied, so that the actual color of a piece occupying it does not matter, which keeps the BDD smaller. All in all, we need at most two BDD nodes for each cell, resulting in a total of $\mathcal{O}(2wh) = \mathcal{O}(wh) = \mathcal{O}(n)$ BDD nodes (cf. Figure 4.14).

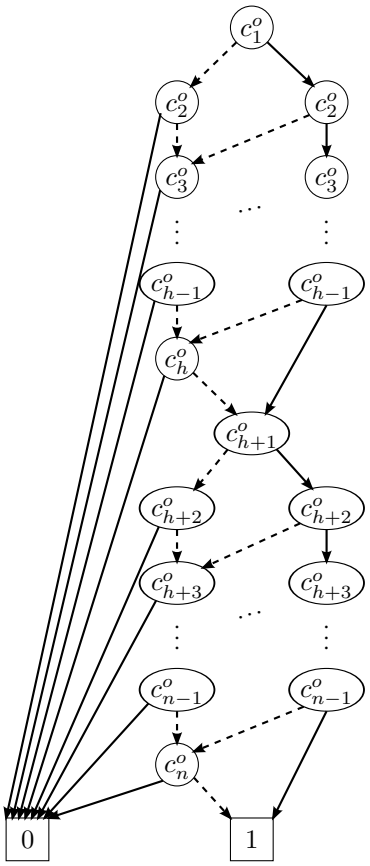


Figure 4.14: BDD for representing the fact that all pieces are located at the bottom in the CONNECT FOUR domain. Each node c_i^o represents the decision for cell i if it is occupied. Dashed edges stand for empty cells, solid ones for occupied cells.

Calculating the conjunction of two BDDs of sizes s_1 and s_2 results in a new BDD of size at most $\mathcal{O}(s_1 s_2)$, so that the conjunction of three polynomial sized BDDs is still polynomial. In this case, the total BDD for each layer l is of size $\mathcal{O}\left(l^2 n^3\right)$. As l is at most n we arrive at a total size of at most $\mathcal{O}\left(n^5\right)$ BDD nodes for each layer. Storing each layer in a separate BDD results in n BDDs of that size, so that overall we need $\mathcal{O}\left(n^6\right)$ BDD nodes to store the desired superset of reachable states. □

Unfortunately, Lemma 4.4 does not help much in analyzing the size of the BDD for representing the precise set of reachable states in a layer l. A way for representing this set we can envision is to use the BDD for the preceding layer, so that gravity is not delayed. This brings the disadvantage that we have to somehow combine the BDDs of all the layers, which will lead to an exponential blowup of the BDDs' sizes, as for the last layer we have to combine the BDDs of all n layers.

Nevertheless, we can use a part of the proof of the lemma for proving that the BDDs for representing the precise sets of reachable states for all layers l of a k-in-a-row game without gravity, such as GOMOKU, are of polynomial size.

Lemma 4.5 (Polynomial bound for k-in-a-row games without gravity and without termination)**.** *There is a binary state encoding and an associated variable ordering for which the BDD size for representing the precise set of reachable states with l pieces on the board of a k-in-a-row game without gravity is polynomial in $n = wh$ if we do not cut off the search after a terminal state.*

Proof. The proof is actually the same as in Lemma 4.4, but we do not need the third BDD for the gravity. This way each layer's BDD consists of at most $\mathcal{O}\left(n^4\right)$ nodes and the total sum of nodes required to represent all the layers of reachable states in separate BDDs is bounded by $\mathcal{O}\left(n^5\right)$. □

4.3.2 Exponential Lower Bound for the Termination Criterion

Integrating the terminal states is a lot more difficult in CONNECT FOUR than in the other domains we have analyzed so far. Most contain only a small number of terminal states (the correct pattern in the $n^2 - 1$-PUZZLE and BLOCKSWORLD, all balls in room B in GRIPPER, or all boxes on the target locations in SOKOBAN). For CONNECT FOUR the termination criterion is fulfilled if one player has succeeded in placing four own pieces in a row (horizontally, diagonally, or vertically), which results in a comparatively complex Boolean formula.

Reducing the termination criterion to those cases where two pieces (not necessarily of the same color) are placed next to each other horizontally or vertically already yields a BDD of exponential size.

Definition 4.6 (Reduced Termination Criterion in CONNECT FOUR)**.** *The* reduced termination criterion *in* CONNECT FOUR $\mathcal{T}_r$ *is the reduction of the full termination criterion* $\mathcal{T}$ *to the case where only two horizontally or vertically adjacent cells must be occupied, i. e.,*

$$\mathcal{T}_r = \bigvee_{1 \leq i \leq w, 1 \leq j \leq h-1} (x_{i,j} \wedge x_{i,j+1}) \vee \bigvee_{1 \leq i \leq w-1, 1 \leq j \leq h} (x_{i,j} \wedge x_{i+1,j}),$$

with $x_{i,j}$ *specifying that the cell at column* i *and row* j *is occupied.*

To proof this claim we start by representing the game board by a grid graph $G = (V, E)$ with V representing the $n = wh$ cells of the board and edges only between horizontally or vertically adjacent nodes. The reduced termination criterion corresponds to the fact that two nodes connected by an edge are occupied. In the course of the proof we also make use of the disjoint quadratic form DQF, which we introduced in Definition 2.7 on page 15.

Lemma 4.7 (Exponential bound for $\mathcal{T}_r$ in CONNECT FOUR)**.** *For* CONNECT FOUR *in the cell-wise encoding from Lemma 4.4 with a board size of* $w \times h$ *the size of the BDD for representing* $\mathcal{T}_r$ *is exponential in* $\min(w, h)$*, independent of the chosen variable ordering.*

Proof. To prove this claim we show that any variable ordering can be split in two, so that $\Omega(\min(w, h))$ variables are in the one part and connected to $\Omega(\min(w, h))$ variables in the other part. Retaining at most one connection for each variable, we will show that $\Omega(\min(w, h)/7)$ edges still connect the two sets. These represent a DQF with a bad variable ordering as in Proposition 2.9 on page 15, which will give us the exponential bound.

Given any ordering π on the variables we divide it into two sets π_1 and π_2 with π_1 containing the first $\lceil wh/2 \rceil$ variables of this order and π_2 the remaining $\lfloor wh/2 \rfloor$ variables. These sets correspond to a partitioning of the nodes of the grid graph G into two sets.

The minimal number of edges crossing between these partitions is in $\Omega(\min(w, h))$. This minimal number is achieved when each partition is connected and forms a rectangle of size $(\max(w, h)/2) \cdot \min(w, h)$ (cf. Figure 4.15), so that the cut between the two partitions crosses the grid in the middle but orthogonal to the longer edge and thus crosses $\min(w, h)$ edges.

We might view the two partitions along with the connecting edges $E' \subseteq E$ as a bipartite graph $G' = (V_1, V_2, E')$, $|E'| = \Omega(\min(w, h))$, with V_i being the nodes corresponding to the variables in partition π_i. For this grid graph we want to find a (not necessarily maximal) matching. To find one, we choose one edge from E' and the corresponding nodes from V_1 and V_2. The other edges connected to one of these nodes are removed (there are at most six of them, as each node can be connected to at most four adjacent nodes, one of which is chosen). We repeat this step until we cannot remove any more edges. As we choose one edge in each step and remove at most six, the final matching M contains at least $|E'|/7 = \Omega(\min(w, h)/7)$ edges.

All variables for which the corresponding node of the grid graph is not part of the matching are set to false[8], while the others are retained. Thus, the remainder of the formula for $\mathcal{T}_r$ is the DQF over the two sets of variables representing the nodes that are part of the matching M. All variables for the one set $(V_1 \cap M)$ appear before all variables of the other set $(V_2 \cap M)$, so that we have a bad ordering for these variables and the corresponding BDD is of exponential size. Thus, according to Proposition 2.9 on page 15 the BDD will contain at least $\Omega\left(2^{\min(w,h)/7}\right) = \Omega\left(\left(2^{1/7}\right)^{\min(w,h)}\right) = \Omega\left(1.1^{\min(w,h)}\right)$ nodes. □

[8] Replacing a variable by a constant does not increase the number of BDD nodes.

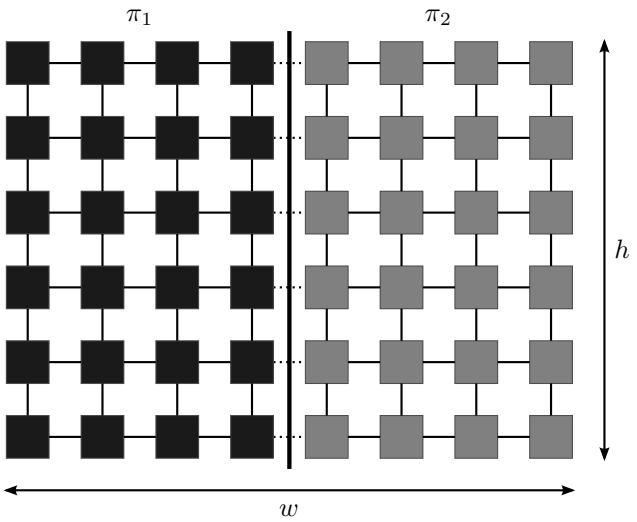


Figure 4.15: Partitioning of the grid graph of CONNECT FOUR minimizing the number of connecting edges.

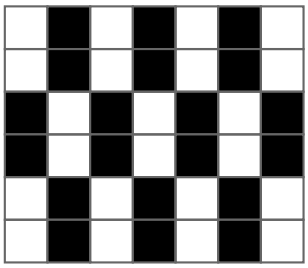


Figure 4.16: Coloring of the game board for the analysis of CONNECT FOUR in Lemma 4.9.

This result immediately implies that for square boards the number of BDD nodes is exponential in the square root of the number of cells.

Corollary 4.8 (BDD size for $\mathcal{T}_r$ in CONNECT FOUR on a square board)**.** *For a* CONNECT FOUR *game on a square board $w \times w$ with a total of $n = w^2$ cells the BDD representing $\mathcal{T}_r$ contains at least $\Omega\left(1.1^{\sqrt{n}}\right)$ nodes.*

With these results we can show that the case of four adjacent cells in a horizontal or vertical line being occupied results in a BDD of exponential size as well.

Lemma 4.9 (Exponential bound for four pieces in a horizontal or vertical line in CONNECT FOUR)**.** *Any BDD representing the part of the termination criterion $\mathcal{T}$ that states that four cells in a horizontal or vertical line must be occupied is exponential in $\min(w, h)$ for a board of size $w \times h$.*

Proof. We can devise a two-coloring of the game board as shown in Figure 4.16. Using this, any line of four pieces resides on two white and two black cells. If we replace all variables representing the black cells by true we can reduce this part of $\mathcal{T}$ to $\mathcal{T}_r$. As this replacement does not increase the BDD size, the BDD is exponential in $\min(w, h)$ as well.

□

In a similar fashion we can show that any BDD for the fact that two (or four) diagonally adjacent cells are occupied is exponential for any variable ordering.

The combination of all these results illustrates that any BDD representing the termination criterion in CONNECT FOUR given the cell-wise encoding is of exponential size, no matter what variable ordering is chosen.

Considering other encodings, it is hard to find a good one. In several other games it helps to switch to a piece-wise encoding, i. e., one where each piece can be identified uniquely and the cell it occupies is stored, whereas in the cell-wise encoding the type of piece occupying a certain cell is stored. The piece-wise encoding typically helps if we have very few equal pieces, such as in CHESS (apart from the pawns each piece exists at most twice) or in the $n^2 - 1$-PUZZLE, where each piece is unique. In CONNECT FOUR the piece-wise encoding would immediately increase the size for representing a single state from $2n + 1$ bits to $2n \lceil \log (n + 1) \rceil + 1$ (each piece can reside on one of the n cells, or nowhere, if it was not yet played). Furthermore, here we face the problem that a line of four pieces concerns pieces that might be arbitrarily far away from each other in this encoding, so that we will very likely suffer from an exponential size in terms of BDD nodes as well, independent of the variable ordering.

4.3.3 Experimental Evaluation

For a column-wise variable ordering we have evaluated the number of BDD nodes for representing the termination criterion for four pieces of one player being placed in a horizontal or vertical line for several board sizes (cf. Figure 4.17). For a fixed number of rows (cf. Figure 4.17a) the growth of the number of BDD nodes is linear in the number of columns, while for a fixed number of columns (cf. Figure 4.17b) it is exponential in the number of rows. Though it first might seem like a contradiction to our result, it actually shows that the result holds for the chosen variable ordering.

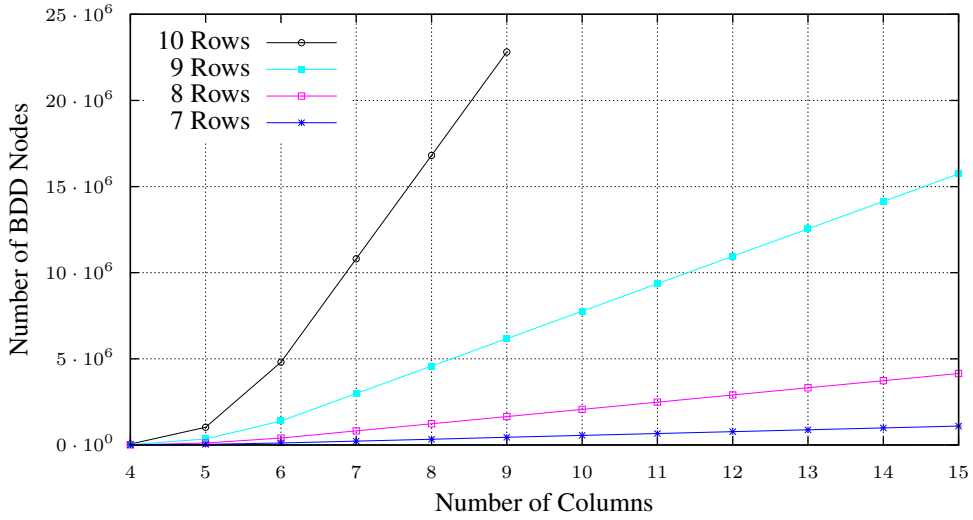
In this variable ordering the partitions induced by the split of the variable ordering cut at least h edges. Thus, the resulting BDD's size is exponential in h. For the case of a fixed height, h does not change, so that the size increases by a constant amount of nodes. On the other hand, when fixing the width the size changes exponentially in the height, no matter if the height is smaller or larger than the width. This happens because in the cases that the height is actually larger than the width the used variable ordering is not the best choice—in those cases it should have been in a row-wise manner.

Additionally, we have evaluated the default instance of 7×6 along with the full termination criterion in two complete searches, one stopping at terminal states and one continuing after them (cf. Figure 4.18). Again we used a column-wise variable ordering. We can see that the number of states in the first case is smaller than in the second, while the number of BDD nodes needed to represent these states is larger in the first case. Thus, for this variable ordering we can clearly see that the use of the termination criterion increases the sizes of the BDDs. Concerning runtimes, performing full symbolic BFS with usual handling of terminal states takes nearly five hours, while finding the states reachable when ignoring terminal states takes only 23 seconds.

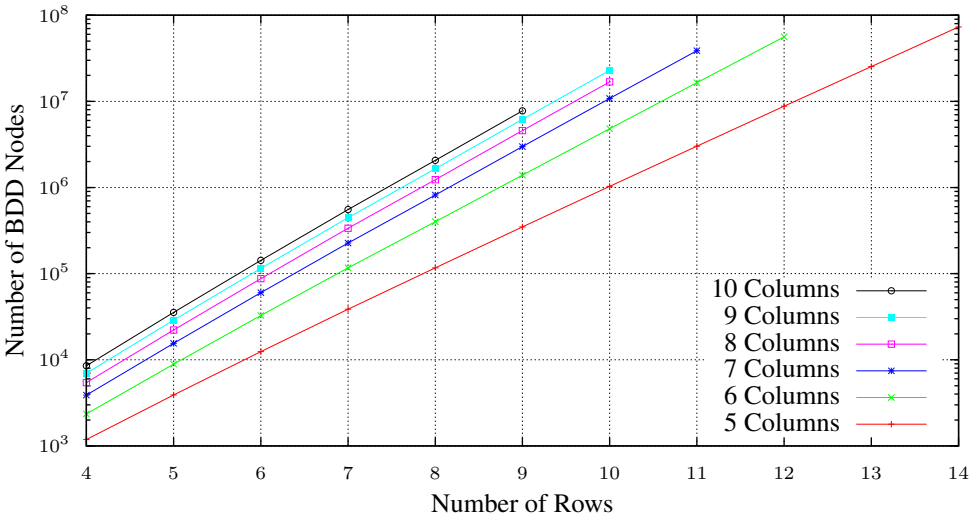
Another fact this experiment shows is that the set of reachable states still requires quite large BDDs, though we cannot really say whether they are of exponential size or not. To validate Lemma 4.5 we slightly adapted the game to no longer use gravity, so that we actually analyzed the game of 4-in-a-row. For this game we performed the same experiments as for CONNECT FOUR (cf. Figure 4.19). Here we can see that the BDD size is polynomial in the case of continuing after terminal states, while the number of states is clearly exponential. In this setting, in layer l ($0 \leq l \leq wh$) we have exactly $\frac{wh!}{(wh-l)!\lceil l/2 \rceil!\lfloor l/2 \rfloor!} = \binom{wh}{wh-l, \lceil l/2 \rceil, \lfloor l/2 \rfloor}$ states, as l of the wh cells are occupied (leaving $wh - l$ cells empty), one half by the first, the other half by the second player. When stopping at terminal states we were not able to fully analyze the game, as after only 16 steps the available memory was no longer sufficient to hold the entire BDD for the next layer and the machine started swapping. Concerning the runtimes, ignoring the terminal states we can find the reachable states for each layer in roughly two seconds, while calculating the first 16 layers in the normal way takes about 2:18 hours. Thus, in this domain we can clearly see the exponential blow-up induced by integrating the termination criterion in the search process.

4.4 Conclusions and Future Work

In this chapter we have determined bounds for the BDD sizes for representing the (supersets of) reachable states in a number of well-known benchmark problems from planning as well as game playing. In the first problems, containing simple termination criteria, the representation of the reachable states can be easy or



(a) Different heights.



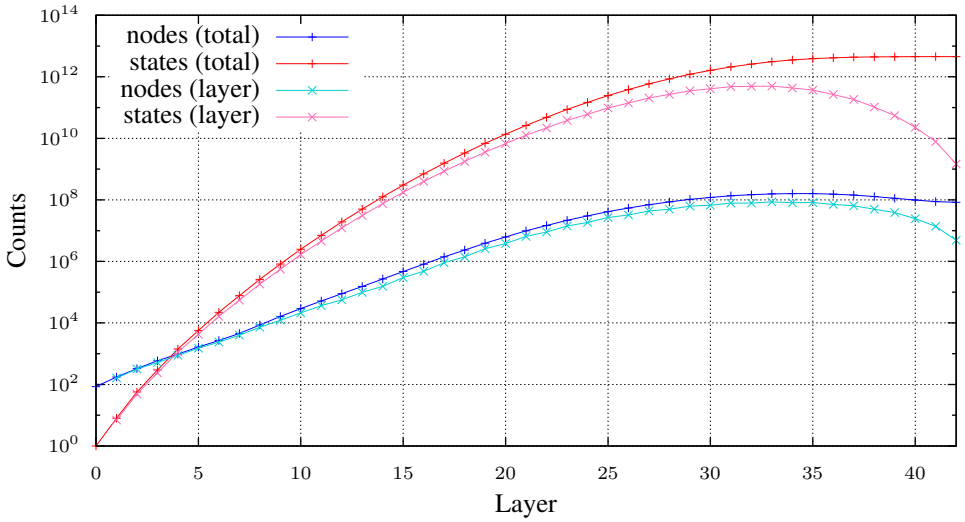
(b) Different widths.

Figure 4.17: Number of BDD nodes for the termination criterion for one player in the CONNECT FOUR domain. The variable ordering is according to the columns.

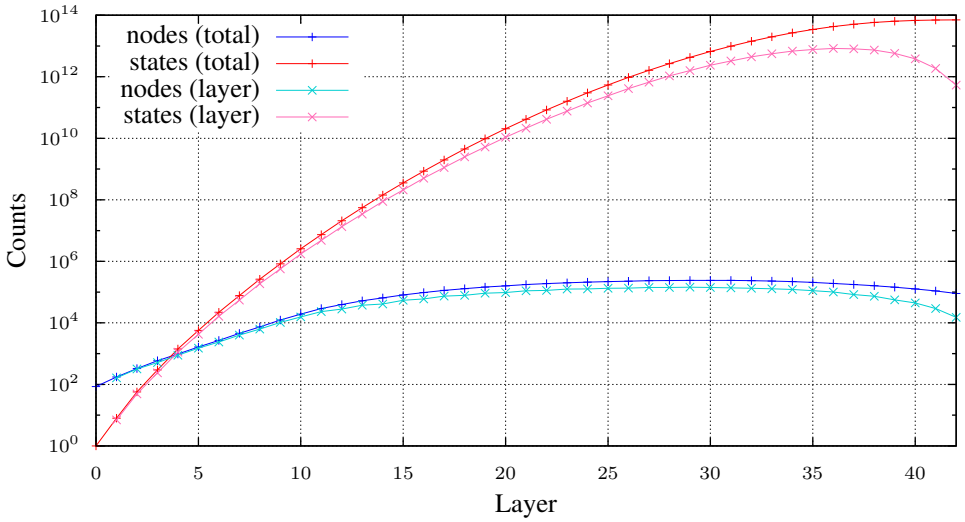
hard when using BDDs, i. e., result in BDDs of polynomial or exponential size. In the last problems, CONNECT FOUR and k-in-a-row games in general, the termination criterion is hard to represent as a BDD, while the set of reachable states can be represented efficiently, if we omit the evaluation of terminal states—and use no gravity in case of CONNECT FOUR.

With these problems we now have a classification for the efficiency of a BDD representation (cf. Table 4.2). Note that one bucket of this classification is not yet filled. It remains future work to find a problem for which the representation of the set of reachable states as well as the termination criterion results in a BDD of exponential size.

Lemma 4.7 is very general by stating that for any variable ordering the BDD representation of the fact that any two adjacent cells of a game board are occupied , no matter by what piece, is exponential. Thus, we might expect games such as CHESS or AMERICAN CHECKERS to be rather difficult for the BDD



(a) Stopping at terminal states.



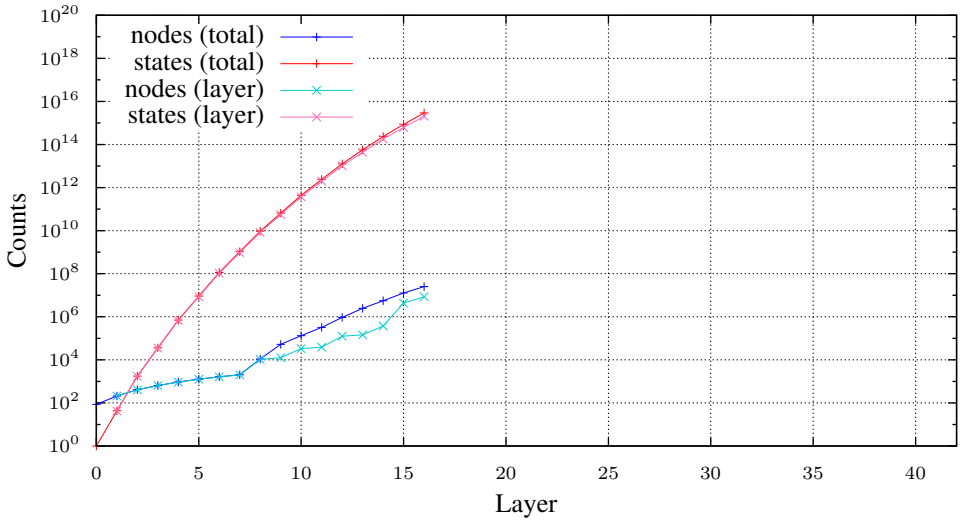
(b) Continuing after terminal states.

Figure 4.18: Comparison of the number of states and BDD nodes for different layers for a 7×6 board in CONNECT FOUR. The variable ordering is according to the columns.

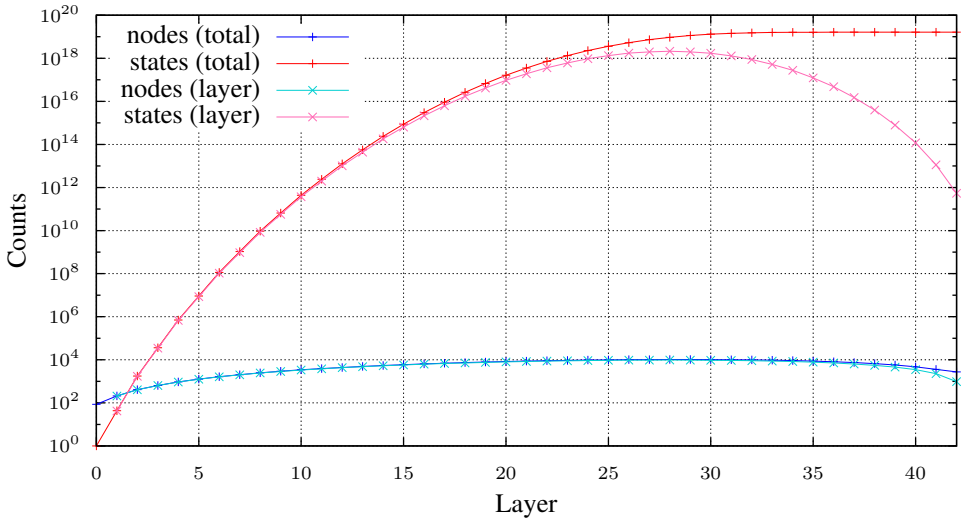
representation as well.[9] In those cases the representation of the transition relation might be overly complex. All moves depend on at least two cells; if we perform a capture two of the relevant cells must be occupied, so that the BDD representation of the precondition of such a move will likely be of exponential size in the dimension of the board. If that can be proved it might be necessary to extend the classification to a three-dimensional table, one dimension for the complexity of the representation of the set of reachable states, one for the termination criterion and the third for the transition relation.

Using the same argumentation as above it seems likely that the termination criterion for CHESS at least for the case of a *check-mate*, will need an exponential sized BDD as well, as there we must be certain that the king is under attack and all cells adjacent to the king's are either occupied by own pieces or are under attack by the opponent.

[9] At least for CHESS this actually seems to be the case, according to work by Kristensen (2005) and Hurd (2005), who worked on the compression and verification of endgame databases in CHESS.



(a) Stopping at terminal states.



(b) Continuing after terminal states.

Figure 4.19: Comparison of the number of states and BDD nodes for different layers for a 7×6 board in the 4-in-a-row game. The variable ordering is according to the columns.

Table 4.2: Classification of some chosen benchmarks with respect to the BDD sizes.

	Polynomial Reachability	Exponential Reachability
Polynomial Termination Criterion	GRIPPER SOKOBAN	BLOCKSWORLD SLIDING TILES PUZZLE $n^2 - 1$-PUZZLE
Exponential Termination Criterion	CONNECT FOUR TIC-TAC-TOE GOMOKU	

Apart from filling the empty buckets of the classification it also remains future work to prove bounds for more benchmark problems and to find other general criteria that immediately decide whether some part of the problem can be represented by a BDD of polynomial or exponential size. Once we have such criteria, we might be able to automatically decide whether to prefer symbolic or explicit search.

Part II

Action Planning

Chapter 5

Introduction to Action Planning

Awaiting my last breath
the mirror calls my name
it's showing me the way
into the dark
the bridge appears
I jump into the dark side
and hear the voice it's cold as ice
"Welcome to reality"

Blind Guardian, *Lost in the Twilight Hall*
from the Album *Tales from the Twilight World*

Action planning is a special case of general problem solving, and as such it is an important part of research in artificial intelligence. The problem solvers (in this context called *planners*) need to find a solution for the problem at hand without interaction of a human being and also without prior knowledge of the given problem. Thus, the developer of such a planner cannot insert any domain-dependent improvements.

All the problems are specified in a common format, which the planners must analyze in order to understand the dynamics. Typical examples for such inputs are STRIPS (Stanford Research Institute Problem Solver) by Fikes and Nilsson (1971) and PDDL (Planning Domain Definition Language) by McDermott (1998). The latter allows for a more natural description of the desired problems, so that it is now the most commonly used language for describing planning problems, especially in the course of the International Planning Competition (IPC), which is held every second or third year since 1998.

These inputs correspond to an implicit description of a state space. Thus, a planning problem consists at least of the elements described in Definition 1.4 on page 4, though they are often called somewhat differently. In the most basic form we are given a set of fluents or *predicates* needed to describe a state, a number of operators (in the context of planning also called *actions*) to transform states to their successors, an initial state, and a set of terminal states (in planning typically called *goal* states). The planners are then supposed to find a solution (in this context called a *plan*), which corresponds to a sequence of actions that transforms the initial state to one of the goal states.

In action planning we distinguish several different subareas (called *tracks*). While this basic idea still holds in most of these various tracks, some are more complex by specifying further inputs or further restrictions for a plan.

In the following (Section 5.1) we will give some insight into a number of the different tracks that are often part of the planning competition as well as to PDDL (Section 5.2). In the following chapters we will provide algorithms for optimally solving problems from two of the tracks, namely classical planning (Chapter 6) and net-benefit planning (Chapter 7).

5.1 Planning Tracks

There are a number of different planning tracks, e. g., classical planning, over-subscription planning, net-benefit planning, non-deterministic planning, and temporal planning, to name but a few. The core idea is similar for all of these, though the descriptions as well as the plans—or *policies*, as the solution of a non-deterministic planning problem is typically called—can vary significantly. Furthermore, in most of these tracks we distinguish between optimal and satisficing planning. In the former we are interested in finding plans optimal according to some optimization criterion depending on the precise problem at hand, while in the latter we are interested in finding a legal, not necessarily optimal, plan, though it should be as good as possible according to the same optimization criterion given certain time and memory constraints.

In *classical planning* we are provided a number of actions, consisting of preconditions and effects. An action can be applied in a state if the precondition is satisfied. In that case, the effect is evaluated to update the state. The plan is a sequence of the available actions that, executed one after the other, transform the initial state to one of the goal states. The optimization criterion is the length of the plan, i. e., the number of actions it consists of (the fewer the better). If action costs are defined the total cost of the plan, i. e., the sum of the costs of all the actions within the plan, is to be minimized.

Over-subscription planning (or *preference planning*) allows for *preferences* or *soft goals*, i. e., we can specify which additional fluents we would like to achieve when a goal state is reached. The achieved preferences then give a certain reward (called *benefit* or *utility*). Here, we are typically interested in finding a plan to satisfy a goal that gives the maximal utility. If we also take the length of the plan or its total cost into account, we speak of *net-benefit planning*, where the *net-benefit*, which is the achieved utility minus the cost of the plan needed to achieve it, is to be maximized.

In case of *non-deterministic planning* we only know the possible effects that might occur when an action is applied, but not exactly which of these will actually happen. Thus, the solution here consists of finding a *policy*, which specifies the action to choose in every state that might be reached. Following Cimatti et al. (2003) we distinguish three kinds of policies. A *strong* one will enable us to reach a goal state, no matter what happens due to non-determinism. A *strong-cyclic* solution corresponds to a policy that allows us to reach a plan if we can wait for an unbounded number of steps, i. e., it might happen that due to non-determinism we end up running in circles, but in the limit we expect due to a fairness condition that the non-determinism will not always choose the effects in such a way that we remain on the circle and thus will be able to reach a goal state. Finally, a *weak* policy allows us to reach a goal state if the non-determinism does not play against us. Of course, not all problems allow for a strong or strong-cyclic solution. However, a weak policy should always be possible to find.

Adding a probability for the occurrence of any of the possible effects results in *probabilistic planning*, where we are interested in finding policies that reach a goal state with the highest probability possible. Uncertainty about the initial state, i. e., a number of possible initial states, is another form of non-determinism and introduced in *conformant planning*. Here, the planner aims at finding a policy that works for any of the possible initial states and reaches a goal state.

The idea of *temporal planning* is the possibility of executing some actions in parallel if they operate on disjoint resources. Furthermore, the actionsf have a certain duration and the aim is to find a solution that minimizes the *makespan*, which is the time needed from the initial state until the last action is finished.

In this work we are only interested in classical and net-benefit planning. For these, we provide approaches to find optimal plans using symbolic search.

5.2 The Planning Domain Definition Language PDDL

For automatic planners to be able to handle planning problems it is important to have a unified language. Currently, the Planning Domain Definition Language PDDL is the language of choice in most planning tracks. Starting with a brief overview of the history of PDDL (Section 5.2.1) we continue with an introduction to the structure of a typical PDDL description along the running example in Section 5.2.2.

5.2.1 The History of PDDL

Though PDDL is currently the language of choice in action planning, it is not the first approach to a unified language for describing planning problems. In this section we will briefly outline the history of such unified languages and also the evolution of PDDL.

STRIPS

One of the first approaches came with the *Stanford Research Institute Problem Solver* (STRIPS) by Fikes and Nilsson (1971), whose input language was later also called STRIPS.

A STRIPS description is essentially a four-tuple $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$ with $\mathcal{V}$ being a set of binary variables, $\mathcal{A}$ a set of actions, $\mathcal{I}$ the initial state and $\mathcal{G}$ a set of goal states. In STRIPS a state is described by the variables that are currently true, i. e., the closed world assumption holds, so that all variables in $\mathcal{V}$ that are not part of the state are assumed to be false.

The initial state description $\mathcal{I} \subseteq \mathcal{V}$ thus fully specifies the state where the planning process starts.

The goal states $\mathcal{G} \subseteq \mathcal{V}$ are described by a *partial state description*, so that the variables within $\mathcal{G}$ must be true in a given state for it to be a goal state, while the other variables may take any value, i. e., a state s is a goal state if $\mathcal{G} \subseteq s$.

An action $a \in \mathcal{A}$ is tuple $a = \langle pre_a, add_a, del_a \rangle$. The *precondition* $pre_a \subseteq \mathcal{V}$ is a partial state description similar to the goal states $\mathcal{G}$, so that an action a is applicable in the current state s if the precondition's variables are true in s (i. e., if $pre_a \subseteq s$). The *effects*, i. e., the changes to the current state, of an action a are described by add_a and del_a. The *add-effects* $add_a \subseteq \mathcal{V}$ contain all variables that will be made true after application of the action, the *delete-effects* $del_a \subseteq \mathcal{V}$ those that will be made false afterward. The frame, i. e., the set of variables that do not change, is not modeled explicitly. All variables not in $add_a \cup del_a$ retain their value. Let s be the current state with $pre_a \subseteq s$, so that action a is applicable. Then the successor state s' is specified by the set of variables $s' = (s \setminus del_a) \cup add_a$.

Even though STRIPS is a rather restricted approach the decision if a plan can be found for a given problem is PSPACE-complete (Bylander, 1994).

ADL

Another approach at a unified language is the *Action Description Language* (ADL) by Pednault (1989, 1994). Using ADL, planning problems can be described in a more concise way than with STRIPS.

There are a number of differences between ADL and STRIPS, starting at the way states are represented. While in STRIPS a state is a set of binary variables (those that are currently true), in ADL a state is fully described by binary *facts* and their values. Additionally, in ADL an open world is assumed so that any fact whose value is not explicitly stated has an unknown value (not false as in the case of STRIPS). Furthermore, the formulas in the preconditions and the description of the goal states can be more complex. While STRIPS supports only sets of variables (which can be translated to conjunctions of positive facts in ADL), here literals (positive and negative facts) and disjunctions are supported as well. Additionally, (existential and universal) quantifications can be used in the formulas. The effects only allow for conjunctions of literals as well as universal quantifications; a disjunction in an effect would mean that the outcome of an action is not entirely certain.

Another important extension is the use of *conditional effects*. A conditional effect can greatly decrease the size of a planning description as it can distinguish the effects an action has depending on the state it is applied in, i. e., a conditional effect can be seen as an action within an action. It consists of an *antecedent* (the precondition of the sub-action), which may be a Boolean formula in the same way a normal action's precondition is, and a *consequence*, which describes the additional effects that take place if the antecedent is satisfied in the current state.

Though the language seems more powerful, actually it is possible to translate planning problems described in ADL to ones described in STRIPS (Gazen and Knoblock, 1997; Hoffmann et al., 2006).

A first step in such a translation is to transform the description of the goal states $\mathcal{G}$ to an action with the precondition being $\mathcal{G}$ and the only effect the addition of a new variable $goal\text{-}achieved$ to the current state. Thus, the goal states are reached once the variable $goal\text{-}achieved$ is present in the current state.

The conditional effects are what make things most difficult. For a conditional effect in an action it is possible to generate two actions, one for the case that the conditional effect is applicable, one for the case that it is not. Thus, the first action's precondition is the conjunction of the original action's precondition with the conditional effect's antecedent, while the effects are the conjunction of the original effects with the consequence of the conditional effect. The second action's precondition is the conjunction of the original precondition with the negation of the conditional effect's antecedent, while the effects remain unchanged—if the conditional effect is not applicable in the current state then it will not change anything about that state. This process can be iterated until no conditional effects are present anymore in any action. For one action the resulting description contains a number of actions exponential in the number of conditional effects in the original action. Nebel (1999, 2000) proved that it is not possible to come up with a shorter STRIPS description.

The quantifiers can be expanded using the constants that might replace the quantified variables, so that they result in a number of disjunctions in case of existential quantifiers or conjunctions in case of universal quantifiers.

By calculating the disjunctive normal form (DNF) of an action's precondition the result is a disjunction of conjunctions of literals. This way it is possible to split the action into a number of smaller actions containing the same effects but different preconditions—one for each operand of the disjunction.

Finally, to remove the negations within the actions' preconditions it is best to establish negation normal form (NNF), which is immediate if in the last step the DNF was created. In NNF any negations appear only immediately in front of a fact. For such a negated fact fac an additional (positive) fact $not\text{-}fac$ is inserted to represent the negation of fac, so that it must always take the opposite value of fac. Thus, if fac holds in the initial state $\mathcal{I}$ $not\text{-}fac$ must not and vice versa. Also, whenever the value of fac is changed in an action, so must the value of $not\text{-}fac$, i. e., if fac is an add effect then $not\text{-}fac$ must be a delete effect and vice versa. Any appearance of the negated fac in a precondition is replaced by the (non-negated) $not\text{-}fac$.

All in all, as problems in ADL can be translated into equivalent problems in STRIPS the decision problem if a plan exists in an ADL description is also PSPACE-complete.

PDDL

Finally, the *Planning Domain Definition Language* (PDDL) was proposed by McDermott (1998) to be on the middle-ground between STRIPS and ADL. In a PDDL description it is possible to specify a number of requirements a planner must be able to handle in order to correctly work with this description. The requirements range from pure STRIPS to full ADL. Some of them are the following:

:strips Basic STRIPS add and delete effects (by means of a conjunction of positive and negative facts) should be used.

:disjunctive-preconditions Preconditions can contain disjunctions.

:existential-preconditions Preconditions can contain existential quantifications.

:universal-preconditions Preconditions can contain universal quantifications.

:quantified-preconditions Preconditions may contain any kind (existential or universal) of quantifications.

:conditional-effect The effects may contain conditional effects.

:adl A meta-requirement that encapsulates all of the above.

A PDDL description consists of two parts (typically stored in separate files), the domain description and the problem description. The domain description specifies the general behavior, while the problem description determines the precise objects that can appear in the problem at hand. In more detail, the domain description is mainly concerned with the actions, in many cases independent of the actual facts present. The problem description is concerned with the problem-dependent initial state and goal states as well as the actual objects—constants for replacing possible variables within the action descriptions—that must be handled.

In many cases the domain description does not contain any specific facts but is modeled using a number of variables. When bringing a domain and a problem description together these variables can be replaced by the actual objects, i. e., the planning description can be *instantiated* (or *grounded*) for easier reasoning (see, e. g., Edelkamp and Helmert, 1999; Helmert, 2008). Using this instantiation and the rules for translating ADL to STRIPS from the previous section it is possible to transform a PDDL description so that it equals a STRIPS description.

PDDL was extended to handle *numerical variables* and a concurrent execution of actions by Fox and Long (2003) resulting in PDDL 2.1, which is divided into three different levels.

Classical Planning The idea behind classical planning is essentially the same as in the original PDDL specification, i. e., states are considered as being the conjunction of positive or negative facts and a plan consists of a sequence of actions that transforms the initial state to one of the goal states.

Metric Planning In metric planning numbers are added in form of the real-valued numerical variables. The classical planning state is extended to allow for simple arithmetic expressions using these variables. Such expressions might be the addition, subtraction or assignment of a constant or another numerical variable.

Temporal Planning To handle temporal planning problems, *durative actions* were added. These actions contain a duration, but may also have certain conditions holding at specified points of time. Especially, the preconditions are transformed to conditions at some time point. If this time point is at the start of the action it corresponds to a classical precondition.

The effects might also take place at different points of time. So it might be possible to create actions that result in some effect immediately after the action has been started, but also in other effects that take place sometime later, e. g., once the action has ended.

The goal is to find a plan that executes the actions in such an order (possibly some even in parallel), so that a goal state is reached after the shortest possible time.

After that PDDL was further extended. In PDDL 2.2 (Hoffmann and Edelkamp, 2005) so called *derived predicates* were integrated. These are great for decreasing the size of a description, as they might be seen as utility functions. Each derived predicate depends on other facts or derived predicates. It is simply a short way to describe some aspects of the planning state, somewhat similar to functions in programming, which encapsulate code that can be used in several locations within other functions.

Another extension introduced with PDDL 2.2 were *timed initial literals*. These are facts that are set to a certain value (true or false) at some predetermined point of time. This is done automatically without calling any action. Of course, this extension is only needed for temporal planning.

The next step in the development of PDDL was the proposal of PDDL 3.0 (Gerevini and Long, 2005, 2006). The new version now also included the definition of *constraints* and *preferences*. The first ones are used to constraint the set of plans (using a temporal logic to describe them) while the latter ones are used to express soft constraints. These constraints are not mandatory to be satisfied, but if they are not the plan's quality will decrease.

Finally, in the context of the sixth International Planning Competition (IPC) 2008 PDDL 3.1[10] was proposed. It allows to define *functional variables*, which can no longer only be true or false (classical planning) or numerical (metric planning), but can be of any finite domain. Especially it is possible to model variables whose domains are other objects of the domain. In principle, this is designed according to the STRIPS extension allowing for functional state variables (Geffner, 2000).

5.2.2 Structure of a PDDL Description

In the following we will show how a PDDL description is organized. For this we will show the important parts of the domain and problem descriptions for our running example introduced in Section 1.6. In this domain we are confronted with a desert, consisting of a number of locations between which the gunslinger

[10] `http://ipc.informatik.uni-freiburg.de/PddlExtension`; the BNF description of it is due to Kovacs (2011)

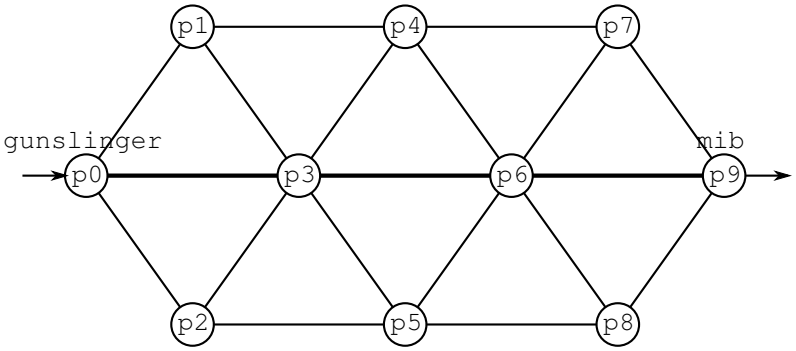


Figure 5.1: The running example in classical planning with unit-cost actions. The optimal solution is denoted by the thicker edges.

can move. The goal for the gunslinger is to reach the man in black at the end of the desert (position `p9`), who cannot move in this setting.

We will discuss the structure of the domain and the problem along this example in the context of classical planning with unit-cost actions. The changes needed for classical planning with general action costs as well as for net-benefit planning will be described in the corresponding chapters. The complete PDDL descriptions for all three can be found in Appendix A.

Here we assume the situation as in Figure 5.1. We have the same desert as before with locations `p0` to `p9`, some of which are connected, so that the protagonist can move from one location to an adjacent one. In this case the protagonist is the `gunslinger`, which starts at location `p0`, the entrance to the desert. As classical planning does not use an opponent, the man in black (`mib`) is static at location `p9`, the exit from the desert. The goal is to move the gunslinger from `p0` to the location where the man in black hides and to catch him there using as few actions as possible.

The domain description (cf. Figure 5.2) stores all the information concerning the predicates as well as the actions. It starts with the specification of the domain's name—`(domain <domain-name>)`.

Next, some requirements are stated. The first one, `:strips`, means that we can use only conjunctions in the actions' preconditions as well as the goal description and the effects are in STRIPS style, i. e., a set of add-and delete-effects. The `:typing` requirement means that we can have different types, so that certain objects can replace only those variables of a predicate that have the same type.

After these the types are specified. In this case we use two types, one—`person`—to denote the people running through the desert, the second one—`location`—to denote the possible locations within the desert.

Following these, some constants may be specified. These essentially can be used to replace variables of the same type. Typically, we need constants (or objects) in the problem description to specify a certain instance of the domain at hand. Here, we know that no matter what the desert might look like we have only two people, the `gunslinger` and the man in black (`mib`).

Then, a list of all predicates in the domain is given, along with their (typed) parameters. The parameters are given in form of variables, which in PDDL are always denoted by the prefix `?`. In this example we have three different predicates—the two-ary `position`, which denotes the location the corresponding person is currently in, the two-ary `adjacent` relation, which denotes which locations are next to each other and can thus be reached by a single move, and the 0-ary `caughtMIB`, which denotes whether the man in black has been caught or not.

Finally, the list of all the actions is given. Each action consists of the action's name, a list of (typed) parameters, the precondition, and the effect. While in principle the precondition might be an arbitrary formula, given the necessary requirements, in this example we are only allowed to use conjunctions of predicates. The effects are STRIPS-like, i. e., they represent the add-and delete-lists by being simple conjunctions of positive (add) or negative (delete) predicates.

```
(define (domain desertClassic)
 (:requirements :typing)
 (:types person location)
 (:constants
    gunslinger mib - person
 )
 (:predicates
    (position ?p - person ?l - location)
    (adjacent ?l1 ?l2 - location)
    (caughtMIB)
 )

 (:action move
    :parameters (?l1 ?l2 - location)
    :precondition
    (and
        (position gunslinger ?l1)
        (adjacent ?l1 ?l2)
    )
    :effect
    (and
        (not (position gunslinger ?l1))
        (position gunslinger ?l2)
    )
 )

 (:action catchMIB
    :parameters (?l - location)
    :precondition
    (and
        (position gunslinger ?l)
        (position mib ?l)
    )
    :effect
    (caughtMIB)
 )
)
```

Figure 5.2: The domain description for the classical planning version with unit-cost actions of the running example.

The first action—`move`—allows the gunslinger to move from one location to an adjacent one. If the gunslinger is currently at some location `?l1` he can move to an adjacent location `?l2`. If he does, the effect simply states that he is no longer at location `?l1` but rather at location `?l2`.

The second action—`catchMIB`—allows the gunslinger to actually catch the man in black. He can do this if he has reached a location `?l`, which must be the same as the location where the man in black resides. If he decides to catch the man in black, the predicate `caughtMIB` is set to true, while no other predicates are affected.

The problem description (cf. Figure 5.3) captures all the stuff needed to generate one instance of the domain at hand, i. e., the objects to replace the variables from the definitions in the domain description, the initial state, and the specification of the goal states.

```
(define (problem desertClassicProb)
 (:domain desertClassic)
 (:objects
    p0 p1 p2 p3 p4 p5 p6 p7 p8 p9 - location
 )

 (:init
    (position gunslinger p0)
    (position mib p9)
    (adjacent p0 p1)   (adjacent p1 p0)
    (adjacent p0 p2)   (adjacent p2 p0)
    (adjacent p0 p3)   (adjacent p3 p0)
    (adjacent p1 p3)   (adjacent p3 p1)
    (adjacent p1 p4)   (adjacent p4 p1)
    (adjacent p2 p3)   (adjacent p3 p2)
    (adjacent p2 p5)   (adjacent p5 p2)
    (adjacent p3 p4)   (adjacent p4 p3)
    (adjacent p3 p5)   (adjacent p5 p3)
    (adjacent p3 p6)   (adjacent p6 p3)
    (adjacent p4 p6)   (adjacent p6 p4)
    (adjacent p4 p7)   (adjacent p7 p4)
    (adjacent p5 p6)   (adjacent p6 p5)
    (adjacent p5 p8)   (adjacent p8 p5)
    (adjacent p6 p7)   (adjacent p7 p6)
    (adjacent p6 p8)   (adjacent p8 p6)
    (adjacent p6 p9)   (adjacent p9 p6)
    (adjacent p7 p9)   (adjacent p9 p7)
    (adjacent p8 p9)   (adjacent p9 p8)
 )

 (:goal
    (caughtMIB)
 )
)
```

Figure 5.3: The problem description for the classical planning version with unit-cost actions of the running example.

It starts by specifying a name for the problem—`(problem <problem-name>)`—and a reference to the corresponding domain—`(:domain <domain-name>)`. The domain name mentioned here must equal the one in the domain description.

Next, some objects may be defined. These objects are similar to the constants of the domain description in that they can be inserted as the arguments of certain predicates, but they may not be used in the domain description, only in the problem description, as they build the basis for this specific instance. If the corresponding requirement is set in the domain description, these objects may be typed. In the example the only objects are the ten locations `p0` to `p9`.

After that, the initial state is specified. In this, no variables are allowed. Also, a closed world assumption is made, i. e., any predicate not listed here is supposed to be false in the initial state. Apart from the positions of the gunslinger and the man in black it also captures all the information concerning the adjacency of the locations.

Finally, the goal is specified. In this example, it is only a single predicate, denoting that the man in black has been caught. In principle, PDDL supports arbitrary formulas as well, but the corresponding requirements must be set in the domain description.

In classical planning we are typically interested in finding a plan which contains as few actions as possible. Thus, in this case we must find the shortest path from `p0` to `p9` and then immediately catch the man in black. Though a plan bringing the gunslinger from `p0` to `p9`, moving it further to `p7` and back to `p9` before finally catching the man in black is valid, it is clearly not the optimal one and thus the other one should be generated.

Here, the shortest path is simply following the straight line from the left to the right. In other words, the optimal plan is

```
0: (move p0 p3)
1: (move p3 p6)
2: (move p6 p9)
3: (catchMIB p9)
```

which we have also depicted by the thicker edges in Figure 5.1.

Chapter 6

Classical Planning

I love it when a plan comes together.

Col. John "Hannibal" Smith in *The A-Team*

In *classical planning* we are confronted with structurally rather simple planning problems. All we get are descriptions of the initial state, the goal states and the available actions. Based on these we want to find a *plan* containing as few actions as possible (in case of unit-cost actions, or if no costs are specified at all) or whose *total cost* is as small as possible (in case of more general action costs) (see, e. g., Russell and Norvig, 2010; Edelkamp and Schrödl, 2012).

Definition 6.1 (Classical Planning Problem)**.** *A* classical planning problem *is described by a tuple* $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ *with* $\mathcal{V}$ *being the variables that build a state*[11]*,* $\mathcal{A}$ *a set of actions consisting of precondition and effects (i. e., an action* $a \in \mathcal{A}$ *is given by a pair* $\langle pre_a, eff_a \rangle$*),* $\mathcal{I}$ *an initial state,* $\mathcal{G}$ *a set of goal states, and* $\mathcal{C} : \mathcal{A} \mapsto \mathbb{N}_0^+$ *a cost function specifying a certain non-negative integral cost for each action in* $\mathcal{A}$*.*[12] *In some domains* $\mathcal{C}$ *might be omitted. In this case all actions are supposed to have a unit cost of* 1*. Thus, even when no action costs are specified we speak of* unit-cost actions*, whereas we speak of* general action costs *if at least two actions have different costs.*

While the PDDL specification allows only for binary variables, it is possible to find sets of mutually exclusive variables, i. e., sets of binary variables of which at most one can hold at any time. Thus, $\mathcal{V}$ can also be viewed as a set of multi-valued variables. This setting is often refered to as *SAS*$^+$ *planning* (Bäckström and Nebel, 1995).

Definition 6.2 (Plan)**.** *A* plan *is a sequence of actions* $\pi = (A_1, A_2, \ldots, A_m) \in \mathcal{A}^m$ *whose application, starting at* $\mathcal{I}$*, results in a goal state, i. e.,* $A_m(A_{m-1}(\ldots A_1(\mathcal{I})\ldots)) \in \mathcal{G}$*.*

Definition 6.3 (Cost of a Plan)**.** *The* total cost $\mathcal{C}(\pi)$ *of a plan* $\pi = (A_1, \ldots, A_n)$ *is the sum of the costs of all actions within* π*, i. e.,* $\mathcal{C}(\pi) := \mathcal{C}(A_1) + \mathcal{C}(A_2) + \cdots + \mathcal{C}(A_n)$*.*

Note that in case of unit-cost actions the cost of the plan actually corresponds to the number of actions within the plan.

Definition 6.4 (Optimal Plan)**.** *A plan* π *is called* optimal*, if there is no plan* π' *with* $\mathcal{C}(\pi') < \mathcal{C}(\pi)$*.*

We can model a classical planning problem as a directed graph with nodes u corresponding to the states and edges $e = (u, v)$ corresponding to the actions leading from u to v. One of those nodes, the root node, corresponds to the initial state and a subset of them, the terminal nodes, to the goal states. A plan then corresponds to a path from the root to a terminal node. If we are confronted with general action costs, these

[11] In case of PDDL these are given in terms of the predicates as well as the constants and objects defined to replace the variables within the predicates.

[12] In the actual planning descriptions the costs are typically non-negative rational numbers, which we can scale to integer values.

Algorithm 6.1: Symbolic Breadth-First Search

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$.
Output: Step-optimal plan or "no plan".

```
1  reach_0 ← I                                          // First layer contains only I.
2  closed ← ⊥                                           // So far, no states expanded.
3  layer ← 0                                            // Initial layer's index is 0.
4  while reach_layer ∧ G = ⊥ do                         // Repeat until goal state found.
5  |  reach_layer ← reach_layer ∧ ¬closed               // Before expansion, remove states already expanded.
6  |  if reach_layer = ⊥ then return "no plan"          // If no new states left to expand, return "no plan".
7  |  closed ← closed ∨ reach_layer                     // Add new states to set of all expanded states.
8  |  reach_layer+1 ← image(reach_layer)                // Calculate successor states.
9  |  layer ← layer + 1                                 // Increase layer's index for next iteration.
10 reach_layer ← reach_layer ∧ G                        // Retain only reached goal states in final layer.
11 return RetrieveBFS(P, reach, layer)                  // Retrieve plan and return it.
```

can be modeled by inserting edge weights, so that the cost of a plan in this scenario equals the sum of the weights of the edges taken by the corresponding path, while in case of unit-cost actions the cost corresponds to the length of the path. An optimal plan is a path with minimal total weight (or length).

In this chapter we will show how to optimally solve classical planning problems by symbolic search algorithms. In Section 6.1 we introduce algorithms for solving problems with unit-cost actions, while in Section 6.2 we deal with general action costs. For the latter we typically need some heuristic to provide an estimate on the minimal cost required to reach a goal state from each reachable state. We briefly introduce some of the most commonly used heuristics as well as the one we use, namely pattern databases, in Section 6.3. Afterward we describe how we implemented our planner, called GAMER, along with a number of improvements to the algorithms (Section 6.4) before we provide some experimental results, some of those of the International Planning Competitions in 2008 and 2011, some for evaluating our improvements (Section 6.5).

6.1 Step-Optimal Classical Planning

We start with rather simple symbolic algorithms that solve classical planning problems with unit-cost actions optimally. In other words, they find a plan containing the minimal number of actions.

6.1.1 Symbolic Breadth-First Search

There are several approaches to find step-optimal plans, e. g., breadth-first search (BFS), depth-first search (DFS), and iterative-deepening depth-first search (ID-DFS) to name but a few. We are only interested in algorithms efficiently utilizing BDDs, so the easiest approach is to perform a BFS starting at the initial state $\mathcal{I}$ and stopping once a goal state $g \in \mathcal{G}$ is found.

As we have seen in Chapter 3, in symbolic search such a BFS is easy to implement. All we need to do is iterate the image calculations until the BDD representing the current BFS layer contains at least one goal state. To be able to retrieve a plan we store all BFS layers in separate BDDs $reach_{layer}$ (cf. Algorithm 6.1). In order to guarantee termination in case of no existing plan we store the set of all states expanded so far in the BDD $closed$, so that we can stop the search when we have found all states without ever reaching a goal state, i. e., when the layer to be expanded next contains only states already expanded before. In that case we return the text "no plan".

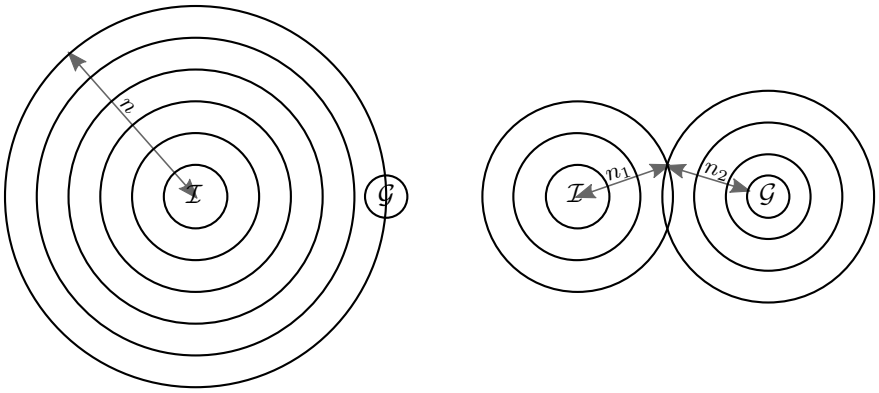
Retrieval of a Plan

Given the BDDs for all the BFS layers, the last of which contains the first reached goal states, and the index of the last layer n, we want to compute an optimal plan containing n actions. In order to determine this

Algorithm 6.2: Retrieval of a Plan for Symbolic BFS ($RetrieveBFS$)

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$.
Input: Set of BDDs $reach$ representing the BFS layers.
Input: Index n of layer containing goal states.
Output: Step-optimal plan π.

```
1  π ← ()                                                  // Plan is initially empty.
2  current ← reach_n                                       // Start with the states in the final layer.
3  for layer ← n to 1 do                                   // Iterate through all layers, in reverse order.
4      for all a ∈ |A| do                                  // Iterate through all actions.
5          pred ← pre-image_a(current)                     // Calculate predecessors of current states.
6          if pred ∧ reach_{layer−1} ≠ ⊥ then               // If some predecessors are within previous layer...
7              current ← pred ∧ reach_{layer−1}            // Retain only predecessor states also in previous layer.
8              add action a to the front of π              // Store current action in the plan.
9              break                                       // Ignore remaining actions.
10 return π                                                // Return the generated plan.
```



(a) Unidirectional. (b) Bidirectional.

Figure 6.1: The search spaces of uni- and bidirectional BFS.

plan we must know which actions to take, so that the normal pre-image operator cannot be used, because that operates over all actions. Instead we apply the single-operator pre-image operator $pre\text{-}image_a$ (cf. Definition 3.7 on page 21) for each action $a \in \mathcal{A}$.

The pseudo-code for the retrieval is provided in Algorithm 6.2. We start at the reached goal states in layer n and apply the single-operator pre-image operator for one action after the other. Once one such pre-image results in a set of predecessor states of which some are part of the previous BFS layer reached in forward direction (line 6) we use those as the new starting point and switch to the previous layer. We repeat these steps until finally we reach the initial state (the only one in layer 0). Store all the found actions in reverse order results in a plan of optimal length.

6.1.2 Symbolic Bidirectional Breadth-First Search

During the experiments with several of the older IPC domains we found some for which forward search is faster than backward search, others where both are of similar speed, and some for which backward search is the faster approach. To automatically decide whether to prefer forward or backward search we also implemented a bidirectional BFS. In it we have two search frontiers, one starting at the initial state $\mathcal{I}$, the other one starting at the goal states $\mathcal{G}$. These frontiers are extended by using the image and the pre-image operator, respectively, until they overlap (cf. Figure 6.1).

Algorithm 6.3: Symbolic Bidirectional Breadth-First Search

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$.
Output: Step-optimal plan or "no plan".

1 $freach_0 \leftarrow \mathcal{I}$ // First (forward) layer contains only $\mathcal{I}$.
2 $breach_0 \leftarrow \mathcal{G}$ // First backward layer contains all goal states $\mathcal{G}$.
3 $fclosed \leftarrow bclosed \leftarrow \bot$ // So far, no states expanded in either direction.
4 $flayer \leftarrow blayer \leftarrow 0$ // Indices of forward and backward layers, both starting at 0.
5 $ftime \leftarrow btime \leftarrow 0$ // Forward and backward runtime initialized to 0.
6 **while** $freach_{flayer} \wedge breach_{blayer} = \bot$ **do** // Repeat until forward and backward layers overlap.
7 **if** $ftime \leq btime$ **then** // Perform forward step.
8 perform one step of Symbolic BFS in forward direction. // Lines 5 to 9 in Algorithm 6.1.
9 update $ftime$ // Store time for this forward step.
10 **else** // Perform backward step.
11 perform one step of Symbolic BFS in backward direction. // Lines 5 to 9 in Algorithm 6.1.
12 update $btime$ // Store time for this backward step.
13 $freach_{flayer} \leftarrow breach_{blayer} \leftarrow freach_{flayer} \wedge breach_{blayer}$ // Retain only states from the // intersection of forward and backward frontier in the final layers.
14 **return** $RetrieveBBFS\,(\mathcal{P}, freach, flayer, breach, blayer)$ // Retrieve plan and return it.

In the pseudocode (cf. Algorithm 6.3) we keep track of the BFS layers, the set of expanded states, the index of the current layer and the time of the last step in forward direction ($freach$, $fclosed$, $flayer$, and $ftime$) as well as in backward direction ($breach$, $bclosed$, $blayer$, and $btime$). Every step corresponds to a step in unidirectional BFS (lines 5 to 9 in Algorithm 6.1), but in backward direction we must use the pre-image instead of the image operator. After each step we compare the runtimes for the last forward and backward steps and continue in the direction where the last step was performed faster. Thus, for problems that are easier searched in forward direction the search will be performed mostly forwards and vice versa.

In the bidirectional case we will typically expand a smaller (total) number of states, as we can stop when the two search frontiers overlap. In most cases each frontier is a lot smaller than the largest unidirectional one, so that we end up with a shorter total running time, though this is not necessarily the case. As a counter-example take a problem that is hard in backward direction (e. g., SOKOBAN). Though we mostly perform forward search, every once in a while we will also perform a backward step. The last of these backward steps might very well dominate the total runtime, so that the bidirectional search might actually take longer than pure forward search.

Retrieval of a Plan

For the bidirectional case the solution reconstruction is divided into two parts (cf. Algorithm 6.4). At first, it handles the layers generated in forward direction as shown in Algorithm 6.2. Afterward, it starts at the initial state and applies all the returned actions, resulting in the one state $inter$ of the intersection between the forward and backward layers that is reached using these actions. Then the solution reconstruction for the backward layers starts. It looks the same only that it is performed in forward direction starting at $inter$. In the end, both partial plans are concatenated and the resulting overall plan is returned.

6.2 Cost-Optimal Classical Planning

Optimally solving planning problems with general action costs requires more sophisticated algorithms. An optimal plan is no longer necessarily one of minimal length but rather one with the minimal total cost, i. e., the sum of the costs of all actions within the plan must be minimal.

As we have pointed out earlier, in planning the full state space can be seen as a graph $G = (V, E, w)$, with the states being the nodes V of the graph and the actions the connecting edges E, though given in an implicit fashion. An edge's weight w is thus the corresponding action's cost. Dijkstra's single-source

Algorithm 6.4: Retrieval of Plan for Symbolic Bidirectional BFS (*RetrieveBBFS*)

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$.
Input: Set of BDDs $freach$ representing the BFS layers found in forward direction.
Input: Index n_1 of forward layer containing intersection of the two frontiers.
Input: Set of BDDs $breach$ representing the BFS layers found in backward direction.
Input: Index n_2 of backward layer containing intersection of the two frontiers.
Output: Step-optimal plan π.

```
perform RetrieveBFS (P, freach, n_1) in default direction      // Generate plan for forward layers.
result: π_1 = (A_1, ..., A_{n_1})
inter ← I                                                      // Start finding intermediate state from I.
for step ← 1 to n_1 do inter ← image_{A_step} (inter)          // Find intermediate state after performing π_1.
perform RetrieveBFS (P, breach, n_2) in reverse direction      // Generate plan for backward layers.
result: π_2 = (A_{n_1+1}, ..., A_{n_1+n_2})
return π ← (A_1, ..., A_{n_1}, A_{n_1+1}, ..., A_{n_1+n_2})    // Return combined plans.
```

shortest-paths search (1959) is one of the classical algorithms used to search for shortest paths in graphs with edge weights. We will introduce it in more detail and provide a symbolic version of it in Section 6.2.2.

The main problem of Dijkstra's algorithm is that it is blind, i. e., it searches in all directions at once. To overcome this problem heuristics can be used, which provide an estimate to the distance to a goal state and thus can be used to direct the search, so that fewer states must be expanded in order to find an optimal plan. An extension of Dijkstra's algorithm that makes use of heuristic estimates is A* search (Hart et al., 1968). In Section 6.2.3 we will introduce a symbolic version of A*.

A* can only work well if we are using a good heuristic, which guides the search in a sensible way. In Section 6.3 we will briefly introduce to a number of heuristics that are often used in action planning, and provide more details to the one we actually use, namely (symbolic) pattern databases (Culberson and Schaeffer, 1998; Edelkamp, 2002b) in Section 6.3.1 and (symbolic) partial pattern databases (Anderson et al., 2007; Edelkamp and Kissmann, 2008d) in Section 6.3.2.

However, before we introduce these possibilities to solve classical planning problems cost-optimally we first of all extend the running example from Sections 1.6 and 5.2.2 to this setting in Section 6.2.1 and show how to adapt the PDDL description accordingly.

6.2.1 Running Example with General Action Costs

In order to extend the running example of the gunslinger following the man in black through the desert to the setting of general action costs all we need to do is to specify the cost of each action. If some action is defined without a cost it is treated as a zero-cost action, i. e., an action of cost zero, which does not change the value of a plan. In this example we assume the costs of the moves from one location to an adjacent one to be as according to the graphical representation in Figure 6.2.

To extend the PDDL description given in Figures 5.2 and 5.3 in Section 5.2.2 to the setting with general action costs we need an additional requirement—`:action-costs`. Furthermore, apart from the objects and the predicates we must define some functions in the domain description, in this case the one for calculating the total cost of a plan—`total-cost`—and also one for specifying the cost for each move—`moveCost`:

```
(:functions
    (total-cost) - number
    (moveCost ?l1 ?l2 - location) - number
)
```

The latter we need to keep the domain description independent of the actual instance, as we can define the move costs in the initial state. Both functions are of type `number`, so that we can use arithmetic operators such as comparisons or assignments. In this case, we must only specify the initial values in the initial state, e. g., `(= (total-cost) 0)` (and similarly specify the costs for all the edges by using the

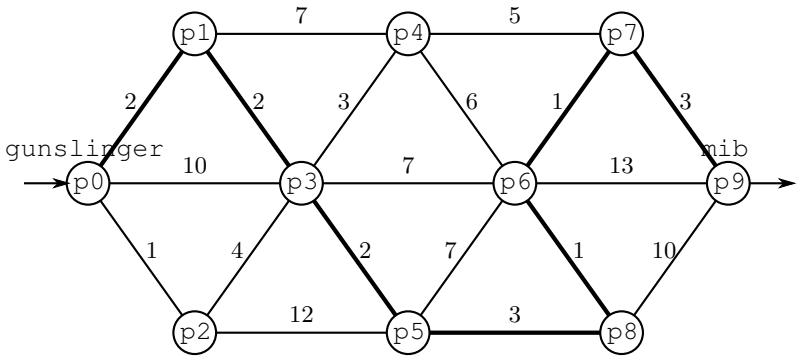


Figure 6.2: The running example in classical planning with general action costs. The optimal solution is denoted by the thicker edges.

`moveCost` function), and the increase of the `total-cost` in the effect of each action, e. g., `(increase (total-cost) (moveCost ?l1 ?l2))`. In this example, we do not specify any cost for catching the man in black, so that this action can be performed without influencing the overall cost of the plan.

Apart from the initialization of the total action costs and the move costs for the adjacent locations we also need a metric in the problem description. In this case, it only says to minimize the total action costs:

```
(:metric minimize (total-cost))
```

The full PDDL description can be found in the appendix of this thesis (Section A.2).

In this setting an optimal solution has a total cost of 14 and uses a total of eight actions:

```
0: (move p0 p1)    [2]
1: (move p1 p3)    [2]
2: (move p3 p5)    [2]
3: (move p5 p8)    [3]
4: (move p8 p6)    [1]
5: (move p6 p7)    [1]
6: (move p7 p9)    [3]
7: (catchMIB p9)   [0]
```

The square brackets denote the costs of the actions. This solution is also depicted in Figure 6.2 by use of the thicker edges.

6.2.2 Symbolic Dijkstra Search

The general idea of Dijkstra's single-source shortest-paths algorithm (1959) in the setting of graph search is to find each node's minimal distance g from the root node. It starts with the root node and then always expands a node u with minimal distance $g(u)$ from the root among all unexpanded nodes. After expansion the distances of the successors of u are updated in order to reflect that a path along that node might be cheaper than another path along which it was reached before, i. e., for each v with $(u, v) \in E$ the distance from the root node is set to

$$g(v) = \min\{g(v), g(u) + w(u, v)\}, \tag{6.1}$$

with $w(u, v)$ being the weight of the edge (u, v).

The algorithm is usually performed by using a *priority queue*. A priority queue stores data along with a certain priority. In this context we need two functions, $deleteMin$ and $decreaseKey$. The former allows us to remove the date with minimal priority, the latter lets us decrease the priority of the specified date. In case of Dijkstra's algorithm, the priority queue stores the nodes along with their distance to the root node, interpreted as the priority. Initially, the distance of the root node is 0 and that of the other nodes ∞.

The node u to be expanded next is the one with minimal distance $g(u)$, which is returned by calling the priority queue's $deleteMin$ function. The priorities of all nodes that can be reached from u must be

Algorithm 6.5: Symbolic Version of Dijkstra's Algorithm

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$.
Output: Cost-optimal plan or "no plan".

$open_0 \leftarrow \mathcal{I}$ // $\mathcal{I}$'s total cost is 0.
$closed \leftarrow \bot$ // So far, no states expanded.
$g \leftarrow 0$ // Current total cost is 0.
while $open_g \wedge \mathcal{G} = \bot$ **do** // Repeat until a goal state is found.
  $open_g \leftarrow open_g \wedge \neg closed$ // Remove all states already expanded.
  if $\bigvee_{i=g-\mathcal{C}_{max}+1}^{g} open_i = \bot$ **then** // If $\mathcal{C}_{max}$ consecutive layers are empty...
    return "no plan" // return "no plan".
  if $open_g \neq \bot$ **then** // If unexpanded states with total cost g exist...
    if zero-cost actions present **then** perform BFS in $open_g$ using only zero-cost actions
    $closed \leftarrow closed \vee open_g$ // Add states to be expanded next to already expanded states.
    for $c \leftarrow 1$ **to** $\mathcal{C}_{max}$ **do** // For all (non-zero) action costs c ...
      $open_{g+c} \leftarrow open_{g+c} \vee image_c\left(open_g\right)$ // Find successors and add them to the priority queue.
  $g \leftarrow g + 1$ // Iterate to next total cost.
$open_g \leftarrow open_g \wedge \mathcal{G}$ // Retain only goal states in final bucket.
return $RetrieveDijkstra\left(\mathcal{P}, open, g\right)$ // Retrieve plan and return it.

updated using the $decreaseKey$ function according to equation 6.1 in order to reflect the new knowledge of the weight of a path from the root via u to that state.

In our setting we are confronted with an implicit search problem, so that we cannot insert all reachable states into the priority queue from the start. Instead we use two structures, one being the priority queue $open$, which stores the already generated but not yet expanded states, the other the set of all states already expanded ($closed$). As all costs are non-negative and thus the priorities of the states we expand are monotonically increasing we can be certain that a state that has been expanded once will not be found on another path with a lower cost. After generation of a successor state we check if it was already expanded, i. e., if it is within the closed list. If it is we can discard the state, otherwise we check for it in the priority queue. If it is present there we might need to update its priority, otherwise we insert it with the corresponding priority.

In the symbolic version of Dijkstra's algorithm (cf. Algorithm 6.5) we need a way to access the states having a certain total cost from $\mathcal{I}$. This might be done by extending the states by their total costs, i. e., by adding $\lceil \log n + 1 \rceil$ additional binary variables if we can be sure that the maximal total cost ever achievable is n, so that the total costs are stored in the BDD representing the set of reachable states. Another possibility is to partition the priority queue into buckets, resulting in a simple list (Dial, 1969) in which we store all the states that have been reached so far in the bucket representing the total cost from $\mathcal{I}$ that was needed to reach them.

We initialize the priority queue $open$ to contain only $\mathcal{I}$ with a total cost of 0. Starting there, we calculate all the successors having a cost of $c \in \{1, \ldots, \mathcal{C}_{max}\}$, with $\mathcal{C}_{max}$ being the greatest action cost in the problem at hand. In contrast to the full image or the $image_a$ operator used in the algorithms for retrieving a plan, which contains only the transition relation for one action $a \in \mathcal{A}$, the equal-cost image operator $image_c$ contains all the transition relations for actions with an equal cost of c.

Definition 6.5 (Equal-Cost Image Operator)**.** *Given a set of states $current$, specified in the variable set $\mathcal{V}$, the successor states resulting from the application of the actions $a \in \mathcal{A}$ with the same cost c are calculated by the* equal-cost image operator

$$
\begin{aligned}
image_c\left(current\right) &:= \bigvee_{a \in \{a \mid a \in \mathcal{A} \wedge \mathcal{C}(a) = c\}} \left(\exists \mathcal{V} \,.\, \mathcal{TR}_a\left(\mathcal{V}, \mathcal{V}'\right) \wedge current\left(\mathcal{V}\right)\right)\left[\mathcal{V}' \rightarrow \mathcal{V}\right] \\
&= \bigvee_{a \in \{a \mid a \in \mathcal{A} \wedge \mathcal{C}(a) = c\}} image_a\left(current\right)
\end{aligned}
$$

Algorithm 6.6: Retrieval of a Plan for Symbolic Dijkstra ($RetrieveDijkstra$)

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$.
Input: Set of BDDs $open$ representing all states reached in forward direction in their corresponding cost buckets.
Input: Total cost g of found goal states.
Output: Cost-optimal plan π.

```
1  π ← ()                                                  // Initial plan is empty.
2  current ← open_g                                        // Start with final bucket.
3  while I ∧ current = ⊥ do                                // Repeat, until I is reached.
4      if zero-cost actions present then retrieve BFS plan in open_g using only zero-cost actions
5      for c ← C_max to 1 do                               // For all possible costs c...
6          for all a ∈ A with C(a) = c do                  // For all actions a with cost c...
7              pred ← pre-image_a(current)                 // Calculate predecessors using action a.
8              if open_{g−c} ∧ pred ≠ ⊥ then               // If predecessor has total cost of g − c...
9                  current ← open_{g−c} ∧ pred             // Take predecessors as current states.
10                 add action a to front of plan π         // Extend plan by found action.
11                 g ← g − c                               // Update index of cost layer for next iteration.
12                 goto line 3                             // Continue in predecessor's bucket.
13 return π                                                // Return the complete plan.
```

We add the successors to the priority queue (line 12) into their corresponding buckets by calculating the disjunction with the BDD representing the states already in that bucket. In case some bucket does not yet exist in the priority queue, instead of calculating the disjunction with the BDD stored at that bucket we need to create the required bucket and insert a new BDD consisting only of the successors.

For the closed list a single BDD $closed$ is sufficient. It is initially empty (line 2) and new states are inserted when they are expanded (line 10). Before actually expanding some bucket the already expanded states are removed (line 5) in order to prevent expanding any state twice.

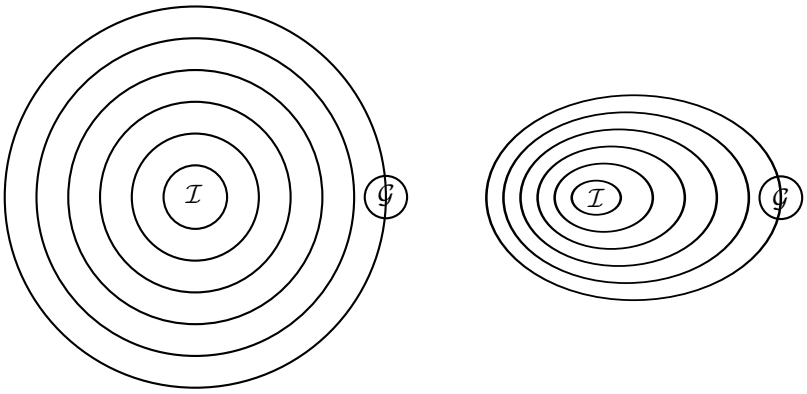
The search ends when a goal state has been found (line 4). In that case the last bucket is updated to contain only the goal states in preparation of the retrieval of a plan. In problems where no plan can be found the search also terminates when $\mathcal{C}_{max}$ consecutive buckets are empty (line 6). In that case all states have been expanded because the action costs are at most $\mathcal{C}_{max}$ so that any successor must reside in one of the next $\mathcal{C}_{max}$ buckets.

If actions with a cost of 0 are present the algorithm gets more complicated. The problem is that their successors reside in the same bucket as the states being expanded, so that this bucket must be expanded several times. To prevent this we start a BFS using only the zero-cost actions for the bucket to be expanded next (line 9) and afterward expand all states reached in this BFS to generate the successors in other buckets. Note that in order to retrieve the full plan the layers of the BFS should be stored within the bucket, so that we actually need a two-dimensional structure.

Retrieval of a plan

If we can keep the complete priority queue in main memory the final calculation of the plan works as shown in Algorithm 6.6. We start at the bucket in which a goal state was found (line 2). From this we determine the predecessors starting with the actions with highest cost and check if one is present in the corresponding bucket (line 8). If it is we continue with the predecessor states in that bucket (line 9), otherwise we check the actions with smaller cost. This we iterate until the initial state $\mathcal{I}$ is reached (line 3).

In case of zero-cost actions the retrieval gets more complicated as well. Before calculating the predecessors of a bucket we must first of all find a plan within the bucket, i. e., along the BFS layers calculated in forward direction, that transforms a state from the first layer within this bucket to one of the current states.



(a) Dijkstra's algorithm. (b) A* search.

Figure 6.3: Possible search spaces of Dijkstra's algorithm and A* search.

Then we can continue to find predecessors of the relevant states in the first layer, which reside in some of the previous buckets.

We start with the highest cost actions in order to try to find a short plan, in terms of number of actions. Nevertheless, the resulting plan may still be longer than some other plan, e. g., due to an immense number of zero-cost actions on this path or several actions with small cost versus few actions with higher cost on some other path. Still, the resulting plan is optimal, as the only criterion we have here is its total cost; the length of it is of no concern.

If we cannot keep the entire priority queue in main memory not all is lost. One possibility is to use an approach similar to Zhou and Hansen's breadth-first heuristic search (BFHS) (2006). For this we would store some relay buckets and apply a divide-and-conquer based approach to find a plan from one relay bucket to another, ultimately resulting in a complete plan. Alternatively, we can also make use of external storage, e. g., hard disks, if we can store at least some complete buckets (or layers of one bucket in case of zero-cost actions) internally. In that case we can store the entire priority queue on the external storage device and load only those buckets that we currently investigate. This brings the advantage that we do not have to expand parts of the search space over and over again, so that the planning and retrieval should be faster.

6.2.3 Symbolic A* Search

One of the disadvantages of Dijkstra's algorithm is that it is an uninformed search algorithm. Due to this it searches in all directions at once, no matter if it actually gets closer to a goal state. That is why it is often also referred to as *blind search*. To remedy this, the A* algorithm (Hart et al., 1968) has been developed (as illustrated in Figure 6.3). This uses a *heuristic function* h to estimate the remaining cost to a goal state to guide the search.

Definition 6.6 (Heuristic Function)**.** *A* heuristic function *(or* heuristic *for short)* h *is a mapping from the set of states* $\mathcal{S}$ *in the problem to a non-negative real-valued number, i. e.,* $h : \mathcal{S} \mapsto \mathbb{R}_0^+$.

A* expands those states v that have the smallest $f(v) = g(v) + h(v)$ value first, with g being the total cost from the initial state as it was used in Dijkstra's algorithm. This way, typically a vastly smaller amount of states must be expanded in order to reach a goal state, if a good heuristic is chosen. However, using a bad heuristic, such as the constant 0, it results in no improvement (actually, using the constant 0 heuristic A* will expand exactly the same states as Dijkstra's algorithm, as A*'s f value in that case equals Dijkstra's g value).

In general, the heuristic function performs a *re-weighting* of the costs, i. e., A* corresponds to Dijkstra's algorithm with the new costs $\hat{c}(u, v) = c(u, v) - h(u) + h(v)$. For the optimality of A* we need the notion of *admissible* and *consistent* heuristics (see, e. g., Russell and Norvig, 2010; Edelkamp and Schrödl, 2012).

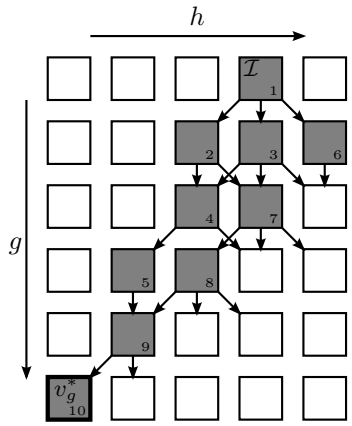


Figure 6.4: The expansion order of BDDA*. Each square stands for a bucket storing a BDD. Buckets in gray are those that are actually expanded. The arrows between the buckets denote positions of successor states. $\mathcal{I}$ is the initial state, v_g^* a goal state with minimal g value. The small numbers in the buckets represent the order in which they are expanded.

Definition 6.7 (Admissible Heuristic)**.** *A heuristic h is called* admissible*, if it never overestimates the cost to a goal state $g \in \mathcal{G}$, i. e., if it is a lower bound on the minimal total cost: $\forall v \in \mathcal{S} \,.\, \forall g \in \mathcal{G} \,.\, h(v) \leq \delta(v, g)$, with $\mathcal{S}$ being the set of all states and $\delta(v, g)$ being the cost of an optimal plan from v to g.*

Definition 6.8 (Consistent Heuristic)**.** *A heuristic h is called* consistent*, if $\forall (u, v) \in \mathcal{A} \,.\, h(u) \leq h(v) + \mathcal{C}(u, v)$, with $\mathcal{A}$ being the set of all actions and $\mathcal{C}$ being the cost of the corresponding action.*

The first plan found by A* is optimal if the heuristic we use is admissible and we do not store which states have already been expanded—the closed list—or we allow re-opening of already expanded states if they are found on a path with a smaller total cost. If the heuristic is consistent both algorithms—Dijkstra's algorithm with re-weighted costs and A*—perform the same and the first found plan is optimal, independent of the use of a closed list.

Note that in case of an admissible heuristic h the heuristic estimate of a goal state $g \in \mathcal{G}$ is always $h(g) = 0$. Also note that the constant 0 is an admissible heuristic, as it can never overestimate the distance to a goal state due to the fact that the action costs may not be negative. While all consistent heuristics with an estimate of 0 for all goal states are always also admissible, the inverse does not hold, so that for a heuristic to be consistent is the stronger requirement.

Some symbolic adaptations of A* have been proposed, e. g., BDDA* by Edelkamp and Reffel (1998) or SetA* by Jensen et al. (2002). In the following, we will describe how BDDA* works.

Similar to the symbolic version of Dijkstra's algorithm in Section 6.2.2 we use buckets to store the BDDs. While for Dijkstra's algorithm a bucket list was sufficient (in the case of no zero-cost actions) here we need a two-dimensional matrix. One dimension represents the total cost from $\mathcal{I}$ (the g value as it is used in Dijkstra's algorithm), the other one the heuristic estimate on the cost to a goal state (the h value).

A* expands states according to $f = g + h$. All states with the same f value are thus stored on the same diagonal in the matrix. Furthermore, as we do not allow negative action costs we know that the g value will never decrease. So, for each f diagonal we start at the bucket with smallest g value, expand it, and go to the next one on the same f diagonal, until either the diagonal has been completely expanded or we find a goal state $v_g^* \in \mathcal{G}$. In the first case we go on to the next f diagonal, in the latter one we are done and can start to retrieve a plan. This behavior is depicted in Figure 6.4.

Apart from the planning problem BDDA* also takes a heuristic function $heur$ as input, along with the maximal possible heuristic value using this function, $heur_{max}$ (cf. Algorithm 6.7). Here, we assume that the heuristic function is a list of BDDs containing all states that have some heuristic estimate h in bucket $heur_h$. This way, we can simply calculate the conjunction of a set of states with a bucket of the heuristic in order to find all states of the given set that have the corresponding heuristic estimate.

Initially, the matrix is supposed to be empty. The first state to be inserted is the initial state $\mathcal{I}$, for which the heuristic estimate is calculated (line 2). To prevent re-expansion of states and also to ensure termination

Algorithm 6.7: Symbolic Version of A* (BDDA*)

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$.
Input: Heuristic function $heur$.
Input: Maximal heuristic value $heur_{max}$.
Output: Cost-optimal plan or "no plan".

```
1  f ← 0                                                          // Initial f value is at least 0.
2  while I ∧ heur_f = ⊥ do f ← f + 1                              // Find f value for the initial state.
3  open_{0,f} ← I                                                 // Insert I into the correct bucket.
4  f_max ← f                                                      // Maximal f value initially is the same as that of I.
5  for h ← 0 to heur_max do closed_h ← ⊥                          // So far, no states expanded.
6  loop                                                           // Repeat...
7      if f > f_max then return "no plan"                         // Stop when all states expanded.
8      for g ← 0 to f do                                          // Follow the f diagonal.
9          h ← f − g                                              // Determine corresponding h value.
10         if h > heur_max then continue                          // We cannot have h values greater than heur_max.
11         open_{g,h} ← open_{g,h} ∧ ¬closed_h                    // Remove all states already expanded with same h value.
12         if open_{g,h} ≠ ⊥ then                                 // If current bucket not empty...
13             if zero-cost actions present then perform BFS in open_{g,h} using only zero-cost actions
14             if h = 0 and open_{g,h} ∧ G ≠ ⊥ then            // If goal state found...
15                 open_{g,h} ← open_{g,h} ∧ G                    // Retain only goal states in final bucket.
16                 return RetrieveA*(P, open, g)                  // Retrieve plan and return it.
17             closed_h ← closed_h ∨ open_{g,h}                    // Add states to be expanded next to closed list.
18             for c ← 1 to C_max do                              // For all (non-zero) action costs c ...
19                 succ ← image_c(open_{g,h})                     // Calculate successors.
20                 for h' ← 0 to heur_max do                      // Check all possible h values.
21                     open_{g+c,h'} ← open_{g+c,h'} ∨ (succ ∧ heur_{h'})   // Insert successors into correct bucket.
22                     if g + c + h' > f_max then f_max ← g + c + h'        // Update maximal f value.
23     f ← f + 1                                                  // Go over to next f diagonal.
```

we again use a closed list. Using the same heuristic function throughout the search we can be sure that equal states will also have the same heuristic estimate, so that instead of storing one BDD to hold the entire set of expanded states we can store it in several—hopefully smaller—BDDs $closed_h$, each of which storing all expanded states that have a certain heuristic estimate h. Initially, this list is empty (line 5). The expansion is similar to the one in Dijkstra's algorithm, i. e., we use the $image_c$ operator, but here must also find the heuristic estimate for all the states (line 21) in order to locate the correct bucket for storing the states (if it exists—otherwise we must insert a new bucket containing only these states).

The main search is performed in the loop (lines 6 to 23). This iterates over the entire matrix and increments the f value, thus ensuring that we visit each f diagonal. The search along one such diagonal is capsuled in the for loop (lines 8 to 22). Here we iterate over all possible g values, ranging from 0 to f. First of all, we remove all states from the current bucket that have been expanded before (line 11). Next, if there are any states left in the bucket, we add those to the closed list (line 17) and expand them (lines 18 to 22).

The algorithm terminates in two situations. The first one arises when all states have been expanded without ever reaching a goal state. To find such a situation we store the maximal f value f_{max} that some generated and stored states have (line 22). If we reach a diagonal with an f value greater than f_{max} we know that we have expanded all buckets containing any states, so that we can stop the search (line 7). The second criterion for termination is achieved when we have found a goal state, in which case we retrieve and return a plan and the search ends successfully (line 16).

We again left out the exact handling of the zero-cost actions (line 13) from the pseudocode in order to keep it more readable. Actually, we can handle them in a similar fashion as we did in Dijkstra's algorithm,

i. e., we can perform a BFS in a bucket using only the zero-cost actions and store all the layers within that bucket, so that we actually require a three-dimensional matrix. If we determine the h values for all successor states during the BFS we can insert them either into the next BFS layer in the current bucket or into the first layer of a bucket with the same g but different h value, so that we can retain the h-wise partitioning of the set of expanded states $closed$. Otherwise we have to switch to a global $closed$ BDD containing all states ever expanded.

Once the BFS is finished we can continue the normal BDDA* search using the other actions.

Theoretical Properties

Similar to normal A* an admissible heuristic that is not consistent might lead to sub-optimal plans if we do not allow re-opening of already expanded buckets, while using a consistent heuristic we can be sure that the first found plan is optimal and we will never have to re-open already expanded buckets.

Proposition 6.9 (BDDA* and admissible heuristics)**.** *Using an admissible heuristic that is not consistent in BDDA* might result in re-opening of already expanded buckets.*

Proof. Let us take two states, u and an unexpanded successor v. Let the heuristic estimate of u be $h(u) > \mathcal{C}(u, v)$, and the heuristic estimate of v be $h(v) = 0$. Then

$$\begin{aligned} f(v) &= g(v) + h(v) \\ &= g(u) + \mathcal{C}(u, v) + 0 \\ &= f(u) - h(u) + \mathcal{C}(u, v) \\ &< f(u) - \mathcal{C}(u, v) + \mathcal{C}(u, v) \\ &= f(u). \end{aligned}$$

In other words, v will reside on a diagonal with an f value smaller than the one u was expanded in, so that the corresponding bucket and possibly large parts that were already expanded before will have to be re-opened and expanded once more. □

Proposition 6.10 (BDDA* and consistent heuristics)**.** *Using a consistent heuristic in BDDA* will result in no re-openings, if we handle zero-cost actions in a sensible manner.*

Proof. If we reverse the inequality from Definition 6.8 for consistent heuristics we know that the heuristic estimate of a successor v from state u is

$$h(v) \geq h(u) - \mathcal{C}(u, v)$$

The corresponding f value is

$$\begin{aligned} f(v) &= g(v) + h(v) \\ &= g(u) + \mathcal{C}(u, v) + h(v) \\ &\geq g(u) + \mathcal{C}(u, v) + h(u) - \mathcal{C}(u, v) \\ &= g(u) + h(u) \\ &= f(u), \end{aligned}$$

so that it will be either in the same bucket, in case of a zero-cost action, or on a diagonal whose f value is at least as large as the one of the predecessor u.

If we handle zero-cost actions by storing all the BFS layers in the original bucket it might be that some of the states are actually inserted in the wrong bucket. Still, the actual h value of the states cannot be smaller than the current one due to the consistency condition. The successors will get the correct h values, so that in such a case the successors' f values will still be greater than the cost of achieving them plus the f value of the states inserted in the wrong bucket, so that the algorithm still works.

Algorithm 6.8: Retrieval of a Plan for BDDA* ($RetrieveA^*$)

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$.
Input: 2-dimensional priority queue $open$ containing all states reached in forward direction.
Input: Total cost g of found goal states.
Output: Cost-optimal plan π.

```
1  π ← ()                                                  // Initial plan is empty.
2  f_max ← g                                               // Initial f diagonal has value g.
3  current ← open_{g,0}                                    // Start with final bucket.
4  while I ∧ current = ⊥ do                                // Repeat, until I is reached.
5      if zero-cost actions present then retrieve BFS plan in open_{g,h} using only zero-cost actions
6      for c ← C_max to 1 do                               // For all possible costs c...
7          for all a ∈ A with C(a) = c do                  // For all actions a with cost c...
8              pred ← pre-image_a(current)                 // Calculate predecessors using action a.
9              for h ← 0 to f_max − (g − c) do             // Search for instance containing pred.
10                 if open_{g−c,h} ∧ pred ≠ ⊥ then          // If some predecessors are in bucket open_{g−c,h}...
11                     current ← open_{g−c,h} ∧ pred       // Take those predecessors as current states.
12                     add action a to front of plan π     // Extend plan by found action.
13                     g ← g − c                           // Update cost for next iteration.
14                     f_max ← g + h                       // Store value of new f diagonal.
15                     goto line 4                         // Continue in predecessor's bucket.
16 return π                                                // Return the complete plan.
```

If we handle them by inserting them into the correct buckets, the consistency condition still holds, so that the found states will be inserted into buckets having an h value that is at least as large as the one of the current bucket.

In both cases the zero-cost successors are either in the same bucket or in a bucket with a higher h value. Because we first calculate the zero-cost fixpoint, in the calculation of the non-zero-cost successors all states are either on the same diagonal, but not within the same bucket due to an increased g value, or on a diagonal with greater f value.

As the g values are not decreasing, we know that we cannot go back on the same f diagonal, so that due the expansion order of one f diagonal we can be certain that no re-openings are necessary. □

Retrieval of a plan

After having reached a goal state in forward direction we can start retrieving a plan (cf. Algorithm 6.8). The reached goal states are in bucket $open_{g,0}$, so that this is where we start the search in backward direction (line 3). In case we are not confronted with zero-cost actions we check for all actions if there is a predecessor stored in one of the corresponding buckets. If we find one, we continue with the search from there (line 11) and add the corresponding action to the front of the plan (line 12), until finally we have found the way back to the initial state $\mathcal{I}$ (line 4). Opposed to the symbolic implementation of Dijkstra's algorithm here it is not enough to check one bucket ($open_{g-c}$), but rather we must check all possible buckets $open_{g-c,h}$, where all buckets with the correct g value to the left or on the current f diagonal are possible (line 9). As we do not support inconsistent heuristics it is safe to ignore all states to the right of the current diagonal. The diagonal initially is the one with an f value of g (line 2), and is updated whenever we have found a predecessor and continue the search from there (line 14).

In case of zero-cost actions (line 5) the actual retrieval depends on the way we handle zero-cost actions in BDDA*. If we store all the BFS layers in the current bucket we once more must find a BFS subplan for the current bucket until we are at the first layer of that bucket and add the subplan to the front of the total plan calculated so far, similar to the procedure in the plan retrieval for Dijkstra's algorithm.

If we distribute the states into the correct buckets we still must retrieve the BFS subplan for the zero-cost actions until we have reached the first layer of a bucket. The states in this layer may have been reached by any action, no matter if zero-cost or not. Thus, to retrieve a plan in this setting we must change line 7 so that actions with a cost of 0 are considered as well when searching for predecessors in other buckets.

6.3 Heuristics for Planning

In the previous section we have seen two algorithms for finding cost-optimal plans, one that blindly searches in all directions simultaneously, and one that tends to search in the direction of a goal state by means of a heuristic function. A question that still remains is how we can come up with a good heuristic. Several approaches for automatically deriving heuristics from the problem description have been proposed.

An nice overview has been published by Helmert and Domshlak (2009). According to that, there are four main classes of heuristics used in planners, *relaxation heuristics*, *critical path heuristics*, *abstraction heuristics*, and *landmark heuristics*. In the following we will briefly describe some approaches of heuristics from these four classes.

Relaxation Heuristics

Relaxation heuristics are based on *relaxing* the planning problem. A relaxation is achieved by removing the delete effects of all actions, so that an action can only add new facts to the current state but never remove any. This way an action that is applicable once will also be applicable in all successor states, no matter what actions have been taken, if we do not allow negation in the preconditions.

One instance of a relaxation heuristic is h^+ (Hoffmann and Nebel, 2001), which is calculated by determining the cost of an optimal plan in the relaxed problem. It has been shown that it is admissible and often very informative (Hoffmann, 2005; Helmert and Mattmüller, 2008), but unfortunately it is NP-hard to compute (Bylander, 1994), so that it is not used in practice.

Several heuristics have been proposed that provide an inadmissible estimate of h^+, probably one of the most prominent ones being the FF heuristic used in the FF planner by Hoffmann and Nebel (2001). Being inadmissible these heuristics of course cannot be used in an optimal planner, but can be quite successful in satisficing planners, i. e., planners that are supposed to find good plans, but not necessarily optimal ones.

Nevertheless, there are also heuristics calculating an admissible estimate of h^+, such as the max heuristic h^{max} (Bonet and Geffner, 2001). For a state s the h^{max} value is the maximal cost $c_s(f)$ over all facts f from the goal description $\mathcal{G}$. This cost $c_s(f)$ is defined as zero, if f already holds in s, and as $c_s(f) = \min_{a \in \mathcal{A} \text{ with } f \in add_a} \left(\mathcal{C}(a) + \max_{p \in pre_a} c_s(p)\right)$ if it does not.

Critical Path Heuristics

For a set of facts f the heuristic h^m as proposed by Haslum and Geffner (2000) determines an estimate on the distance to the initial state $\mathcal{I}$ for those states sharing the variables specified in f:

$$h^m(f) = \begin{cases} 0 & \text{if } f \subseteq \mathcal{I} \\ \min_{\{a \in \mathcal{A} | D_a \cap f = \emptyset\}} h^m((f \setminus A_a) \cup pre_a) + \mathcal{C}(a) & \text{if } |f| \leq m \\ \max_{f' \subseteq f, |f'| \leq m} h^m(f') & \text{otherwise} \end{cases}$$

The h^m heuristic is a so called *critical path heuristic* and is known to be admissible. It provides a lower bound on the cost of achieving m-sized sets of facts and the general idea is that a set of facts is reachable with some cost c if all subsets of size m are reachable with that cost c.

Abstraction Heuristics

Abstraction heuristics are concerned with mapping a planning problem to an abstract planning problem. The heuristic estimate of a state is then the minimal distance from the corresponding abstract state to a goal state in the abstract problem.

In case of *pattern databases* (Culberson and Schaeffer, 1998; Edelkamp, 2001) abstraction is achieved by removing some of the variables from the planning problem. The heuristic is then generated by performing Dijkstra's algorithm in backward direction, starting at the (abstract) goal states, and the minimal distance to one of those is stored for every reached (abstract) state. PDBs provide an admissible estimate on the cost to a goal state for any (non-abstract) state.

In the *merge-and-shrink* abstraction (Helmert et al., 2007), which was originally proposed in the context of (directed) model checking (Dräger et al., 2006, 2009), the idea is to combine a set of abstractions until only one abstraction remains. The initial set of abstractions consists of all the binary facts, i. e., all are atomic abstractions. To come up with one remaining abstraction two functions are applied. The first one is to *merge* two abstractions by calculating the synchronized product of them and replacing them by the result, the second one is to *shrink* an existing abstraction, i. e., to replace it by an abstraction of that abstraction. When only one abstraction is left the minimal cost to a goal state in the abstract space can be calculated and used as the heuristic estimate.

At least in case of explicit search the sizes of the abstract spaces must be controlled in order to be used in context with limited memory. To overcome this problem Katz and Domshlak (2010a) proposed the use of *implicit abstractions*. Instead of storing the abstract state spaces explicitly they proposed to use abstractions of the planning problem at hand that are provably tractable and use them to perform implicit state space search.

Landmark Heuristics

Landmarks in the context of planning correspond to facts that must be true at some point of time for any plan (Porteous et al., 2001). It is possible to find an ordering of the landmarks. For example, in our running example (the gunslinger following the man in black through the desert) the gunslinger must necessarily catch the man in black, i. e., `(caughtMIB)` must hold, as the facts of the goal description are trivial landmarks. In order to catch the man in black the gunslinger must be at the same location. As the man in black does not move it is clear that `(position gunslinger p9)` must hold before `(caughtMIB)` can hold. Porteous et al. (2001) proved that the decision whether some fact is a landmark as well as finding the ordering between two landmarks are PSPACE-complete problems.

A first simple heuristic using landmarks was proposed by Richter et al. (2008). In short, they count the number of landmarks not yet achieved, i. e., the number of all landmarks minus the number of landmarks already achieved plus the number of landmarks that must be achieved again. The latter might be the case if the gunslinger already was in position `p9` but decided to move on instead of catching the man in black. It is trivial to prove that this heuristic is not admissible, as an action might very well achieve two or more landmarks.

Admissible landmark heuristics have been proposed by Karpas and Domshlak (2009). Their idea is to distribute the cost of an action over the landmarks that are achieved by its application. The cost of a landmark is the minimal cost that is assigned to it by an action and the heuristic estimate is the sum of the costs of all unachieved landmarks.

One of the results of the overview paper by Helmert and Domshlak (2009) was that their proofs concerning the dominance of the various heuristic approaches were actually quite constructive and allowed for the implementation of a new heuristic, the so-called *LM-cut* heuristic h^{LM-cut}. The first step in calculating h^{LM-cut} for a state s is to calculate its h^{max} value. In case it is zero or infinite they set $h^{LM-cut}(s) = h^{max}(s)$. Otherwise they calculate an action landmark, which is a set of actions with costs greater than zero of which one must be used in each relaxed plan starting from s. The heuristic value, which is initially zero, is increased by the cost of the cheapest action within the action landmark. At the same time the costs of all actions are decreased by that cost. Based on the new action costs the h^{max} value is recomputed, followed by a computation of a new action landmark and so on, until the h^{max} estimate of the state becomes zero after a reduction of the action costs.

6.3.1 Pattern Databases

Given the set of possible heuristics from the previous section the best fit for symbolic search seems to be the abstraction heuristics. What might be seen as a disadvantage in explicit search, i. e., the need to store all

heuristic values for the entire abstract space, results in a natural handling in case of symbolic search, as sets of states share the same heuristic estimates and thus can be stored together in one BDD. In this section we will focus on *pattern databases* (PDBs) in case of explicit search but also on how to calculate them using symbolic search.

The concept of PDBs was originally introduced by Culberson and Schaeffer (1998). They are generated for an *abstracted* version of the problem at hand.

Definition 6.11 (Restriction Function)**.** *Let $\mathcal{V}$ be a set of variables and $\mathcal{V}' \subseteq \mathcal{V}$. For a state s a* restriction function *$\phi_{\mathcal{V}'} : 2^{\mathcal{V}} \mapsto 2^{\mathcal{V}'}$ is defined as the projection of s to $\mathcal{V}'$ containing only those variables that are also present in $\mathcal{V}'$, while those in $\mathcal{V} \setminus \mathcal{V}'$ are removed. $\phi_{\mathcal{V}'}(s)$ is also denoted by $s|_{\mathcal{V}'}$.*

Such a restriction function is often also called a *pattern* as we choose a subset, the pattern, of the entire set of variables to restrict the search space. PDBs were originally introduced in the context of the 15-PUZZLE, where a subset of the entire set of variables actually corresponds to a pattern of pieces still relevant.

Definition 6.12 (Abstract Planning Problem)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem and $\mathcal{V}' \subseteq \mathcal{V}$ a set of variables. An* abstract planning problem *$\mathcal{P}|_{\mathcal{V}'} = \langle \mathcal{V}', \mathcal{A}|_{\mathcal{V}'}, \mathcal{I}|_{\mathcal{V}'}, \mathcal{G}|_{\mathcal{V}'}, \mathcal{C}|_{\mathcal{V}'} \rangle$ is $\mathcal{P}$ restricted to $\mathcal{V}'$, where $\mathcal{A}|_{\mathcal{V}'} = \{a|_{\mathcal{V}'} = \langle pre_a|_{\mathcal{V}'}, \mathit{eff}_a|_{\mathcal{V}'} \rangle \mid a = \langle pre_a, \mathit{eff}_a \rangle \in \mathcal{A} \wedge \mathit{eff}|_{\mathcal{V}'} \neq \emptyset\}$ and $\mathcal{C}|_{\mathcal{V}'}$ is equal to $\mathcal{C}$ for the actions also present in $\mathcal{P}|_{\mathcal{V}'}$ and is 0 for the others.*

In other words, the abstraction of a planning problem results in a (typically smaller) planning problem where certain variables (those in $\mathcal{V} \setminus \mathcal{V}'$) are ignored. The definition of the abstract actions also takes care of *empty actions*, i. e., those that do not change anything about a state due to an empty effect, as they can be safely removed from the abstract problem description.

Definition 6.13 (Pattern Database)**.** *For an abstract planning problem $\mathcal{P}|_{\mathcal{V}'} = \langle \mathcal{V}', \mathcal{A}|_{\mathcal{V}'}, \mathcal{I}|_{\mathcal{V}'}, \mathcal{G}|_{\mathcal{V}'}, \mathcal{C}|_{\mathcal{V}'} \rangle$ a* pattern database *(or PDB) $\Phi_{\mathcal{V}'}$ stores a pair $\left(u|_{\mathcal{V}'}, \min_{\pi_{u|_{\mathcal{V}'}}} \mathcal{C}|_{\mathcal{V}'}\left(\pi_{u|_{\mathcal{V}'}}\right)\right)$ for each abstract state $u|_{\mathcal{V}'}$ where $\pi_{u|_{\mathcal{V}'}}$ is a plan starting at state $u|_{\mathcal{V}'}$. $\Phi_{\mathcal{V}'}(u)$ returns the minimal cost of a plan starting at $u|_{\mathcal{V}'}$.*

Thus, a PDB specifies the minimal distances to a goal state for all states of an abstract planning problem. In order to use this for the generation of a heuristic estimate we can use one PDB directly, or we can try to find a number of PDBs, i. e., PDBs for different abstractions, so that we can come up with better heuristic estimates.

To calculate the heuristic estimate resulting from a number of PDBs we can take the maximum of all returned values, i. e., for a set of PDBs $\Phi_{\mathcal{V}'_1}, \ldots, \Phi_{\mathcal{V}'_n}$ for the abstract problems defined by the sets of variables $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ the heuristic estimate h of a state $u \in \mathcal{S}$ is $h(u) = \max_{1 \leq i \leq n} \Phi_{\mathcal{V}'_i}(u)$.

This results in an estimate that is at least as good as the one generated by any of the PDBs for each state, i. e., it *dominates* the estimates generated by any single PDB of this set. Given such a set of PDBs it is possible to calculate heuristic estimates that are better than that of each PDB. In the following we will present two of these in more detail. The first one is concerned with *disjoint* PDBs, i. e., PDBs where each action is relevant only in at most one PDB, the second one with *additive* PDBs, i. e., PDBs whose results can be added without ever overestimating the correct cost.

Disjoint Pattern Databases

For a set of *disjoint* PDBs we can easily calculate a combined heuristic estimate that never overestimates the optimal distance to a goal state.

Definition 6.14 (Disjoint PDBs)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem and let $\Phi_{\mathcal{V}'_1}$ and $\Phi_{\mathcal{V}'_2}$ be PDBs for the abstract planning problems $\mathcal{P}|_{\mathcal{V}'_1}$ and $\mathcal{P}|_{\mathcal{V}'_2}$, respectively, with $\mathcal{V}'_1 \cap \mathcal{V}'_2 = \emptyset$. The PDBs are called* disjoint*, if $\mathcal{A}|_{\mathcal{V}'_1}(a) \cap \mathcal{A}|_{\mathcal{V}'_2}(a) = \emptyset$ for all $a \in \mathcal{A}$.*

Given some disjoint PDBs we can determine the heuristic estimate for any state by calculating the sum of the distances to a goal state for all the abstract problems (Korf and Felner, 2002; Felner et al., 2004).

Theorem 6.15 (Disjoint PDBs are admissible)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem, $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ be n disjoint sets of variables with $\mathcal{V}'_i \subseteq \mathcal{V}$ for all $1 \leq i \leq n$, and $\Phi_{\mathcal{V}'_1}, \ldots, \Phi_{\mathcal{V}'_n}$ be disjoint PDBs for the n abstract planning problems $\mathcal{P}|_{\mathcal{V}'_1}, \ldots, \mathcal{P}|_{\mathcal{V}'_n}$, respectively. Then the sum of the abstract distances to an abstract goal state resulting from the PDBs, i. e., $h(u) = \sum_{i=1}^{n} \Phi_{\mathcal{V}'_i}(u)$ for all states u, is an admissible heuristic for $\mathcal{P}$.*

Proof. The following holds

$$h(u) = \sum_{i=1}^{n} \Phi_{\mathcal{V}'_i}(u) = \sum_{i=1}^{n} \min_{\pi_{u|_{\mathcal{V}'_i}}} \mathcal{C}|_{\mathcal{V}'_i}\left(\pi_{u|_{\mathcal{V}'_i}}\right) \leq \min_{\pi_u} \sum_{i=1}^{k} \mathcal{C}|_{\mathcal{V}'_i}(\pi_u)$$

with π_u being a plan starting at u in the original problem $\mathcal{P}$ and $\pi_{u|_{\mathcal{V}'_i}}$ being a plan starting at $u|_{\mathcal{V}'_i}$ in the abstract problem $\mathcal{P}|_{\mathcal{V}_i}$. The inequality follows from the fact that the cheapest plan of each abstraction cannot be any more expensive than any other plan, which includes an optimal plan for the original problem transformed to the abstract one.

Let $\pi_u = (a_1, \ldots, a_l)$ be some plan starting at state u. Then

$$\begin{aligned}
\sum_{i=1}^{n} \mathcal{C}|_{\mathcal{V}'_i}(\pi_u) &= \sum_{i=1}^{n} \sum_{j=1}^{l} \mathcal{C}|_{\mathcal{V}'_i}(a_j) \\
&= \sum_{j=1}^{l} \sum_{i=1}^{n} \mathcal{C}|_{\mathcal{V}'_i}(a_j) \\
&\leq \sum_{j=1}^{l} \mathcal{C}(a_j) = \mathcal{C}(\pi_u). \qquad (6.2)
\end{aligned}$$

Inequality 6.2 holds because for each action a_j at most one abstract problem has the same cost as the original one and all others a cost of 0.

With this the claim follows, i. e., the heuristic generated by the sum of the costs of all abstract problems never overestimates the minimal cost of a plan.

$$h(u) \leq \min_{\pi_u} \sum_{i=1}^{k} \mathcal{C}|_{\mathcal{V}'_i}(\pi_u) \leq \min_{\pi_u} \mathcal{C}(\pi_u). \qquad \square$$

We can even proof that the heuristic generated by the sum of the distances to an abstract goal state of the disjoint PDBs is consistent, so that we can use this as the heuristic in A* search or the symbolic version BDDA*, if we do not want to reopen already expanded states.

Theorem 6.16 (Disjoint PDBs are consistent)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem, $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ be n disjoint sets of variables with $\mathcal{V}'_i \subseteq \mathcal{V}$ for all $1 \leq i \leq n$, and $\Phi_{\mathcal{V}'_1}, \ldots, \Phi_{\mathcal{V}'_n}$ be disjoint PDBs for the n abstract planning problems $\mathcal{P}|_{\mathcal{V}'_1}, \ldots, \mathcal{P}|_{\mathcal{V}'_n}$, respectively. Then the sum of the abstract distances to an abstract goal state resulting from the PDBs, i. e., $h(u) = \sum_{i=1}^{n} \Phi_{\mathcal{V}'_i}(u)$ for all states u, is a consistent heuristic for $\mathcal{P}$.*

Proof. We must proof that

$$h(u) \leq \mathcal{C}(u, v) + h(v)$$

holds for all states u with successor v given the proposed heuristic function h.

With the same argument as for inequality 6.2 we know that

$$\mathcal{C}(u, v) \geq \sum_{i=1}^{n} \mathcal{C}|_{\mathcal{V}'_i}(u, v).$$

With this it follows that

$$\begin{aligned} h(v) + \mathcal{C}(u,v) &\geq h(v) + \sum_{i=1}^{n} \mathcal{C}|_{\mathcal{V}'_i}(u,v) \\ &= \sum_{i=1}^{n} \min_{\pi_{v|_{\mathcal{V}'_i}}} \mathcal{C}|_{\mathcal{V}'_i}\left(\pi_{v|_{\mathcal{V}'_i}}\right) + \sum_{i=1}^{n} \mathcal{C}|_{\mathcal{V}'_i}(u,v) \\ &= \sum_{i=1}^{n} \left(\min_{\pi_{v|_{\mathcal{V}'_i}}} \mathcal{C}|_{\mathcal{V}'_i}\left(\pi_{v|_{\mathcal{V}'_i}}\right) + \mathcal{C}|_{\mathcal{V}'_i}(u,v) \right) \\ &\geq \sum_{i=1}^{n} \min_{\pi_{u|_{\mathcal{V}'_i}}} \mathcal{C}|_{\mathcal{V}'_i}\left(\pi_{u|_{\mathcal{V}'_i}}\right) = h(u) \end{aligned} \tag{6.3}$$

with $\pi_{v|_{\mathcal{V}'_i}}$ being a plan starting at $v|_{\mathcal{V}'_i}$ in the abstract problem $\mathcal{P}|_{\mathcal{V}'_i}$. Inequality 6.3 holds because for each abstraction the triangle inequality holds, i. e., an optimal plan starting at u cannot be more expensive than taking the action from u to v and then following an optimal plan from v. □

Additive Pattern Databases

Calculating heuristic estimates given disjoint PDBs is fairly simple. The difficulty actually resides in finding PDBs that are distinct, i. e., in choosing patterns so that no action appears in more than one abstraction, especially if this is to be done automatically. We might try to establish rules for finding those, or we might actually use any patterns we like and then assign new costs to all the actions. If done correctly we again can simply use the sum of the distances in the abstract problems, as they are stored in the PDBs, as an admissible and consistent heuristic estimate. Such PDBs are called *additive* PDBs (Edelkamp and Kissmann, 2008d; Katz and Domshlak, 2008, 2010b).

To calculate the new costs, first of all we need to find out how many of the abstractions are *valid* for an action $a \in \mathcal{A}$, i. e., in how many of the abstractions action a is present (because it is not empty).

Definition 6.17 (Number of Valid Abstractions)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem and $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ be n sets of variables, $\mathcal{V}'_i \subseteq \mathcal{V}$ for all $1 \leq i \leq n$. Then $\lambda : \mathcal{A} \mapsto \mathbb{N}_0^+$ determines the* number of valid abstractions *for an action $a \in \mathcal{A}$, i. e., the number of abstractions where action a is not empty, by $\lambda(a) = \left|\left\{i \in \{1, \ldots, n\} \mid a|_{\mathcal{V}'_i} \neq noop\right\}\right|$ with $noop$ specifying an action that does not change anything about the state. In the following we will use the extended operator $\lambda' = \max\{\lambda, 1\}$, in order to avoid divisions by zero, in case an action's effect is empty in all abstractions.*

With this we can calculate a weight w with which we scale the original problem's action costs.

Definition 6.18 (Scaled Planning Problem)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem. Then the* scaling weight *w is the least common multiple (lcm) of the number of valid abstractions for all actions $a \in \mathcal{A}$, i. e., $w = lcm_{a \in \mathcal{A}} \lambda'(a)$.*

The scaled planning problem *$\mathcal{P}^s = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C}^s \rangle$ is generated by adjusting the action costs to $\mathcal{C}^s(a) = w \times \mathcal{C}(a)$ for all actions $a \in \mathcal{A}$.*

The total cost of a plan π for $\mathcal{P}^s$ is then $\mathcal{C}^s(\pi) = \sum_{a \in \pi} \mathcal{C}^s(a) = \sum_{a \in \pi} w \times \mathcal{C}(a) = w \times \mathcal{C}(\pi)$, so that a plan that is optimal in the original problem is also optimal in the scaled one, and vice versa.

For n sets of variables $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ with $\mathcal{V}'_i \subseteq \mathcal{V}$ for all $1 \leq i \leq n$ and the abstract problems $\mathcal{P}|_{\mathcal{V}'_1}, \ldots, \mathcal{P}|_{\mathcal{V}'_n}$ we can similarly determine *scaled abstract planning problems* $\mathcal{P}^s|_{\mathcal{V}'_1}, \ldots, \mathcal{P}^s|_{\mathcal{V}'_n}$ by setting the scaled abstract action costs $\mathcal{C}^s|_{\mathcal{V}'_i}(a)$ to

$$\mathcal{C}^s|_{\mathcal{V}'_i}(a) = \begin{cases} w \times \mathcal{C}(a) / \lambda'(a) & \text{if } a|_{\mathcal{V}'_i} \neq noop \\ 0 & \text{otherwise} \end{cases}$$

for all $1 \leq i \leq n$ and all actions $a \in \mathcal{A}$. Note that, due to the fact that w is the least common multiple, all action costs of all scaled abstract planning problems remain integral.

In the following we will prove that the sum of the scaled abstract distances to an abstract goal results in an admissible heuristic, i. e., we will show that this sum is a lower bound on the scaled distance to a goal state in the original space.

Theorem 6.19 (Admissible heuristic for scaled planning problems)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem, $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ be n sets of variables with $\mathcal{V}'_i \subseteq \mathcal{V}$ for all $1 \leq i \leq n$, and $\Phi_{\mathcal{V}'_1}, \ldots, \Phi_{\mathcal{V}'_n}$ be PDBs for the n scaled abstract planning problems $\mathcal{P}^s|_{\mathcal{V}'_1}, \ldots, \mathcal{P}^s|_{\mathcal{V}'_n}$, respectively. Then the sum of the distances to an abstract goal state resulting from the PDBs, i. e., $h(u) = \sum_{i=1}^{n} \Phi_{\mathcal{V}'_i}(u)$ for all states u, is an admissible heuristic for the scaled planning problem $\mathcal{P}^s$.*

Proof. Let $\pi_u = (a_1, \ldots, a_l)$ be any plan in $\mathcal{P}^s$ starting at state u and let

$$\left[a|_{\mathcal{V}'_i} \neq noop\right] = \begin{cases} 1 & \text{if } a|_{\mathcal{V}'_i} \neq noop \\ 0 & \text{otherwise} \end{cases}$$

Then the following holds

$$\begin{aligned}
\sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(\pi_u) = \sum_{i=1}^{n}\sum_{j=1}^{l} \mathcal{C}^s|_{\mathcal{V}'_i}(a_j) &= \sum_{i=1}^{n}\sum_{j=1}^{l} w \times \mathcal{C}(a_j) \times \left[a_j|_{\mathcal{V}'_i} \neq noop\right] / \lambda'(a_j) \\
&= \sum_{j=1}^{l}\sum_{i=1}^{n} w \times \mathcal{C}(a_j) \times \left[a_j|_{\mathcal{V}'_i} \neq noop\right] / \lambda'(a_j) \\
&= \sum_{j=1}^{l} \left((w \times \mathcal{C}(a_j) / \lambda'(a_j)) \times \sum_{i=1}^{n} \left[a_j|_{\mathcal{V}'_i} \neq noop\right] \right) \qquad (6.4) \\
&\leq \sum_{j=1}^{l} w \times \mathcal{C}(a_j) = \mathcal{C}^s(\pi_u). \qquad (6.5)
\end{aligned}$$

The inner sum in equation 6.4 evaluates exactly to $\lambda(a_j)$ for every action $a_j \in \pi_u$. To proof that inequality 6.5 holds we must analyze two cases. In the first one $\lambda(a_j) = 0$, so that the jth summand of the outer sum will be 0 as well, which is no higher than $w \times \mathcal{C}(a_j)$ as we do not allow negative action costs. In the second case $\lambda(a_j) = \lambda'(a_j)$, so that they can be canceled, which results exactly in $w \times \mathcal{C}(a_j)$.

As this holds for any plan, it also holds for an optimal plan, i. e.,

$$\min_{\pi_u} \sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(\pi_u) \leq \min_{\pi_u} \mathcal{C}^s(\pi_u). \qquad (6.6)$$

For the costs of a plan optimal in $\mathcal{P}^s|_{\mathcal{V}'_i}$ it holds that $\min_{\pi_{u|_{\mathcal{V}'_i}}} \mathcal{C}^s|_{\mathcal{V}'_i}\left(\pi_{u|_{\mathcal{V}'_i}}\right) \leq \mathcal{C}^s|_{\mathcal{V}'_i}(\pi_u)$ for any plan π_u. It follows that

$$\sum_{i=1}^{n} \min_{\pi_{u|_{\mathcal{V}'_i}}} \mathcal{C}^s|_{\mathcal{V}'_i}\left(\pi_{u|_{\mathcal{V}'_i}}\right) \leq \min_{\pi_u} \sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(\pi_u). \qquad (6.7)$$

All together, we have now proved that

$$h(u) = \sum_{i=1}^{n} \min_{\pi_{u|_{\mathcal{V}'_i}}} \mathcal{C}^s|_{\mathcal{V}'_i}\left(\pi_{u|_{\mathcal{V}'_i}}\right) \overset{6.7}{\leq} \min_{\pi_u} \sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(\pi_u) \overset{6.6}{\leq} \min_{\pi_u} \mathcal{C}^s(\pi_u),$$

which concludes the proof. □

We can even proof that the heuristic estimate calculated based on PDBs generated from scaled abstract planning problems is a consistent heuristic for the scaled original problem.

Theorem 6.20 (Consistent heuristic for scaled planning problems)**.** *Let $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$ be a planning problem, $\mathcal{V}'_1, \ldots, \mathcal{V}'_n$ be n sets of variables with $\mathcal{V}'_i \subseteq \mathcal{V}$ for all $1 \leq i \leq n$, and $\Phi_{\mathcal{V}'_1}, \ldots, \Phi_{\mathcal{V}'_n}$ be PDBs for the n scaled abstract planning problems $\mathcal{P}^s|_{\mathcal{V}'_1}, \ldots, \mathcal{P}^s|_{\mathcal{V}'_n}$, respectively. Then the sum of the distances to an abstract goal state resulting from the PDBs, i. e., $h(u) = \sum_{i=1}^{n} \Phi_{\mathcal{V}'_i}(u)$ for all states u is a consistent heuristic for the scaled planning problem $\mathcal{P}^s$.*

Proof. To proof the claim we must show that $h(u) \leq \mathcal{C}^s(u, v) + h(v)$ for all states u with successor v.

Following the argument for inequality 6.5 we know that

$$\mathcal{C}^s(u, v) \geq \sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(u, v).$$

With this it follows that

$$\begin{aligned} h(v) + \mathcal{C}^s(u, v) &\geq h(v) + \sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(u, v) \\ &= \sum_{i=1}^{n} \min_{\pi_{v|_{\mathcal{V}'_i}}} \mathcal{C}^s|_{\mathcal{V}'_i}\left(\pi_{v|_{\mathcal{V}'_i}}\right) + \sum_{i=1}^{n} \mathcal{C}^s|_{\mathcal{V}'_i}(u, v) \\ &= \sum_{i=1}^{n} \left(\min_{\pi_{v|_{\mathcal{V}'_i}}} \mathcal{C}^s|_{\mathcal{V}'_i}\left(\pi_{v|_{\mathcal{V}'_i}}\right) + \mathcal{C}^s|_{\mathcal{V}'_i}(u, v) \right) \\ &\geq \sum_{i=1}^{n} \min_{\pi_{u|_{\mathcal{V}'_i}}} \mathcal{C}^s|_{\mathcal{V}'_i}\left(\pi_{u|_{\mathcal{V}'_i}}\right) = h(u) \end{aligned} \tag{6.8}$$

with $\pi_{v|_{\mathcal{V}'_i}}$ being a plan starting at $v|_{\mathcal{V}'_i}$ in the scaled abstract problem $\mathcal{P}^s|_{\mathcal{V}'_i}$. Inequality 6.8 holds because for each abstraction the triangle inequality holds, i. e., an optimal plan starting at u cannot be more expensive than taking the action from u to v and then following an optimal plan from v. □

Symbolic Pattern Databases

In the non-symbolic PDB generation algorithms, originally most authors assumed that the problem at hand contains only one goal state, which of course is not helpful in the context of action planning. The adaptation to the case of multiple goals is straight-forward by starting at all the goal states at once, so that the minimal distance of a state to a goal state is known when it is reached for the first time (Korf and Felner, 2007).

The construction of *symbolic pattern databases* (Edelkamp, 2002b) works exactly the same by performing symbolic Dijkstra search (see Algorithm 6.5) in backward direction. It takes the abstract (and possibly scaled) planning problem as input. Then it starts at the abstract goal states and operates in backward direction (and omits the calculation of a plan) until all states are inserted into the database, which is essentially a list of BDDs, the BDD within each bucket of this list representing the abstract states that need a corresponding cost to reach an abstract goal state.

Here the use of BDDs is advantageous, as they handle sets of states in a native way, so that starting at all goal states is no problem at all.

Given the PDBs $\Phi_{\mathcal{V}'_1}, \ldots, \Phi_{\mathcal{V}'_n}$ for the abstract problems $\mathcal{P}|_{\mathcal{V}'_1}, \ldots, \mathcal{P}|_{\mathcal{V}'_n}$ we can calculate the set of states that have the same heuristic estimate h in the following way if the BDDs in all PDBs use the full set of variables (those not in the pattern are replaced by *don't cares*, i. e., variable v_i is replaced by $v_i \vee \neg v_i$) and the same variable ordering. Let $\Phi_{\mathcal{V}'_i}(j)$ be the bucket of the PDB for the abstract problem $\mathcal{P}|_{\mathcal{V}'_i}$ where all abstract states with a cost of j to an abstract goal state are stored. Then we must combine the abstract states of all PDBs in the following way

$$\bigvee_{\{(j_1, \ldots, j_n) | j_1 + \cdots + j_n = h\}} \bigwedge_{i=1}^{n} \Phi_{\mathcal{V}'_i}(j_i).$$

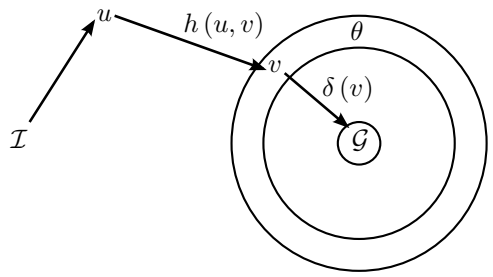


Figure 6.5: Perimeter Search. θ is the perimeter, u a state outside the perimeter, v a state on the perimeter, $h(u,v)$ a heuristic estimate on the distance from u to v and $\delta(v)$ the exact distance from v to a goal state in $\mathcal{G}$.

In other words, for a combination of values j_i whose sum over all PDBs is exactly h we calculate the conjunction of the corresponding buckets of the PDBs. This we do for all such combinations and unite the results by calculating the disjunction.

We can either calculate the full heuristic function in a preprocessing step, or we can calculate it on the fly, whenever we need it. In the latter case we can also insert the BDD $current$ for which we want to find the states that have a certain heuristic estimate into the calculation, i. e.,

$$\bigvee_{\{(j_1,\ldots,j_n)|j_1+\cdots+j_n=h\}} \bigwedge_{i=1}^{n} \Phi_{\mathcal{V}'_i}(j_i) \wedge current.$$

This way the BDDs often remain a lot smaller and the calculation is faster than the preprocessing, though similar calculations will have to be done several times. Thus, it is hard to know beforehand which approach actually will work better in terms of runtime.

6.3.2 Partial Pattern Databases

Even though abstractions can decrease the state space sizes, an abstract problem might still be too complex, so that the computation of a PDB takes too long. In such cases we should omit the complete calculation and thus ignore some of the information we could generate. This is where partial PDBs (Anderson et al., 2007) come into play. Partial PDBs are one of several approaches of combining classical PDBs with *perimeter search*.

Perimeter Search

Perimeter search (Dillenburg and Nelson, 1994) is a two-step approach. In a first step, the *perimeter* is constructed. This represents all states that have a distance of at least d to a goal state and whose successors have a distance of less than d, with d being specified beforehand. The perimeter is constructed by BFS or Dijkstra's algorithm, which is started at the goal states $\mathcal{G}$ and performed in backward direction. Whenever a state with a total cost of at least d is reached, this state is inserted into the perimeter and its predecessors are not processed.

In a second step, some (heuristic) search is performed in forward direction until a state on the perimeter is reached. Note that to find a path from a state outside the perimeter to a goal state it is always necessary to cross the perimeter, i. e., to find a path over some state v that is on the perimeter. Typical algorithms to be used are A* and IDA* (Korf, 1985). The authors named their resulting algorithms PS* and IDPS*, respectively. The heuristic estimate for a state u outside the perimeter is $h(u) = \min_{v\in\theta}(h(u,v) + \delta(v))$ with θ being the calculated perimeter, v a state on the perimeter, $h(u,v)$ a heuristic estimate on the distance from u to v, and $\delta(v)$ the exact distance from v to a goal state (cf. Figure 6.5).

A disadvantage of this approach is that for each generated state many different heuristic estimates—one for each state on the perimeter—must be calculated.

Independently, Manzini (1995) came up with a similar approach, called BIDA*, that generates the same nodes as IDPS* but uses more efficient techniques for pruning sub-optimal paths. The main difference is

that in BIDA* not all states on the perimeter are treated for the calculation of the heuristic estimate. It calculates only the heuristic estimate to each state within some *active set* of states.

It is also possible to generate the states for the perimeter not by performing BFS (or another backward search algorithm) up to a maximal depth, but to stop the backward search after a certain time. In this case, the generated but not yet expanded states will form the perimeter. If BFS was used and we are confronted only with unit-cost actions—or Dijkstra's algorithm for problems with general costs—we know the exact distance for each of these states on the perimeter.

Combining Perimeter Search and Pattern Databases

Instead of only using perimeter search or only PDBs it is actually possible to combine the two. Two such approaches are *perimeter PDBs* and *partial PDBs*, both of which we will briefly describe in the following.

Felner and Ofek (2007) proposed two algorithms for combining perimeter search and PDBs. The first one, called *simplified perimeter PDB* (SP_PDB) uses a normal PDB and a perimeter for a distance of d. All states outside the perimeter are known to have a distance of at least $d+1$. Thus, the heuristic estimate for a state u outside the perimeter can be set to $h(u) = \max(\Phi(u), d+1)$ with $\Phi(u)$ denoting the heuristic estimate that would be calculated by the PDB alone. Thus, the perimeter is used merely to correct too low heuristic estimates from the PDB up to the depth bound of $d+1$.

The second approach, which they called *perimeter PDB* (P_PDB), performs a perimeter search up to a depth of d as a first step, followed by a PDB calculation starting at the states on the perimeter. The heuristic estimates result from the PDB and give estimates on the distance to the perimeter. The forward search is then performed using these heuristic estimates until a state on the perimeter is reached and chosen for expansion.

The idea of *partial PDBs* is due to Anderson et al. (2007). A partial PDB is a pattern database that is not calculated completely. Similar to perimeter search it is created up to a maximal distance of d to a goal state, which in this case is an abstract goal state. For all states that have been reached during the PDB construction process the heuristic estimate is calculated according to the PDB, while for states not within the PDB the estimate can be set to d (or even to $d+1$, if we know for certain that all states up to distance d are within the PDB).

An adaptation to symbolic search is again straight-forward (Edelkamp and Kissmann, 2008d). All we need to do is perform symbolic Dijkstra search starting at the (abstract) goal states up to a maximal distance of d and then stop the PDB creation process. In our approach we do not set the value of d explicitly but generate the PDB until a certain timeout is reached. The maximal distance up to which states are within the PDB, i. e., states that are expanded, is then denoted by d. As we are concerned with symbolic search and have one BDD for all states sharing the same distance to the goal states we are sure that all states not within the database have a distance that is greater than d, so that we assign all such states a distance of $d+1$.

Note that a heuristic based on partial PDBs is admissible and consistent as well. Concerning admissibility, we find the same heuristic estimates for all states that are within the PDBs, while for those outside we now find an estimate that is smaller than in the case of full PDBs. Thus, as the heuristic derived from full PDBs never overestimates the true distance to a goal state, the heuristic based on partial PDBs does not do so either. Concerning consistency, the most important property, i. e., the triangle inequality, still holds for all PDBs, so that the proof from Theorem 6.20 requires only slight adaptations and then still works in the case of partial PDBs.

6.4 Implementation and Improvements

We have implemented a planner, GAMER (Edelkamp and Kissmann, 2008a, 2009), based on the algorithms we presented in the previous sections. That is, in case of unit-cost actions the planner performs symbolic bidirectional BFS, while in case of general action costs it performs BDDA*. The heuristic is generated by using a single symbolic partial PDB, where the pattern is actually the full variable set, so that we do not use any abstraction.

For the bucket structure needed in BDDA* we chose to use a simple matrix (two-dimensional, or three-dimensional if zero-cost actions are present), which has a fixed width (the h axis) of $heur_{max}$ and a variable

height (the g axis)—whenever a state is to be inserted with a g value higher than the currently highest possible value in the matrix we extend the matrix by some fixed amount. This way, we have a bucket for every possible g-h-combination, no matter if the bucket actually contains some states or not.

We use a similar structure for Dijkstra's algorithm, which we use for the PDB generation. Here, a one-dimensional—or two-dimensional, in case of zero-cost actions—list is sufficient.

While in case of BFS the bidirectional search is comparatively simple to implement we did not find any way to do so in case of BDDA* and PDBs, at least not in an interleaved manner, as we did in case of BFS. The problem is that we need the heuristic generated in backward direction in the forward search, and we also expect the heuristic to be fixed, i. e., not to change in the course of the search. Thus, for stopping the PDB generation we chose to set a fixed timeout. In the competition each planner was allowed to take up to 30 minutes for each problem, so that we thought it best to set the time for the PDB generation to half that time, i. e., 15 minutes. This way we still cannot perform an interleaved bidirectional search, but rather we weight forward and backward search equally, which appears to be a reasonable compromise.

In our implementation it is not possible to stop a BDD operation from within the program, so that we chose to start one process for the PDB generation, kill it after the timeout and then—or after the PDB generation has finished, if that happens earlier—start the BDDA* process.

We implemented the algorithms in Java, using the native interface JavaBDD[13] to access the BDD library, in our case Fabio Somenzi's efficient C++ library CUDD[14], so that the time consuming BDD operations can be performed most efficiently.

We need the precise set of variables (ground predicates) when we create the first BDD, but the input in PDDL typically makes use of variables in the predicates, so that we cannot use it immediately. Instead we used a grounding utility (Edelkamp and Helmert, 1999), which not only instantiates the input by transforming it to a grounded representation, but also finds an efficient SAS$^+$ encoding (Bäckström and Nebel, 1995) of the set of binary variables used in the BDDs. In the SAS$^+$ encoding some variables—those that are mutually exclusive—are grouped together, so that it enables us to use fewer BDD variables to represent a state. Such a group of n mutually exclusive predicates can be represented by only $\lceil \log n \rceil$ BDD variables instead of the n we would need in a default *one-shot* encoding.

Additionally to the grouping the ground predicates are ordered lexicographically. This reflected our hope that predicates sharing the same name might also affect each other and thus bringing them close together might help in reducing the BDDs' sizes.

With this planner we participated in the sequential optimal track of the International Planning Competition (IPC) 2008 and were able to win that track (we will give detailed results on this in Section 6.5).

For the following competition in 2011 we came up with a number of improvements, all of which are rather simple but in the end result in an impressive boost in GAMER's performance (Kissmann and Edelkamp, 2011). In the following we will present these improvements in detail.

6.4.1 Replacing the Matrix

During IPC 2008 we noticed that for one domain (namely PARC-PRINTER) the matrix-based approach used in BDDA* (cf. Algorithm 6.7) did not work well. GAMER was not able to solve a single instance, while the other planners did not have such problems. The main difficulty of this domain lies in the action costs, which are immensely large. While the action costs in most domains do not exceed 100, here some actions have costs of more than 100,000. Due to this, the matrix becomes very large and at the same time very sparse, so that finding the next non-empty bucket takes a long time and after only very few steps it does not fit into the available memory any more.

Therefore, we decided to replace the matrix by a map of maps of BDDs. A map takes a key and returns the object stored with the specified key. For the outer map we chose to take the f value as the key and for the inner one the g value. This way, all states on the same f diagonal are stored within the same position of the outer map, so that we only need to find the smallest key stored in the map. The same holds for the inner map, where we also always take the BDD stored under the smallest key in order to go along an f diagonal with increasing g values. After expanding a BDD it is removed from the inner map (along with its

[13] http://javabdd.sourceforge.net
[14] http://vlsi.colorado.edu/~fabio/CUDD

key). Once an f diagonal is fully expanded, i. e., when the inner map for the specified f value is empty, it is removed from the outer map.

This brings the disadvantage that we need an additional structure for the retrieval of a plan but the overhead in memory is small, as the BDD is moved completely from one two-dimensional map to the other.

The advantage, of course, is that we no longer need to store all positions of the matrix, which helps to reduce the memory usage in case of a sparse matrix. Also, we do not need to search several empty buckets of the matrix in order to find the next non-empty one. Rather, we can simply use the $deleteMin$ function of the priority queue on which the map structure is based and thus find the next BDD to be expanded a lot faster—again, this advantage only catches if the matrix is rather sparse.

A similar procedure is possible in the PDB generation process (and thus also in the Dijkstra search). We can replace the list used there by a one-dimensional map with the distance being the key and the corresponding BDD the value. Though before we did not run out of memory, this actually helps a lot as now we are able to faster find the next BDD to be expanded and thus are able to generate larger PDBs resulting in more informed heuristics.

6.4.2 Reordering the Variables

The variable ordering used in the IPC 2008 version of the planner is by no means optimal, so that we decided to investigate how to improve it. Many of the current BDD packages support dynamic variable reordering. The shortcoming of these approaches is the need of *sifting* operations on existing BDDs, so that they are often too slow to be of much use in planning. Additionally, for us another problem is that we store the PDBs on the hard disk and load them from there at the start of BDDA*, so that they must have the same ordering as the one we use in BDDA*. The most reasonable approach to achieve this is to find one variable ordering at the very start of the planning process and then stick to it.

One way to find a better ordering is to exploit the knowledge on the dependencies between the variables we can deduce from the planning descriptions. This can be done by evaluating the dependencies between the SAS$^+$ groups, which we can determine by constructing the *causal graph* (see, e. g., Knoblock, 1994; Helmert, 2004).

Definition 6.21 (Causal Graph)**.** *The* causal graph *of a planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$ is a directed graph $G = (V, E)$ with $V = \{gr(v) \mid v \in \mathcal{V}\}$ and $(u, u') \in E$ if and only if $u \neq u'$ and there is an action $a \in \mathcal{A}$ with an element of u' in the effect and an element of u in the precondition or the effect, with $gr(v)$ being the SAS$^+$ group variable v resides in. This implies that an edge is drawn from one group to another if the change of the second group's value is dependent on the current assignment of the first group's. The same structure can also be generated for planning problems with general action costs $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$.*

We use what we call a *symmetric causal graph*, i. e., a causal graph with the dependencies applied in both directions, so that we arrive at a symmetrical relation.

Using these dependencies we approximate the *simple linear arrangement problem* (Garey et al., 1976).

Definition 6.22 (Linear Arrangement Problem)**.** *Given a weighted graph $G = (V, E, c)$, the goal of the* linear arrangement problem *is to find a permutation $\pi : V \mapsto \{0, \ldots, |V| - 1\}$ that minimizes $\sum_{e=(u,v)\in E} c(e) \times |\pi(u) - \pi(v)|$.*

Definition 6.23 (Simple Linear Arrangement Problem)**.** *The linear arrangement problem on graphs without (or with unitary) weights c is called the* simple linear arrangement problem*.*

By reduction to Max-Cut and Max-2-Sat, Garey et al. proved the following result.

Theorem 6.24 (Complexity simple linear arrangement problem)**.** *Finding an optimal solution for the simple linear arrangement problem is NP-hard*

Furthermore, Devanur et al. (2006) proved that it is even hard to provide approximations within any constant factor.

There are different known generalizations to the linear arrangement problem, e. g., the *quadratic assignment problem* $\sum_{e=(u,v)\in E} c(e) \times c'(\pi(u), \pi(v))$ for some weight c' is a generalization, which includes

Algorithm 6.9: Variable Reordering

Input: Symmetric causal graph $G = (V, E)$, $|V| = n$.
Output: Variable ordering π_{best}.

1 $\vartheta_{best} \leftarrow \infty$ // Best pairwise distance of entire ordering wrt. the causal graph, initialized to ∞.
2 $\pi_{best} \leftarrow (0, \ldots, n-1)$ // Default permutation.
3 **for** 1 **to** ρ **do** // Restart ρ times with new permutation.
4 $\quad \pi \leftarrow randomPermutation\,(n)$ // Generate random permutation.
5 $\quad \vartheta_{old} \leftarrow \vartheta \leftarrow evaluate\,(\pi)$ // Evaluate entire current permutation.
6 $\quad$ **for** 1 **to** η **do** // Perform η transpositions.
7 $\quad\quad swapInd_1 \leftarrow random\,(0, n-1)$ // Determine index of first element for transposition.
8 $\quad\quad swapInd_2 \leftarrow random\,(0, n-1)$ // Determine index of second element for transposition.
9 $\quad\quad$ **for** $i \leftarrow 0$ **to** $n-1$ **do** // Iterate once through permutation.
10 $\quad\quad\quad$ **if** $i = swapInd_1$ **or** $i = swapInd_2$ **then continue** // No change for transposition elements.
11 $\quad\quad\quad$ **if** $(\pi(i), \pi(swapInd_1)) \in E$ **then** // If elements on positions i and $swapInd_1$ dependent...
12 $\quad\quad\quad\quad \vartheta \leftarrow \vartheta - (i - swapInd_1)^2 + (i - swapInd_2)^2$ // Update distance for this pair.
13 $\quad\quad\quad$ **if** $(\pi(i), \pi(swapInd_2)) \in E$ **then** // If elements on positions i and $swapInd_2$ dependent...
14 $\quad\quad\quad\quad \vartheta \leftarrow \vartheta - (i - swapInd_2)^2 + (i - swapInd_1)^2$ // Update distance for this pair.
15 $\quad\quad$ **if** $\vartheta < \vartheta_{old}$ **then** // If improvement in pairwise distance...
16 $\quad\quad\quad \vartheta_{old} \leftarrow \vartheta$ // Store current distance.
17 $\quad\quad\quad swap\,(\pi, swapInd_1, swapInd_2)$ // Apply transposition.
18 $\quad\quad$ **else** // Otherwise...
19 $\quad\quad\quad \vartheta \leftarrow \vartheta_{old}$ // Retain distance of previous permutation.
20 $\quad$ **if** $\vartheta_{old} < \vartheta_{best}$ **then** // If improvement on best pairwise distance...
21 $\quad\quad \vartheta_{best} \leftarrow \vartheta_{old}$ // Store current permutation's distance.
22 $\quad\quad \pi_{best} \leftarrow \pi$ // Store current permutation as best one.
23 **return** π_{best} // Return best permutation found.

the traveling salesman problem (TSP) as one case (Lawler, 1962). We consider the optimization function $\vartheta(\pi) = \sum_{(u,v)\in E \vee (v,u)\in E} d(\pi(u), \pi(v))$, subject to the metric $d(x, y) = \|x - y\|_2^2 = \sum_{i=1}^{n} (x_i - y_i)^2$. The graph we use is the symmetric causal graph.

Due to the problem's complexity we apply a greedy search procedure for optimization with two loops (cf. Algorithm 6.9). The outer loop calculates a fixed number ρ of random permutations, while the inner loop performs a fixed number η of transpositions on the current permutation by choosing two indices to be swapped randomly. Before the transpositions are actually applied they are evaluated (lines 9 to 14). If the result is an improvement, i. e., ϑ decreases, the transposition is applied (line 17) and the next pair of indices is chosen. Otherwise, it is discarded (line 19). At the end, the ordering π_{best} with smallest ϑ is chosen.

In a naïve approach the function ϑ would be evaluated in each step on the full permutation π. This results in a runtime of $\mathcal{O}\left(|V|^2\right)$, because in the worst case we must determine the distance for each pair of indices. To increase the values of ρ and η we decided to incrementally compute $\vartheta(\pi)$ as follows. Let $\tau(i, j)$ be the transposition to be applied to the permutation π to obtain the new permutation π' and let x and y be the variables associated to the indices i and j in π. Then we have

$$\begin{aligned}\vartheta(\pi') &= \sum_{(u,v)\in E} d(\pi'(u), \pi'(v)) \\ &= \vartheta(\pi) + \sum_{(w,x)\in E} (d(\pi(w), j) - d(\pi(w), i)) + \sum_{(w,y)\in E} (d(\pi(w), i) - d(\pi(w), j)).\end{aligned}$$

This incremental evaluation is already present in Algorithm 6.9 (lines 9 to 14). With it the complexity for calculating $\vartheta(\pi')$ reduces to linear time $\mathcal{O}(|V|)$ for most of the evaluations—ϑ has to be calculated

anew at a cost of $\mathcal{O}\left(|V|^2\right)$ only after a restart with a new (random) ordering. In total, the runtime reduces from $\mathcal{O}\left(\rho\,(\eta+1)\,|V|^2\right)$ to $\mathcal{O}\left(\rho\,|V|^2+\rho\eta\,|V|\right)$. Thus, we can perform millions of updates in a matter of seconds instead of minutes—of course depending on the domain and the corresponding causal graph.

6.4.3 Incremental Abstraction Calculation

Another improvement we implemented is concerned with the heuristic construction. While the version of GAMER we implemented for IPC 2008 does not perform any abstraction, we chose to extend it by one, in order to faster generate some PDB or to find partial PDBs with higher maximal cost.

For explicit search, an automated selection of patterns for PDBs (and thus an automated calculation of an abstraction) has been considered by Haslum et al. (2007). For one PDB they greedily construct a pattern $\mathcal{V}''$ as an extension of a previously used pattern $\mathcal{V}'$ by adding one variable $v \in \mathcal{V} \setminus \mathcal{V}'$ of the unchosen ones at a time. For the greedy selection process the quality of the unconstructed PDBs $\mathcal{V}' \cup \{v\}$ is evaluated by drawing and abstracting n random samples in the original state space. These subproblems are solved using heuristic search with respect to the already constructed PDB $\mathcal{V}'$. The decision criterion for selecting the next candidate variable is the search tree prediction formula for IDA* proposed by Korf et al. (2001). If v is fixed then the PDB for $\mathcal{V}' \cup \{v\}$ is constructed and the process iterates.

Our symbolic approach works in a similar manner (cf. Algorithm 6.10). The initial abstraction uses a pattern containing all the variables that appear in the description of the goal states (line 1). For the greedy extension of this pattern we try all variables on which variables in the current pattern depend, i. e., that share an edge in the causal graph. However, we neither use sampling nor heuristics but construct the symbolic PDBs for all candidate variables. That is, if the pattern currently contains the variables $\mathcal{V}' \subset \mathcal{V}$, we construct the symbolic PDBs for $\mathcal{V}' \cup \{v\}$ for all variables $v \in \mathcal{V} \setminus \mathcal{V}'$ if the causal graph contains an edge from v to any of the variables in $\mathcal{V}'$ (lines 8 to 15).

To determine with which pattern to continue we evaluate the PDB in terms of average cost (in the abstract problem), motivated by the idea that a heuristic with a higher average cost provides a stronger lower bound on the actual cost in the original problem and is thus a more informed heuristic (line 11). In order to determine this average cost we use model counting (*Satisfy-count* as proposed by Bryant (1986), see Section 2.2.3 on page 14), an operation linear in the size of the constructed PDB and easier to compute than evaluating the predication formula.

After we are done with testing the insertion of all candidate variables we actually add all those that achieved the best mean heuristic value (and those that were worse by a small factor of ω), if that is greater than the mean heuristic value of $\mathcal{V}'$ (line 18). In case the pattern is extended by only one variable we can immediately continue with new candidate variables, as the PDB for this pattern has already been generated and evaluated. If more than one new variable was added we must generate the PDB for this new pattern and evaluate it before actually continuing (lines 19 to 24).

Though in many domains the description of the goal states contains only few variables, there are some in which it already contains the full set of variables, so that we actually do not arrive at any abstraction. Even if it consists of fewer than there are in the entire set it might still be too large to calculate the full PDB in time, so that in such a case we use the partial PDB generated until the timeout as the basis of our heuristic.

While in Haslum et al.'s approach (2007) the result was a number of PDBs, we still use only one, namely the PDB that resulted in the highest average cost over all PDBs generated.

All PDBs are stored on disk, so that it is sufficient to store only the index of the currently best PDB. When starting BDDA* we then load the PDB that has the highest average cost, which is just the one that corresponds to the last stored index.

6.4.4 Amount of Bidirection

In preliminary experiments we found that for some domains the use of abstraction works better while for others the backward search without abstraction yields better results. This is mainly due to the difficulty of some domains in the context of backward search, in which the backward search tends to generate lots of

Algorithm 6.10: Incremental Abstraction

Input: Planning problem $\mathcal{P} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{C} \rangle$.
Input: Causal graph $G = (V, E)$.
$\mathcal{V}' \leftarrow \{v \mid v \text{ in goal description } \mathcal{G}\}$ // Calculate initial pattern using all variables in goal description.
$index \leftarrow index_{best} \leftarrow 0$ // Best PDB so far is the first one, index 0.
store $index_{best}$ // Store index of the currently best PDB on disk.
$avg_{best} \leftarrow generatePDB\,(index_{best}, \mathcal{P}, \mathcal{V}')$ // Generate PDB for selected pattern and // determine the average cost.
while $index_{best} \neq -1$ **do** // While improvement over previous pattern is found...
  $index_{best} \leftarrow -1$ // Re-initialization of stopping criterion.
  $averages \leftarrow$ new map of size $\mathcal{V} \setminus \mathcal{V}'$ // Storage for all average costs for final selection.
  for all $v \in \mathcal{V} \setminus \mathcal{V}'$ **with** $\exists u \in \mathcal{V}' \,.\, (v, u) \in E$ **do** // Check all candidate variables.
    $index \leftarrow index + 1$ // Increase the current PDB's index.
    $\mathcal{V}'' \leftarrow \mathcal{V}' \cup \{v\}$ // Extend pattern.
    $averages\,[v] \leftarrow generatePDB\,(index, \mathcal{P}, \mathcal{V}'')$ // Generate PDB and store average cost.
    if $averages\,[v] > avg_{best}$ **then** // If new average cost better than currently best one...
      $avg_{best} \leftarrow averages\,[v]$ // Store new average cost as best one.
      $index_{best} \leftarrow index$ // Store current PDB's index as best one.
      store $index$ // Store index of currently best PDB on disk.
  if $index_{best} \neq -1$ **then** // If improvement over old best PDB found...
    $size_{old} \leftarrow |\mathcal{V}'|$ // Store size of old pattern.
    $\mathcal{V}' \leftarrow \mathcal{V}' \cup \{v \mid v \in \mathcal{V} \setminus \mathcal{V}' \wedge averages\,[v] \geq \omega \times avg_{best}\}$ // Extend pattern by those variables // that generate a PDB with sufficient average cost.
    if $|\mathcal{V}'| > size_{old} + 1$ **then** // If more than one variable was added...
      $index \leftarrow index + 1$ // Increase index to generate new PDB.
      $avg \leftarrow generatePDB\,(index, \mathcal{P}, \mathcal{V}')$ // Generate PDB and determine average cost.
      if $avg > avg_{best}$ **then** // If PDB's average cost is better than currently best one...
        $avg_{best} \leftarrow avg$ // Store new average cost as best one.
        store $index$ // Store index of currently bet PDB on disk.

states unreachable in forward direction. In case of the PDB generation for the abstract problems this effect is less prominent due to the reduced complexity achieved by restricting the variables to a smaller subset.

Using BFS we found similar results. For some domains forward search is faster, for others backward search, while the bidirectional approach tries to find the best ratio between the two.

In most domains that are difficult in backward direction already the first backward expansion (from $\mathcal{G}$) takes considerably longer than the first forward expansion (from $\mathcal{I}$). Thus, we decided to calculate one image from $\mathcal{I}$ and one pre-image from $\mathcal{G}$ and compare the runtimes. If the backward step takes considerably longer, i. e., longer than β times the forward step's time, we turn off bidirectional search. This means that in case of unit-cost actions we perform unidirectional forward BFS instead of bidirectional BFS, and in case of general action costs we use abstractions for the PDB, while we use no abstraction if the ratio is below β.

6.5 International Planning Competitions (IPCs) and Experimental Evaluation

We participated in the international planning competition IPC 2008[15] with the original version—the version without the improvements proposed in the previous section—of our planner GAMER (Edelkamp and Kissmann, 2008a, 2009). The setting of the competition was the following. The planners were limited to use

[15] http://ipc.informatik.uni-freiburg.de

at most 2 GB of (internal) memory and 30 minutes real-time for each problem. Parallel planners were not allowed. The whole competition was run by the organizers on a cluster, so that no team had an advantage based on the used hardware.

The sequential optimal track, i. e., the one handling optimal classical planning, contained a total of eight new domains—ELEVATORS, OPENSTACKS, PARC-PRINTER, PEG-SOLITAIRE, SCANALYZER, SOKOBAN, TRANSPORT, and WOODWORKING —with 30 problem instances each, all of which were newly designed for this competition. All domains contain general action costs, a novelty of IPC 2008, as in previous competitions the classical planning problems always came with unit-cost actions. For some domains there were formulations satisfying different requirements of PDDL, but for the comparison for each planner only that formulation where it performed best was taken into account.

In the optimal track the planners were required to find optimal plans. Finding a sub-optimal one resulted in the planner getting 0 points for the entire domain, no matter how many optimal plans it found in that domain. To compare the planners only the number of solved instances for each domain was taken into account, not the runtime. Thus, to win it was necessary to solve the highest number of problems in total.

A total of nine planners participated in the competition, though two of them were rated only non-competitively as they either could not handle general action costs and thus returned suboptimal plans, or the returned plans were only optimal with regard to some planning horizon but not necessarily globally optimal.

After implementing the improvements proposed in the previous section we submitted the planner to IPC 2011[16]. There the setting was similar to that of 2008. Again the competition was run on a cluster by the organizers, and again the time-limit for each problem were 30 minutes real-time. Only the amount of available memory was increased, from 2 to 6 GB. For that competition a total of 14 domains were used, each with 20 problem instances. The domains were the eight of IPC 2008 and six new ones, namely BARMAN, FLOORTILE, NOMYSTERY, PARKING, TIDYBOT, and VISITALL. Of the new domains, only BARMAN and FLOORTILE contain general action costs.

In that competition a total of twelve planners participated. Most of these were actually based on the same basic planner, namely Helmert's FAST DOWNWARD (2006; 2009), though most implemented their own heuristics (or portfolios thereof) or their own search procedures.

In the following (Section 6.5.1), we will briefly outline the ideas of each of the six competitors of IPC 2008. Then, in Section 6.5.2 we will show the results achieved in the competition. In Section 6.5.3 we will compare our new improvements to the original version of GAMER to see what an impact they actually have. After that we briefly outline the planners of IPC 2011 in Section 6.5.4 and show results achieved in the course of the competition and also afterward in Section 6.5.5.

6.5.1 IPC 2008: Participating Planners

In the following, we will provide brief descriptions of the other participating planners.

CFDP

CFDP (Grandcolas and Pain-Barre, 2008) is based on FDP (*Filtering, Decomposition and Search Space Reduction*) (Grandcolas and Pain-Barre, 2007), a sequential optimal planner performing planning as constraint satisfaction. FDP basically uses an iterative deepening DFS procedure. Thus, it searches for a plan of length k using DFS starting from the initial state $\mathcal{I}$ and increases the bound k each time the DFS is unsuccessful, ensuring the optimality (minimal number of actions) of the first found plan. It makes use of several methods (e. g., filtering of inconsistent values and actions, a divide-and-conquer approach for searching for a plan in their planning structure, or decomposition procedures such as enumerating actions, assigning literal variables or splitting action sets).

The extension of CFDP over FDP is that it can handle general action costs and thus calculates cost-optimal plans. It operates in three phases.

In the first phase, it calculates a step-optimal plan using FDP. This plan establishes a lower bound on the length of a cost-optimal plan, as well a an upper bound on its total cost.

[16] http://www.plg.inf.uc3m.es/ipc2011-deterministic

The second phase starts at the goal states $\mathcal{G}$ and performs iterative DFS in backward direction to enumerate all sequences of length less than a given bound l_{max}. For each length the minimal costs are memorized during the search, which helps to prune the search in the third phase.

In the final phase again the FDP search procedure is used. In its search structure the goal states are set as undefined. The structure is extended in each step until it can be proved that no longer plan of cost less than the current minimal cost exists. It terminates when all dead ends are caused by the violation of the cost constraint, which aborts search in all states that cannot achieve a plan with a total cost smaller than the best one found so far.

CO-PLAN

Most SAT-based planners, such as SATPLAN (Kautz and Selman, 1996; Kautz et al., 2006) or MAXPLAN (Chen et al., 2007), so far cannot handle general action costs, which were an important novelty of IPC 2008. Thus, the idea behind CO-PLAN (Robinson et al., 2008), a cost-optimal SAT-based planner, is to remedy this.

CO-PLAN performs two steps. In the first step it constructs plangraphs (Blum and Furst, 1997) of increasing length. At this point, action costs are completely ignored, but reachability and neededness analysis are performed. Problems of the ith plangraph are compiled into conjunctive normal form (CNF). Their solutions correspond to parallel plans with i steps. During this first step, a modified version of RSAT (Pipatsrisawat and Darwiche, 2007), which the authors call CORSAT, is used to work on the CNF formulas. The modifications enable it to identify whether a plan exists and, if one does, to identify the one with minimal total action cost.

The second step is started once the first one finds a plan. It performs a forward search in the problem space, which is bounded by the cost of the returned (possibly sub-optimal) plan.

CPT3

The *Constraint Programming Temporal Planner* CPT (Vidal and Geffner, 2006) calculates optimal temporal plans based on constraint programming. It combines a partial order causal link (POCL) branching scheme with several pruning rules. In the temporal track it optimizes the makespan by a branch-and-bound approach.

For the sequential optimal track a total ordering of the actions is enforced. To treat general action costs it translates classical actions to temporal actions by transforming the costs to durations.

HSP$^*_0$ and HSP$^*_F$

The two planners HSP$^*_0$ and HSP$^*_F$ (Haslum, 2008) are essentially the same planner, only that the first one operates in backward direction while the latter operates in forward direction. Haslum submitted both planners as it is suggested that some planning domains are better suited for forward search based planners while others are better for backward search. This has not been sufficiently analyzed before, especially as often core procedures of the planners were different, such as, e. g., the heuristics used. Thus, these two planners are designed to be as similar as possible, hoping to gain some more insight into this.

Both planners work with STRIPS input (Fikes and Nilsson, 1971), at least after a pre-calculation step where some features, e. g., the newly introduced functional variables, are compiled into a format the planners can work with.

The planners use the critical path heuristic h^m (Haslum and Geffner, 2000), which we have briefly sketched in Section 6.3. It is designed to be used in backward search. Thus, for HSP$^*_F$ to work in forward direction the planning problem $\mathcal{P}$ is reversed into $R(\mathcal{P})$. This way, if a (reversed) plan $R(\pi) = a_1, \ldots, a_n$ is found for $R(\mathcal{P})$ the reversal of this plan $\pi = a_n, \ldots, a_1$ results in a plan for $\mathcal{P}$. Such a reversal was originally proposed by Massey (1999) for STRIPS planning problems. Similarly, Pettersson (2005) created a reversed planning graph, coming up with a version of GRAPHPLAN operating in forward direction.

Table 6.1: Numbers of solved problems for all domains of the sequential optimal track of IPC 2008, competition results.

Domain	BASE	GAMER	HSP^*_F	HSP^*_0	CO-PLAN	MIPS XXL	CPT3	CFDP
ELEVATORS (30)	11	**22**	8	8	6	2	1	0
OPENSTACKS (30)	19	20	**21**	6	4	6	1	6
PARC-PRINTER (30)	10	0	16	14	5	7	**17**	3
PEG-SOLITAIRE (30)	**27**	22	**27**	21	25	24	10	8
SCANALYZER (30)	**12**	9	6	11	6	6	3	3
SOKOBAN (30)	**19**	18	14	4	12	6	3	0
TRANSPORT (30)	**11**	**11**	10	9	8	6	3	0
WOODWORKING (30)	7	**13**	9	9	5	5	8	4
Total (240)	**116**	115	111	82	71	62	46	24

MIPS XXL

The only external-memory planner of IPC 2008 was MIPS XXL (Edelkamp and Jabbar, 2008). This stores the reached states on a hard disk, which typically is orders of magnitude slower than RAM, but its size is large compared to the limit of 2 GB used for the internal memory.

In external memory algorithms the main bottleneck is the access to the hard disk. Furthermore, hard disks are especially bad in performing random access (reads or writes). Thus, according to the external memory model by Aggarwal and Vitter (1988) the complexity of these algorithms is measured in the number of scans and the number of sortings of files.

The algorithm used in MIPS XXL is an external version of Dijkstra's single source shortest path algorithm, which looks similar to the symbolic version we have presented in Algorithm 6.5.

6.5.2 IPC 2008: Results

The results are publicly available on the competition's website[17] and are also given in Table 6.1.

For the evaluation the organizers provided an additional baseline planner (BASE), which performs A* with a zero-heuristic, i. e., Dijkstra's algorithm, but it participated only non-competitively.

In the end, GAMER was able to win the competition with a slight advantage over HSP^*_F, but was actually beaten by the organizer's baseline planner. When looking at the results it is apparent that GAMER did not find any solution in the PARC-PRINTER domain, as we have pointed out before, which lead to the replacement of the matrix by a two-dimensional map. Afterward we were able to solve nine problems on a machine similar to the one used in the competition, so that GAMER would have won more clearly and also outperformed the baseline planner.

We give a more detailed analysis of the different improvements of GAMER in the next section.

6.5.3 Evaluation of the Improvements of GAMER

In the course of the preparation of GAMER for IPC 2011 we came up with a number of improvements (Kissmann and Edelkamp, 2011), as detailed in Section 6.4. In this section we will investigate the influence of each of the improvements on the total performance and compare it to the performance of the original version of GAMER, which participated in IPC 2008.

For this, we ran all the problems from IPC 2008 on our own machine (Intel $i7$ 920 CPU with 2.67 GHz and 24 GB RAM) using a timeout of 30 minutes, but no limit on the usable main memory. The different versions of GAMER are the following.

[17] http://ipc.informatik.uni-freiburg.de/Results

Table 6.2: Numbers of solved problems for all domains of the sequential optimal track of IPC 2008, own results.

Domain	BASE	GAMER	GAMER $^{m}_{p}$	GAMER $^{m}_{a}$	GAMER $^{m}_{b}$	GAMER $^{m,r}_{p}$	GAMER $^{m,r}_{a}$	GAMER $^{m,r}_{b}$
ELEVATORS (30)	17	**24**	**24**	21	**24**	**24**	20	**24**
OPENSTACKS (30)	23	22	22	22	22	**30**	**30**	**30**
PARC-PRINTER (30)	11	0	11	**12**	**12**	11	11	11
PEG-SOLITAIRE (30)	**28**	25	25	25	25	27	27	27
SCANALYZER (30)	**12**	9	9	10	10	9	9	9
SOKOBAN (30)	**24**	19	19	23	20	21	**24**	22
TRANSPORT (30)	**11**	**11**	**11**	10	**11**	**11**	10	**11**
WOODWORKING (30)	9	14	14	17	14	22	**24**	22
Total (240)	135	124	135	140	138	155	155	**156**

GAMER The original version of the planner, as it participated in IPC 2008.

GAMER $^{m}_{p}$ The original version with a map instead of a matrix.

GAMER $^{m}_{a}$ The original version with a map and with automated abstraction.

GAMER $^{m}_{b}$ The original version with a map and with automatic choice between abstraction and PDB generation in the original space, i. e., the amount of bidirection.

GAMER $^{m,r}_{p}$ The original version with a map and with reordering of the variables enabled.

GAMER $^{m,r}_{a}$ The original version with a map, reordering enabled, and abstraction.

GAMER $^{m,r}_{b}$ The version of GAMER used in IPC 2011, which contains all improvements (map, reordering, and choice between abstract or non-abstract PDB generation).

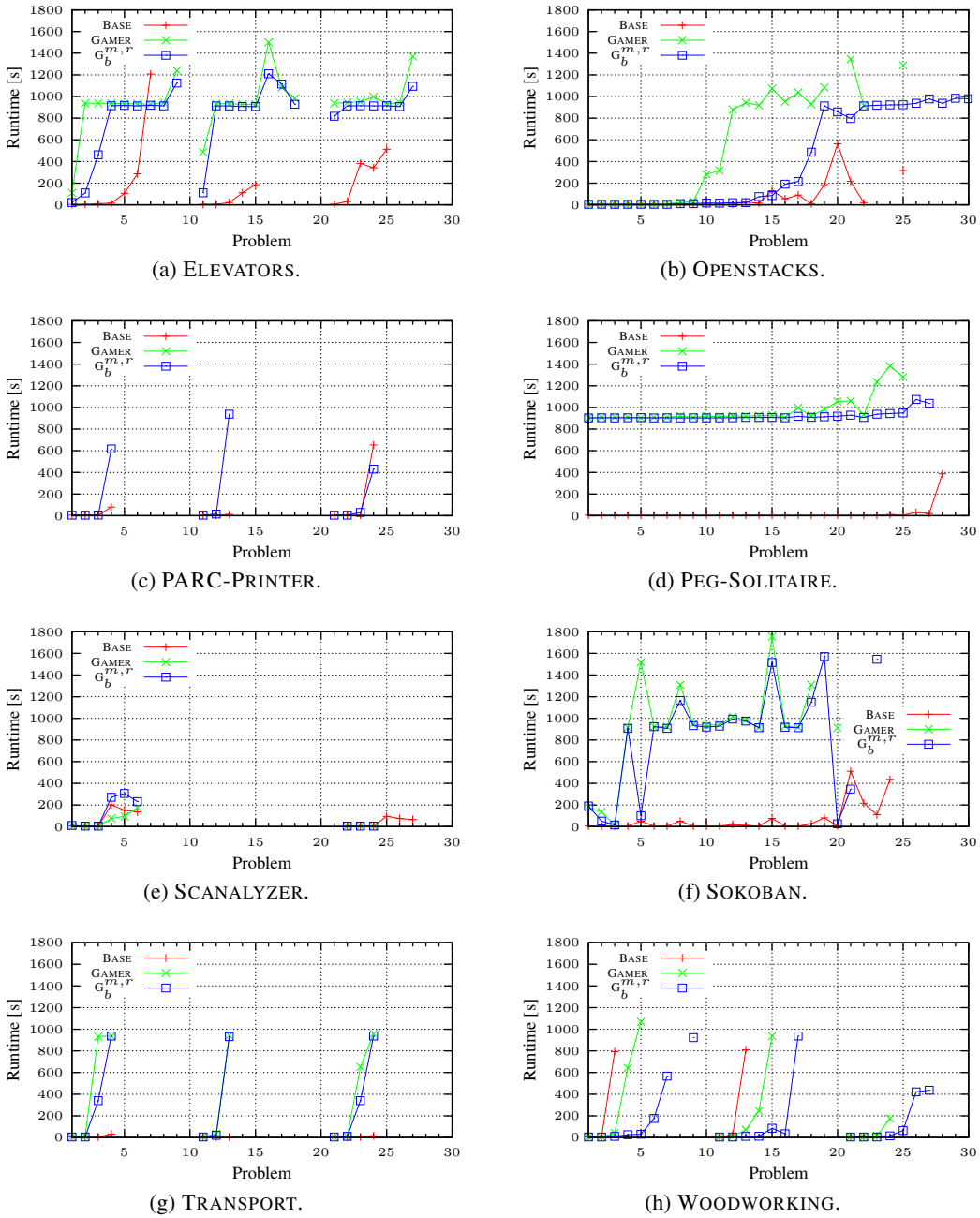
For our comparison we also used the baseline planner BASE from the competition, to get a feeling how good we compare to an explicit search planner. The results, i. e., the number of solved instances for each domain, are detailed in Table 6.2.

On our machine the advantage of BASE against GAMER is a lot greater than it was in the real competition, as the difference in the number of solved instances increased from one to eleven. This indicates that BASE is much more limited by the memory than GAMER is. Replacing the matrix by the two-dimensional map structure increases the number of solved instances of the PARC-PRINTER domain from zero to eleven, while the numbers in all other domains remain unchanged.

Overall, we can see that the version of GAMER that uses the map, reordering and the bidirectional choice is the best of all the tested versions. The biggest advantage comes with the reordering, which helps especially in the OPENSTACKS and the WOODWORKING domains by increasing the number of solved instances by up to eight in each domain. It might be that we would be able to find even more solutions in the OPENSTACKS domain if there were more problems, as we were able to find solutions for all 30 instances. While it does not bring much of an advantage in the other domains, it also does not hinder the planner, the number of solved instances remaining largely the same.

Concerning the abstraction we can see that in SOKOBAN and WOODWORKING the number of solved instances increases by up to four, while in the ELEVATORS domain the number decreases by up to four.

Finally, the bidirectional choice works to some extent, but is not yet really satisfactory. Too often it chooses the version without abstraction, no matter if that actually is the better version, though sometimes it really makes the right decision in choosing the version with abstraction. It seems that the criterion of the runtimes for the first step is not yet sufficient, because in some of the problems where the abstraction works better we did not really see a difference in the runtimes.



(a) ELEVATORS. (b) OPENSTACKS.

(c) PARC-PRINTER. (d) PEG-SOLITAIRE.

(e) SCANALYZER. (f) SOKOBAN.

(g) TRANSPORT. (h) WOODWORKING.

Figure 6.6: Runtime results for all domains of the sequential optimal track of IPC 2008 on our own machine.

Concerning the runtimes for finding the plans we provide some results in Figure 6.6. This shows the results for the baseline planner BASE, the IPC 2008 version of GAMER and the IPC 2011 version of GAMER, denoted by $G_b^{m,r}$. However, any comparison to the explicit state planner BASE is unfair, as this does not perform backward search, which often takes a long time in GAMER, but still we inserted it as a reference.

Recall that for the PDB generation we allow up to half the available time, i. e., 15 minutes or 900 seconds. In several of the domains, e. g., ELEVATORS, OPENSTACKS, PEG-SOLITAIRE, and SOKOBAN, we can see that the two versions of GAMER finish after roughly 900 seconds. This happens because the generation of the PDBs was not yet finished, but nevertheless the heuristic is good enough, so that the forward search finishes in a matter of seconds.

Overall, we can see that in most domains the version with all improvements is faster than the 2008 version, but only in OPENSTACKS, PEG-SOLITAIRE, SOKOBAN, and WOODWORKING the difference is quite big. Surprisingly, in the SCANALYZER domain the new version actually is slower than the old one. When looking once more at Table 6.2 we can see that there the number of solutions for the versions with abstraction or automated bidirectional choice was worse in the setting with variable reordering than in the one without. Thus, it seems that the reordering might actually increase the runtime in this domain, though it is unclear why that is the case.

To sum up, we have seen that most of the improvements actually bring an advantage to the planner, though the automated bidirectional choice is not yet optimal and can very likely be improved in a future version. This overall advantage can not only be found in the number of solved instances of the several domains, but also in the time it took to find the solutions, which in many cases is shorter than with the old version of GAMER without the improvements.

6.5.4 IPC 2011: Participating Planners

In 2011 including GAMER a total of twelve planners participated in the sequential optimal track. In the following we will briefly describe all of them.

BJOLP

The BIG JOINT OPTIMAL LANDMARKS PLANNER BJOLP (Domshlak et al., 2011b) is based on FAST DOWNWARD and makes use of admissible landmark heuristics. As such it uses the ideas of Karpas and Domshlak (2009) for achieving admissible landmark heuristics by distribution of the costs of the actions achieving them. Furthermore, it also makes use of admissible action landmarks as proposed by Keyder et al. (2010). Finally, it uses the landmarks to come up with a path-dependent heuristic, i. e., one which might take different values on different paths, depending on the landmarks that were already achieved and must still be achieved.

This heuristic might work with A* as well, but instead they use a version of A* that better exploits the information of the path-dependent heuristic, namely LM-A* (Karpas and Domshlak, 2009).

CPT4

CPT4 (Vidal, 2011) is the new version of the temporal planner CPT3, which participated in IPC 2008 and as such the newest version of the CPT planner (Vidal and Geffner, 2006). Apart from some minor improvements and bug fixes the biggest change is the handling of classical planning domains. It still translates action costs to durations, but while CPT3 inserted pairwise mutexes on all actions ensuring that the actions can be applied only serially, in CPT4 a new constraint is used. This prevents an action to be added to a (partial) plan if the sum of the costs of the actions in the resulting plan exceed some bound on the makespan, which is here interpreted as the sum of the costs. Thus, it can use the temporal way of handling concurrent actions but optimality in the classical sense is still ensured.

FAST DOWNWARD AUTOTUNE

FAST DOWNWARD AUTOTUNE (FD AUTOTUNE) (Fawcett et al., 2011a,b; Vallati et al., 2011) is also based on FAST DOWNWARD, for which a big number of different configurations are possible. The idea here is to automatically learn a good configuration of the parameters beforehand and to use that for the entire competition hoping that it will work well for the new problems as well.

Given a general-purpose, highly parametric planner, a set of planning problems and some performance metric, to come up with a configuration of the planner's parameters optimized for performance on the given problems with respect to the given metric a generic automated algorithm configuration tool is used. In this case the tool of choice is ParamILS (Hutter et al., 2007, 2009). The performance metric used is the mean runtime required to find an optimal plan (which is to be minimized). The set of planning problems is a subset of the problems of the sequential-optimal track of IPC 2008.

FAST DOWNWARD STONE SOUP 1 and 2

As their names suggest, the two FAST DOWNWARD STONE SOUP (FDSS) planners (Helmert et al., 2011a,b) are both based on FAST DOWNWARD. The authors made two observations concerning heuristic search planning. First, they believe that there is no algorithm and heuristic that dominates all others, secondly, if no solution is found quickly then very likely no solution will be found at all.

Given these two observations the idea of FDSS is to run a set of algorithms and heuristics sequentially, each of them running for a comparatively short time. In the optimal track there is no sharing of any information between the different components, and of course the complete planner stops once one of the components has found an optimal plan.

For the competition they submitted two versions of the same planner, each with somewhat different algorithms, heuristics to use and runtime settings.

FORKINIT, IFORKINIT, and LMFORK

The three planners FORKINIT, IFORKINIT, and LMFORK (Katz and Domshlak, 2011) are all implemented on top of FAST DOWNWARD. They use implicit abstractions (Katz and Domshlak, 2010a) to come up with heuristic estimates. The first one makes use of what the authors call *forks*, the second of *inverted forks* and the last one of forks, but in that case the implicit abstraction is based on the landmark-enriched planning problem (Domshlak et al., 2010b). The first two planners use A* search, the last one LM-A* (Karpas and Domshlak, 2009), the version of A* that can handle path-dependent heuristics, which they are used here, in a better way than classical A* search.

LM-CUT

The planner LM-CUT (Helmert and Domshlak, 2011) is a straight-forward extension of the FAST DOWNWARD framework. It uses its A* search utilizing the authors' LM-Cut heuristic (Helmert and Domshlak, 2009).

MERGE AND SHRINK

The MERGE AND SHRINK (M&S) planner (Nissim et al., 2011b) uses FAST DOWNWARD's A* search along with merge-and-shrink abstractions (Helmert et al., 2007). In this case, two different shrinking strategies are used (Nissim et al., 2011a). One is called *greedy bisimulation*, the other, inspired by Dräger et al.'s DFP (2009), is called *DFP-gop* (with DFP standing for the original authors, g for greedy and op for operator projection).

In the competition both versions were used sequentially, i. e., first the planner using the greedy bisimulation shrinking strategy was started and killed after 800 seconds, afterward—in case the first one did not find a solution—the one using the DFP-gop shrinking strategy was started for 1,000 seconds.

SELMAX

SELMAX (Domshlak et al., 2011a) is a planner that is also implemented in the FAST DOWNWARD framework. It uses the selective max method (Domshlak et al., 2010a) to automatically decide which heuristic to use to evaluate a state. The idea is to use a heuristic that results in the shortest overall search time. In case all heuristic calculations take the same time this reduces to choosing to calculate the value according to the heuristic that results in the maximal value.

In order to determine when to use which heuristic, the algorithm uses a training set that is gained by taking a number of sample states from the planning problem. Based on these and some features representing the sample states, the algorithm decides whether to use the heuristic that is computed faster or one that takes longer but might be more informative. As this decision is not perfect, a confidence is also taken into account. If the decision procedure is confident enough the proposed heuristic is used, otherwise all heuristics are calculated and the maximum value is used as the heuristic value for the current state. In that case the current state can be seen as a new training example and thus used to improve the decision procedure.

Table 6.3: Number of solved problems for all domains of the sequential optimal track of IPC 2011, competition results.

Domain	FDSS-1	FDSS-2	SELMAX	M&S	LM-CUT	FD AUTOTUNE	FORKINIT	BJOLP	LMFORK	GAMER	IFORKINIT	CPT4
NOMYSTERY (20)	**20**	**20**	**20**	**20**	15	15	**20**	**20**	**20**	14	14	9
PARKING (20)	**7**	**7**	4	**7**	2	2	**7**	3	5	0	4	0
TIDYBOT (20)	**14**	**14**	**14**	13	**14**	**14**	**14**	**14**	**14**	0	**14**	0
VISITALL (20)	13	13	10	13	10	10	12	10	12	9	**14**	10
Total (unit-cost) (80)	**54**	**54**	48	53	41	41	53	47	51	23	46	19
BARMAN (20)	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	0
ELEVATORS (20)	18	17	18	11	18	18	16	14	14	**19**	14	0
FLOORTILE (20)	7	7	7	7	7	7	2	2	2	**9**	2	0
OPENSTACKS (20)	16	16	14	16	16	16	16	14	12	**20**	16	0
PARC-PRINTER (20)	14	13	13	14	13	13	11	11	10	7	10	**17**
PEG-SOLITAIRE (20)	**19**	**19**	17	**19**	18	17	17	17	17	17	17	1
SCANALYZER (20)	**14**	**14**	10	9	12	12	10	6	8	6	6	1
SOKOBAN (20)	**20**	**20**	**20**	**20**	**20**	**20**	19	**20**	19	19	**20**	0
TRANSPORT (20)	**7**	**7**	6	**7**	6	6	6	**7**	6	**7**	6	0
WOODWORKING (20)	12	11	12	9	12	12	4	9	5	**17**	3	6
Total (gen. costs) (200)	**131**	128	121	116	126	125	105	104	97	125	98	25
Total (all) (280)	**185**	182	169	169	167	166	158	151	148	148	144	44

In the competition version of SELMAX two heuristics, the LM-Cut heuristic (Helmert and Domshlak, 2009) as well as an admissible landmark heuristic (Karpas and Domshlak, 2009; Keyder et al., 2010) along with LM-A* instead of A* are used.

6.5.5 IPC 2011: Results

The actual competition results of IPC 2011 are depicted in Table 6.3. Unfortunately, GAMER did not score as well as it did in 2008. Of the twelve planners it finished ninth with only 148 solved instances, while one of the FDSS versions won with a total of 185 solutions. If we compare the number of solved instances of the domains with only unit-cost actions and those with general action costs the results are quite peculiar. For the former GAMER found only 23 solutions; only one participant was worse than that, while the best one found 54 solutions. For the latter GAMER found 125 solutions; only three other planners were able to find more (with the maximum being 131).

After the competition we investigated the results and found several problems with GAMER. First of all, there was a small bug in the solution reconstruction for bidirectional BFS. It supposed that at least one forward and at least one backward step was performed. The two easiest problems of VISITALL actually require only a single step, so that the solution reconstruction crashed.

Also, the parser we used was extremely slow. In some cases, parsing the ground input took more than 15 minutes, so that actually no search whatsoever was performed in the domains with general action costs: At first the was parsed in order to generate a PDB, this was killed after 15 minutes, and then the input was parsed again for BDDA*. In the unit-cost domains the parsing sometimes also dominated the overall runtime. Thus, we decided to change the parser and use JavaCC, a parser generator for Java programs, with which the parsing typically takes only a few seconds.

In some domains generating the BDDs for the transition relation takes a lot of time. So far, we had to generate them twice in case of domains with general action costs if we did not use the abstraction, once for the PDB generation and once for BDDA*. To omit this we now store the transition relation BDDs, the

Table 6.4: Number of solved problems for all domains of the sequential optimal track of IPC 2011, own results.

Domain	M&S greedy	M&S DFP-gop	M&S combined	GAMER IPC	GAMER Post-IPC
NOMYSTERY (20)	13	**20**	**20**	14	14
PARKING (20)	**7**	0	**7**	0	0
TIDYBOT (20)	**13**	0	**13**	0	6
VISITALL (20)	13	11	**14**	9	11
Total (unit-cost) (80)	46	31	**54**	23	31
BARMAN (20)	4	4	4	4	**5**
ELEVATORS (20)	0	11	11	**19**	**19**
FLOORTILE (20)	3	7	7	8	**9**
OPENSTACKS (20)	4	16	16	**20**	**20**
PARC-PRINTER (20)	11	**14**	**14**	7	7
PEG-SOLITAIRE (20)	0	**19**	**19**	17	17
SCANALYZER (20)	6	**10**	**10**	6	9
SOKOBAN (20)	1	**20**	**20**	19	19
TRANSPORT (20)	6	**7**	**7**	**7**	**7**
WOODWORKING (20)	9	6	9	**16**	**16**
Total (gen. costs) (200)	44	114	117	123	**128**
Total (all) (280)	90	145	**171**	146	159

BDD for the initial state and that for the goal condition on the hard disk; storing and reading them is often a lot faster than generating them again from scratch.

In two of the new domains, namely PARKING and TIDYBOT, we found that the first backward step takes too long, often even more than 30 minutes, so that the decision whether to use bidirectional or unidirectional BFS could not be finished before the overall time ran out. In these cases, the single images for all the actions were quite fast, but the disjunction took very long. Thus, during the disjunction steps we entered the possibility to check whether too much time, in this case 30 seconds, has passed. If it has we stop the disjunctions and the planner only performs unidirectional BFS (or PDB generation using abstractions). This enabled us to find some solutions in TIDYBOT, where in the competition we failed completely. The problem here is that the goal description allows for too many possibilities, because variables from only very few of the SAS$^+$ groups are present.

The FDSS planners seem out of reach for the time being, so that we did not try to compare against those, but rather chose one of the planners that finished third, MERGE AND SHRINK. We ran two recent[18] versions of MERGE AND SHRINK, one using the greedy bisimulation shrinking strategy (M&S greedy) and one using the DFP-gop shrinking strategy (M&S DFP-gop). The combined results (M&S combined) are then the number of solutions found within 800 seconds by M&S greedy or withing 1,000 seconds by M&S DFP-gop, just like in the competition setting. We also ran two versions of GAMER, namely the competition version (GAMER IPC), and the one with all problems addressed the way we have just described (GAMER Post-IPC). For the experiments we used our own machine (Intel Core $i7$ 920 CPU with 2.67 GHz and 24 GB RAM) with the same settings concerning timeout (30 minutes) and maximal memory usage (6 GB) as in the competition. The results are depicted in Table 6.4.

On our machine, neither of the two versions of MERGE AND SHRINK took more than 600 seconds for any of the problems. The memory limit of 6 GB was what prevented them from finding more solutions. Thus, the competition setting is really reasonable, as it makes the best of both approaches.

From the table we see that the results of the competition version of GAMER have become slightly worse than on the competition machine, showing that our machine is less powerful than the one used in the competition, while those of MERGE AND SHRINK have slightly improved—maybe due to some bugfixes and performance boosts in that planner as well.

[18] Retrieved on February 22nd 2012 from the FAST DOWNWARD repository at `http://hg.fast-downward.org`.

All the small improvements helped mainly in the domains with only unit-cost actions. There we are now able to find the two trivial solutions in VISITALL, as well as six solutions in TIDYBOT. In the domains with general action costs the new parser helped us to find three additional solutions in the SCANALYZER domain.

A last change we tried was running BDDA* in all cases, no matter if we are confronted with general action costs or not. Thus, for the domains with unit-cost actions we treated all actions as if they had a cost of 1. With this we see another small improvement, one solution in the parking domain. Overall, we arrive at 160 solutions in the set of competition problems (32 in the domains with unit-cost actions and 128 in those with general action costs). If we ignore the PARC-PRINTER domain, which is special in its way of having extremely high and very diverse action costs, we are now at the top of all the planners in the domains with general action costs.

Compared with the two versions of MERGE AND SHRINK we can now find the same number of solutions as the one using the DFP-gop shrinking strategy in case of the unit-cost domains. However, the greedy shrinking strategy (and the combination of both) is still ahead of us.

Maybe a downside of GAMER is that so far it uses only a single heuristic. The results of the last IPC indicate that a portfolio of heuristics often helps, at least in case of explicit-state planners. We see the same in case of MERGE AND SHRINK. Each of the two versions alone are worse than the current version of GAMER, while the combination brings it ahead by a total of twelve additional solutions.

Chapter 7

Net-Benefit Planning

1. A robot may not injure a human being or, through inaction, allow a human being to come to harm.
2. A robot must obey the orders given to it by human beings, except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Laws.

Issac Asimov, *The Three Laws of Robotics*

In contrast to classical planning described in the previous chapter, in *net-benefit planning* we are concerned with a planning problem that specifies certain *soft goals* or *preferences*, i. e., goals that do not necessarily need to be achieved. Plans achieving these soft goals are awarded some reward, so that the idea of optimal net-benefit planning is to find a plan that maximizes the sum of the rewards for achieving soft goals minus the total action costs.

Definition 7.1 (Net-Benefit Planning Problem)**.** *A* net-benefit planning problem *is described by a tuple* $\mathcal{P}_{\mathcal{NB}} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{G}_S, \mathcal{C}, \mathcal{U} \rangle$ *with* $\mathcal{V}$ *being the variables that build a state,* $\mathcal{A}$ *a set of actions consisting of precondition and effect, i. e., an action* $a \in \mathcal{A}$ *is given by a pair* $\langle pre, \mathit{eff} \rangle$*,* $\mathcal{I}$ *an initial state,* $\mathcal{G}$ *a set of goal states,* $\mathcal{G}_S$ *a set of soft goals,* $\mathcal{C}$ *a cost function specifying a certain cost for each action in* $\mathcal{A}$*, and* $\mathcal{U}$ *a utility function specifying a reward for each of the soft goals* $\mathcal{G}_S$*.*

A plan $\pi_{\mathcal{NB}}$ for a net-benefit planning problem is defined in the same way as it is for classical planning as being a sequence of actions transforming the initial state to a goal state (see Definition 6.2), but as we have soft goals where plans achieving them get additional rewards we need to define a new optimality criterion. This is called the plan's *net-benefit*.

Definition 7.2 (Net-Benefit of a Plan)**.** *Let* $\pi_{\mathcal{NB}} = (A_1, \ldots, A_n)$ *be a plan for a net-benefit planning problem and* $g_S \subseteq \mathcal{G}_S$ *be the soft goals that are achieved by this plan, i. e.,* $g_S \subseteq A_n(A_{n-1}(\ldots A_1(\mathcal{I})\ldots))$*. The plan's* net-benefit *is then defined as* $\mathcal{NB}(\pi_{NB}) = \mathcal{U}(g_S) - \mathcal{C}(\pi_{NB})$ *with* $\mathcal{U}(g_S) := \sum_{g \in g_S} \mathcal{U}(g)$ *being the total achieved utility (the total reward) and* $\mathcal{C}(\pi_{NB})$ *being the plan's cost, i. e., the sum of the costs of all actions within the plan, as defined in Definition 6.3.*

Definition 7.3 (Optimal Plan)**.** *A plan* $\pi_{\mathcal{NB}}$ *for a net-benefit planning problem is called* optimal*, if there is no plan* π'_{NB} *with* $\mathcal{NB}(\pi'_{NB}) < \mathcal{NB}(\pi_{NB})$*.*

In the following we will first of all extend the running example to the net-benefit planning setting (Section 7.1) before we give a brief overview of existing approaches for finding plans for a subset of net-benefit planning problems, the so-called over-subscription planning problems (Section 7.2). Afterward we present

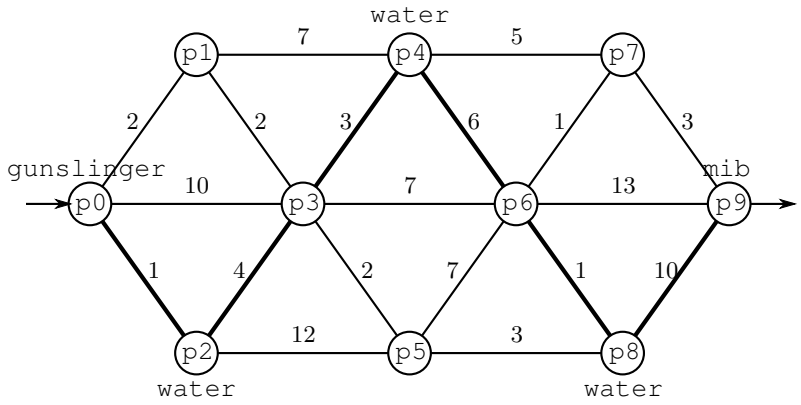


Figure 7.1: The running example in net-benefit planning. The optimal solution is denoted by the thicker edges.

some steps leading to our algorithm for optimally solving net-benefit planning problems, starting with an existing symbolic branch-and-bound algorithm that we iteratively refine until we come up with our optimal net-benefit algorithm (Edelkamp and Kissmann, 2009) in Section 7.3. Finally, we will provide the results of the international planning competition (IPC) 2008 along with an evaluation of a slight improvement (Section 7.4).

7.1 Running Example in Net-Benefit Planning

For the net-benefit version of the running example of the gunslinger following the man in black through the desert we decided to add some idea of thirst to the domain. We distinguish four different degrees of thirst (not thirsty, thirsty, very thirsty, and dead). The thirst of the gunslinger increases with each move he takes, so that he can only make two moves and then must drink something or die, hallucinating that he caught the man in black. Because the gunslinger did not expect to enter the desert he does not carry any water, so he must make use of the water he can find in certain locations of the desert (cf. Figure 7.1).

In order to allow the gunslinger to drink some water we need an additional action:

```
(:action drink
   :parameters (?l - location)
   :precondition
   (and
        (position gunslinger ?l) (waterAt ?l)
        (< (thirst) 3)
   )
   :effect
   (assign (thirst) 0)
)
```

This reduces the thirst level to no thirst, and can be applied if the gunslinger is at a location where he can find water and if he has not yet died. The other actions must specify that they can also only applied when the thirst is not yet deadly, i. e., `(< (thirst) 3)` must be part of the precondition, but the move actions must increase the thirst, i. e., `(increase (thirst) 1)` must be added to the effects.

The hallucinating is modeled by use of a new action:

```
(:action hallucinate
   :parameters (?l - location)
   :precondition
   (>= (thirst) 3)
   :effect
   (and
       (caughtMIB) (isDead gunslinger)
   )
)
```

This is the only action that can be applied when the thirst is already deadly.

A plan ending in the gunslinger's demise is still valid, though he only hallucinated catching the man in black. This is where the actual net-benefit mechanism comes into play. Instead of just trying to reach a goal state we specify certain *preferences*, the soft goals, which should be achieved. Failing one of those will be penalized. In this case, we define four preferences:

```
(:goal
    (and
        (caughtMIB)
        (preference pref_alive (not (isDead gunslinger)))
        (preference pref_notThirsty (thirsty t0))
        (preference pref_thirsty (thirsty t1))
        (preference pref_veryThirsty (thirsty t2))
    )
)
```

The first one states that the gunslinger should not die, so that he really should have caught the man in black, not just in his hallucination. The next three state that he should be not thirsty at all, only a bit thirsty, or very thirsty (an even higher degree results in death, as mentioned earlier).

To weight the preferences, we must add them to the metric definition, which in this case looks as follows:

```
(:metric maximize
    (- 100
      (+
        (total-cost)
        (* (is-violated pref_alive) 50)
        (* (is-violated pref_notThirsty) 25)
        (* (is-violated pref_thirsty) 20)
        (* (is-violated pref_veryThirsty) 10)
      )
    )
)
```

What it tells us is that we wish to maximize the outcome of the difference of 100 and the sum of the `total-cost` and the penalty for each violated preference. This way, we actually want to minimize the sum of violations and cost of the plan. The weights for the violations of the four preferences are given in the multiplication for each of them. Thus, we penalize the failure to stay alive hardest, and we prefer to be less thirsty. This way, given the action costs as before and the water in locations `p2`, `p4`, and `p8`, in an optimal solution the gunslinger can safely find a way through the desert to catch the man in black and not die of thirst on his way there. The optimal solution is as follows, and also displayed in the graph in Figure 7.1, where it is denoted by thicker edges.

```
0: (move p0 p2)     [1]
1: (drink p0)       [0]
2: (move p2 p3)     [4]
3: (move p3 p4)     [3]
4: (drink p4)       [0]
5: (move p4 p6)     [6]
6: (move p6 p8)     [1]
7: (drink p8)       [0]
8: (move p8 p9)    [10]
9: (catchMIB p9)    [0]
```

The cost of the actions sums up to 25, while that for the violated preferences sums up to 35, which results in a total net-benefit of 40. Another possible plan would be to move from p4 via p7 to p9 and there catch the man in black, which would reduce the total action cost to 16, but the violation cost would increase to 45 because in that case the gunslinger is very thirsty and not just thirsty when he catches the man in black, so that the overall net-benefit in this case would be 39 and thus slightly worse. Another possibility is to just move between p0 and p2 until the gunslinger dies, which is the case after three moves, resulting in a total action cost of only 3. But in that case the gunslinger dies, so that we violate all the preferences and the violation cost thus increases to 105, so that the total net-benefit is -8, which is clearly inferior to the other results.

7.2 Related Work in Over-Subscription and Net-Benefit Planning

Net-benefit planning is a comparably new direction so that it has received considerably less interest than classical planning, and still does, as we can see from the participants in the international planning competition. In 2008 there were three teams in the optimizing track, in 2011 only one in the optimizing track and one in the satsificing track, so that these tracks were actually canceled.

Net-benefit planning can be seen as the union of classical planning (with general action costs) and *over-subscription planning* (also called *partial satisfaction planning*). In contrast to net-benefit planning in over-subscription planning the set of goal states is empty ($\mathcal{G} = \emptyset$), so that only soft goals are present. Thus, any possible plan is a valid solution for an over-subscription planning problem whose quality is determined by its net-benefit.

Similar to classical planning, the decision if a plan with a net-benefit that is greater than some positive number k can be found is PSPACE-complete (van den Briel et al., 2004).

For the over-subscription planning problem a number of papers have been published during the last decade. In the following we will briefly introduce some of those proposing algorithms to find optimal solutions.

In the work of van den Briel et al. (2004) three methods for solving over-subscription planning problems are proposed. The first one, which they called OPTIPLAN, is an approach that translates the planning problem into an integer program. In principle it is an extension of Vossen et al.'s (1999) idea to solve planning problems by integer programming—but adapted to the over-subscription case. OPTIPLAN finds optimal plans.

The second one, called ALTALT^{ps}, extends ALTALT (Nguyen et al., 2002; Sanchez Nigenda and Kambhampati, 2003), a heuristic regression planner. This derives a reachability heuristic from planning graphs. The main extension consists of another approach for heuristically selecting a subset of the soft goals that is likely to be most useful. Using these soft goals as the goal state, a classical planning approach can be used to solve the resulting planning problem. Nevertheless, if the heuristic chooses bad soft goals, the approach cannot find an optimal plan. To remedy this, ALTALT^{ps} needs to create all possible subsets of soft goals and find an optimal plan for each of these. To ensure optimality it is necessary to use an admissible heuristic during the search.

The third approach, called SAPA^{ps}, extends the forward state-space planner SAPA (Do and Kambhampati, 2003) in that it does not select a subset of the soft goals before the planning process (as ALTALT^{ps} does) but treats the soft goals as *soft constraints*, i. e., (as no goal states are present) any possible plan is a solution, but its quality is determined by its net-benefit. SAPA^{ps} estimates the g (distance from $\mathcal{I}$) and h

(estimate on the distance to a goal state) values of partial solutions and uses them to guide an A* search. In their proposal, SAPA^{ps} is not necessarily optimal, but it can be made optimal, if an admissible heuristic is used.

An approach to handle net-benefit planning problems proposed by Keyder and Geffner (2009) is to compile the soft goals away and use a planner that can handle classical planning input. The idea is to transform each soft goal $g_S \in \mathcal{G}_S$ into a classical goal and add two actions, both achieving that goal. The actions have cost 0 if g_S holds in the precondition and cost $\mathcal{U}(g_S)$ if it does not. An additional action is required with the precondition being the goal of the net-benefit problem and the effect that some new predicate *end-mode* now holds. The preconditions of all actions from the net-benefit problem are extended by the negation of end-mode, while those for the soft goals take end-mode as a precondition. This way, a plan is actually the plan as it is in the net-benefit problem, followed by the action adding end-mode, followed by a number of actions setting the soft goals, and the cost of that plan corresponds to the negation of the net-benefit of the first part of the plan in the net-benefit specification.

7.3 Finding an Optimal Net-Benefit Plan

In the following, we present our approach to find a plan achieving optimal net-benefit. First of all, we slightly adapt the definition of net-benefit planning and also a plan's net-benefit (Edelkamp and Kissmann, 2009).

Definition 7.4 (Adapted Net-Benefit Planning Problem)**.** *An* adapted net-benefit planning problem *is a tuple $\mathcal{P}_{\mathcal{NB}} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{PC}, \mathcal{C}_a, \mathcal{C}_c \rangle$ with $\mathcal{V}$ being the variables that build a state, $\mathcal{A}$ a set of actions transforming states into states, $\mathcal{I}$ an initial state, $\mathcal{G}$ a set of goal states, $\mathcal{PC}$ a set of preference constraints, $\mathcal{C}_a$ a cost function specifying a certain cost for each action in $\mathcal{A}$, and $\mathcal{C}_c$ a cost function specifying a certain cost when failing to achieve one of the constraints.*

With this change from soft goals with utilities for achieving them to constraints with costs for failing to achieve them the adaptation of a plan's net-benefit is straight-forward.

Definition 7.5 (Adapted Net-Benefit of a Plan)**.** *Let $\pi_{\mathcal{NB}} = (A_1, \ldots, A_n)$ be a plan for an adapted net-benefit planning problem and $pc \subseteq \mathcal{PC}$ be the preference constraints that are missed by this plan, i. e., $pc \cap A_n(A_{n-1}(\ldots A_1(\mathcal{I})\ldots)) = \emptyset$. The plan's* net-benefit *is then defined as $\mathcal{NB}(\pi_{\mathcal{NB}}) = \mathcal{C}(pc) + \mathcal{C}(\pi_{\mathcal{NB}})$ with $\mathcal{C}(pc) := \sum_{p \in pc} \mathcal{C}_c(p)$ being the total cost for missing the constraints and $\mathcal{C}(\pi_{\mathcal{NB}})$ being the cost of all actions within the plan.*

Definition 7.6 (Optimal Plan)**.** *A plan $\pi_{\mathcal{NB}}$ for an adapted net-benefit planning problem is called* optimal*, if there is no plan $\pi'_{\mathcal{NB}}$ with $\mathcal{NB}(\pi'_{\mathcal{NB}}) > \mathcal{NB}(\pi_{\mathcal{NB}})$.*

Thus, we switched over from the problem of finding a plan maximizing the difference between goal utility and action costs to one minimizing the total cost, i. e., costs for missing constraints plus action costs. As maximization problems can be transformed to minimization problems and adding a constant offset does not change the set of optimal plans, any optimal plan for the adapted net-benefit planning problem is also an optimal plan for the original net-benefit planning problem. The evaluations for the two views do not match, but as we are interested in an optimal plan and not its evaluation this is not a problem.

According to the costs $\mathcal{C}_c$ for each of the preference constraints $pc \in \mathcal{PC}$, we can calculate an upper bound on the total violation cost $\mathcal{C}$. As none of the violation costs is negative, the upper bound max_v is simply the sum of all violation costs, i. e., we cannot get any worse than violating all constraints. The case of achieving all of the preference constraints, which equals a total cost of 0, is a lower bound min_v. Thanks to these bounds we can safely represent the entire range of possible values (from min_v to max_v) using $\lceil \log(max_v - min_v + 1) \rceil$ BDD variables.

For each preference constraint $pc \in \mathcal{PC}$ of type $(preference\ pc\ \phi_{pc})$ we associate a variable v_{pc}, which denotes the cost of pc (either $\mathcal{C}_c(pc)$ if it is violated or 0 if it is satisfied). We further use $\Phi_{pc}(v) = ((v_{pc} = \mathcal{C}_c(pc)) \Leftrightarrow \neg\phi_{pc}) \wedge ((v_{pc} = 0) \Leftrightarrow \phi_{pc})$ for evaluating one constraint in the given state and can use the formula $\bigwedge_{pc \in \mathcal{PC}} \Phi_{pc}(v)$ to evaluate all preference constraints in one operation.

Algorithm 7.1: Symbolic Branch-and-Bound

Input: Adapted Net-Benefit Planning Problem $\mathcal{P}_{\mathcal{NB}} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{PC}, \mathcal{C}_a, \mathcal{C}_c \rangle$.
Output: Violation cost optimal plan.

1 $U \leftarrow max_v + 1$ // Initial upper bound on total violation cost (can never reach U).
2 $bound\,(v) \leftarrow \bigvee_{i=0}^{U-1} (v = i)$ // Bound on total violation cost in form of a BDD.
3 $open \leftarrow \mathcal{I}$ // Start at initial state $\mathcal{I}$.
4 $closed \leftarrow \bot$ // So far, no state has been expanded.
5 **loop** // Repeat, until plan with minimal total violation cost is found.
6 **if** $open = \bot$ **or** $U = 0$ **then return** stored plan // All states expanded or all constraints satisfied.
7 $goals \leftarrow open \wedge \mathcal{G}$ // Determine the goal states in the $open$ list.
8 $eval\,(v) \leftarrow goals \wedge \bigwedge_{pc \in \mathcal{PC}} \Phi_{pc}\,(v)$ // Evaluate goal states.
9 $metric\,(v) \leftarrow eval\,(v) \wedge bound\,(v)$ // Retain only values within bound.
10 **if** $metric\,(v) \neq \bot$ **then** // If state with better metric value than currently best one is found...
11 $U \leftarrow 0$ // Start updating upper bound.
12 **while** $(eval\,(v) \wedge (v = U)) = \bot$ **do** // While smallest value is $\neq U$...
13 $U \leftarrow U + 1$ // Increase bound U.
14 $CalculateAndStorePlan\,(eval\,(v) \wedge (v = U))$ // Calculate plan with cost of U.
15 $bound\,(v) \leftarrow \bigvee_{i=0}^{U-1} (v = i)$ // Update bound in form of a BDD according to U.
16 $closed \leftarrow closed \vee open$ // Append states to be expanded to the $closed$ list.
17 $open \leftarrow image\,(open) \wedge \neg closed$ // Calculate all new successor states.

While the total action cost in classical planning is a monotonically increasing function, the net-benefit does not increase monotonically , so that the first found plan is not necessarily optimal. Thus, we perform a symbolic branch-and-bound search (Jensen et al., 2006). In the following, we will show how to adapt that algorithm to finally come up with our net-benefit planning algorithm (Edelkamp and Kissmann, 2009).

7.3.1 Breadth-First Branch-and-Bound

Jensen et al.'s symbolic branch-and-bound algorithm (2006) is outlined in Algorithm 7.1. This ignores any action costs but rather returns a plan that is optimal with respect to the achieved total violation cost. It works by adapting an upper bound U on the total violation cost, which is initialized to $max_v + 1$, so that all possible results are supported. Furthermore, a BDD representation of the bound is necessary, i. e., a BDD that represents the disjunction of all possible values ranging from 0 to $U - 1$ (line 2). Following Bartzis and Bultan (2006), such a BDD can be constructed efficiently.

Starting at the initial state $\mathcal{I}$ the algorithm checks if there are no more unexplored states ($open = \bot$) or the upper bound U equals 0 (line 6). In both cases we cannot get any better as in the first case we have explored all reachable states and in the second we have found a path that satisfies all preference constraints. If we have calculated some plan to this point we can return it, otherwise we return "no plan".

If we are not yet done we determine the goal states that are contained in the current BFS layer (line 7). For these we calculate the total violation costs (line 8) and remove any states with an evaluation that is not smaller than the current bound U (line 9). If the set of these states is not empty, we have found a goal state with a total violation cost smaller than any found before. To find the best one we search from the bottom up, i. e., we start with a violation cost of 0 and increase it until the conjunction with the evaluation is no longer false (lines 11 to 13). This results in all the states we have reached in the current BFS layer with the minimal violation cost. For these we can calculate a plan, but as we are not certain that this plan is in fact optimal we only store it internally and continue the search with adapted bounds. This calculation of a plan works just as we presented it for BFS in classical planning (Algorithm 6.2).

To continue, we calculate the successor states of the current layer and remove those that were already expanded before and stored in the set $closed$ (line 17).

Theorem 7.7 (Optimality of symbolic breadth-first branch-and -bound)**.** *The symbolic breadth-first branch-and-bound algorithm finds an optimal plan if we do not consider action costs or the length of the resulting plan, i. e., it finds the shortest plan that produces the minimal total violation cost.*

Proof. The algorithm stops when one of two conditions holds: Either the set of unexplored states runs empty or a plan satisfying all constraints has been found.

For the second case the optimality is immediately clear, as we are only concerned with the total violation cost. Due to the expansion in the breadth-first manner we are sure that the first found plan satisfying all constraints has the shortest length among all the plans satisfying all constraints.

For the first case we need to proof that at some point the set of unexplored states definitely will run empty and that at that point the last stored plan is an optimal one.

The fact that eventually the set of unexplored states runs empty is inherited from BFS, which we perform here. The planning problems contain only a finite number of different states and we perform a BFS with full duplicate elimination, so that no state is expanded more than once.

Though we do not consider all possible plans, we are still certain that the last generated plan is optimal. This is because we only discard states that are clearly inferior. Either they are discarded because their violation cost is greater than the current bound, so that in one of the previous steps a better plan has been found, or because in the current iteration at least one state with a smaller violation cost exists. □

7.3.2 Cost-First Branch-and-Bound

The problem with net-benefit planning in symbolic search is that the total action cost is not bounded, and thus also the net-benefit is not bounded, so that we cannot construct a BDD for this bound, as we can for the total violation cost. To overcome this problem we use a structure similar to the one we used in Dijkstra and A* search, i. e., one that stores all states having the same distance from $\mathcal{I}$ in the same bucket (Dial, 1969). Thus, we are able to determine the total action cost immediately by checking the bucket's index.

In a first step we adapt the previous breadth-first branch-and-bound algorithm (Algorithm 7.1) to work in a cost-first manner (see Algorithm 7.2), though this still ignores the total action cost, i. e., it finds a plan that is optimal with respect to the total violation cost and is the one with smallest total action cost among all those achieving the minimal total violation cost.

Large parts of this are identical to Algorithm 7.1. Note that we do not use immediate duplicate elimination but a delayed one, i. e., once a bucket is to be expanded, all previously expanded states are removed from this. Also, we cannot stop immediately after having found an empty bucket, but only when the last $\mathcal{C}_{max}$ buckets were empty as well, with $\mathcal{C}_{max}$ being the highest action cost.

Theorem 7.8 (Optimality of symbolic cost-first branch-and-bound)**.** *The symbolic cost-first branch-and-bound algorithm for general action costs $\mathcal{C}(a) \in \{0, \ldots, \mathcal{C}_{max}\}$, $a \in \mathcal{A}$, finds a cheapest plan among all those achieving the minimal total violation cost.*

Proof. The proof is essentially the same as the one for Theorem 7.7. Here, the algorithm also stops after the set of unexplored states runs empty or a plan satisfying all constraints has been found.

In the second case optimality is immediate. Due to the cost-wise approach no other plan satisfying all constraints can have a smaller total action cost.

In the first case we need to proof that the set of unexplored states runs empty (and that the algorithm will not stop before this) and that the last stored plan is an optimal one.

Here, the set of unexplored states runs empty as the basic algorithm used is Dijkstra's algorithm with a $closed$ list, which also expands all reachable states. The algorithm recognizes this by checking the last $\mathcal{C}_{max}$ buckets. If all these are empty all states have been explored. This is sufficient as the maximal action cost is $\mathcal{C}_{max}$, so that all successors of a bucket $open_g$ reside in buckets $open_{g+1}$ through $open_{g+\mathcal{C}_{max}}$.

We do not consider all possible plans, but discard only those that are clearly inferior, i. e., either their violation cost is at least as high as the best one found so far or in the same iteration another plan of smaller violation cost has been found. □

Algorithm 7.2: Symbolic Cost-First Branch-and-Bound

Input: Adapted Net-Benefit Planning Problem $\mathcal{P}_{\mathcal{NB}} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{PC}, \mathcal{C}_a, \mathcal{C}_c \rangle$.
Output: Violation cost optimal plan.

1 $U \leftarrow \max_v + 1$ // Initial upper bound on total violation cost (can never reach U).
2 $bound\,(v) \leftarrow \bigvee_{i=0}^{U-1} (v = i)$ // Bound on total violation cost in form of a BDD.
3 $open_0 \leftarrow \mathcal{I}$ // Start at initial state $\mathcal{I}$.
4 $closed \leftarrow \bot$ // No state expanded so far.
5 $g \leftarrow 0$ // Initial state's distance is 0.
6 **loop** // Repeat, until plan with minimal total violation cost is found.
7 **if** zero-cost actions present **then** // If there are any zero-cost actions...
8 calculate zero-cost fix point for $open_g$ // Calculate states reachable using only zero-cost actions.
9 $open_g \leftarrow open_g \wedge \neg closed$ // Remove all duplicate states from previous expansions.
10 **if** $\bigvee_{i=g-\mathcal{C}_{max}+1}^{g} open_i = \bot$ **or** $U = 0$ **then return** stored plan // All states expanded or // all constraints satisfied.
11 $goals \leftarrow open_g \wedge \mathcal{G}$ // Determine the goal states in the current bucket.
12 $eval\,(v) \leftarrow goals \wedge \bigwedge_{pc \in \mathcal{PC}} \Phi_{pc}\,(v)$ // Evaluate goal states.
13 $metric\,(v) \leftarrow eval\,(v) \wedge bound\,(v)$ // Retain only values within bound.
14 **if** $metric\,(v) \neq \bot$ **then** // If state with better metric value than currently best one is found...
15 $U \leftarrow 0$ // Start updating upper bound.
16 **while** $(eval\,(v) \wedge (v = U)) = \bot$ **do** // While smallest value is $\neq U$...
17 $U \leftarrow U + 1$ // Increase bound U.
18 $CalculateAndStorePlan\,(eval\,(v) \wedge (v = U))$ // Calculate plan with cost of U.
19 $bound\,(v) \leftarrow \bigvee_{i=0}^{U-1} (v = i)$ // Update bound in form of a BDD according to U.
20 **for all** $c \in \{1, \ldots, C\}$ **do** // For all action costs...
21 $open_{g+c} \leftarrow open_{g+c} \vee image_c\left(open_g\right)$ // Calculate successor states.
22 $closed \leftarrow closed \vee open_g$ // Update set of expanded states.
23 $g \leftarrow g + 1$ // Iterate to next g-value.

It might be interesting to note that Algorithm 7.2 essentially behaves the same as Algorithm 7.1, although now the exploration is cost-first instead of breadth-first; especially the total violation costs of the found plans match, as both calculate plans optimal with respect to the same metric, i. e., the total violation cost alone.

Corollary 7.9 (Equivalence of Algorithms 7.1 and 7.2)**.** *The costs of the plans found by both algorithms match.*

7.3.3 Net-Benefit Planning Algorithm

Using the additional information of the stored buckets we can now propose an algorithm that calculates a plan achieving optimal net-benefit (cf. Algorithm 7.3), so that it considers not only the total violation cost but also the total action cost and finds a plan minimizing the sum of both (Edelkamp and Kissmann, 2009). For this we need a second bound, V, on the net-benefit, which is decreased with each plan we find.

Instead of updating the BDD representing the bound on the violation costs only when a plan has been found, we update it before each step (line 11). With this we make sure that only plans that have a violation cost smaller than the current optimum U as well as a net-benefit (which is the violation cost plus g) smaller than the current optimum V.

Theorem 7.10 (Optimality of the symbolic net-benefit planning algorithm)**.** *The symbolic net-benefit planning algorithm finds a plan achieving optimal net-benefit for general action costs $\mathcal{C}\,(a) \in \{0, \ldots, \mathcal{C}_{max}\}$, $a \in \mathcal{A}$.*

Algorithm 7.3: Symbolic Net-Benefit Planning Algorithm

Input: Adapted Net-Benefit Planning Problem $\mathcal{P}_{\mathcal{NB}} = \langle \mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G}, \mathcal{PC}, \mathcal{C}_a, \mathcal{C}_c \rangle$.
Output: Net-benefit optimal plan.

$U \leftarrow \max_v + 1$ // Initial upper bound on total violation cost (can never reach U).
$V \leftarrow \infty$ // Initial upper bound on net-benefit.
$open_0 \leftarrow \mathcal{I}$ // Start at initial state $\mathcal{I}$.
$closed \leftarrow \bot$ // No state expanded so far.
$g \leftarrow 0$ // Initial state's distance is 0.
loop // Repeat, until plan achieving optimal net-benefit is found.
  if zero-cost actions present **then** // If there are any zero-cost actions...
    calculate zero-cost fix point for $open_g$ // Calculate states reachable using only zero-cost actions.
  $open_g \leftarrow open_g \wedge \neg closed$ // Remove all duplicate states from previous expansions.
  if $\bigvee_{i=g-\mathcal{C}_{max}+1}^{g} open_i = \bot$ **or** $U = 0$ **or** $V \leq g$ **then return** stored plan // All states expanded or all constraints satisfied or bound on net-benefit greater than distance to $\mathcal{I}$.
  $bound(v) \leftarrow \bigvee_{i=0}^{\min\{U-1, V-g\}} (v = i)$ // Update bound in form of BDD according to U, V and g.
  $goals \leftarrow open_g \wedge \mathcal{G}$ // Determine the goal states in the current bucket.
  $eval(v) \leftarrow goals \wedge \bigwedge_{pc \in \mathcal{PC}} \Phi_{pc}(v)$ // Evaluate goal states.
  $metric(v) \leftarrow eval(v) \wedge bound(v)$ // Retain only values within bound.
  if $metric(v) \neq \bot$ **then** // If state with better net-benefit than currently best one is found...
    $U \leftarrow 0$ // Start updating bound on total violation cost.
    while $(eval(v) \wedge (v = U)) = \bot$ **do** // While smallest value is $\neq U$...
      $U \leftarrow U + 1$ // Increase bound U.
    $V \leftarrow U + g$ // Update bound on net-benefit.
    $CalculateAndStorePlan(eval(v) \wedge (v = U))$ // Calculate plan with violation cost of U.
  for all $c \in \{1, \ldots, C\}$ **do** // For all action costs...
    $open_{g+c} \leftarrow open_{g+c} \vee image_c(open_g)$ // Calculate successor states.
  $closed \leftarrow closed \vee open_g$ // Update set of expanded states.
  $g \leftarrow g + 1$ // Iterate to next g-value.

Proof. Again we need to proof that the algorithm stops once all reachable states have been considered or when the best plan found so far cannot be improved. Furthermore, we need to proof that the last stored plan is optimal.

Here, in the worst case the algorithm again considers all reachable states. If that happens, the open list runs empty, which is found out by looking at the last $\mathcal{C}_{max}$ buckets. This suffices as the action costs cannot be larger than $\mathcal{C}_{max}$, so that once $\mathcal{C}_{max}$ adjacent buckets are empty all states have been considered.

If not all states are considered, the algorithm nevertheless stops once all constraints are satisfied ($U = 0$), in which case we surely got the optimal result due to the cost-wise expansion, or when the total action cost has reached the smallest total net-benefit found so far ($V \leq g$). In this case we would need a negative violation cost to come up with a net-benefit smaller than V, which is impossible, as all violation costs are assumed to be non-negative.

What remains is proving that the last stored plan is optimal. In each iteration we only consider those goal states whose violation cost is smaller than U and also at most as large as $V - g$. The second bound is necessary to discard those states that achieve a better violation cost but still cannot beat the currently best net-benefit. Thus, we discard only those states clearly inferior; all others are considered, so that at any time the last stored plan is the best one we can achieve with the given total action cost. □

7.4 Experimental Evaluation

We implemented the symbolic net-benefit algorithm in another version of our planner GAMER. This version is implemented in C++ and uses the BDD package BuDDy[19] to handle the BDDs. With it we participated in the optimal net-benefit track of the International Planning Competition (IPC) 2008 and won that track as well.

The setting is similar to the one used in the sequential-optimal track (see Section 6.5). The planners were also limited to 2 GB of main memory and 30 minutes for each problem. Each of the six domains (CREW PLANNING, ELEVATORS, OPENSTACKS, PEG-SOLITAIRE, TRANSPORT, and WOODWORKING) contain 30 problem instances. For many domains several formulations were given and the results of the one a planner performed best in chosen for the comparison.

The comparison was also based solely on the number of solved problems. If a planner found a sub-optimal solution it achieved 0 points for the entire domain. Otherwise, the number of solved instances for each domain was counted and summed up to build the final score.

The net-benefit track was considerably less popular, so that only three planners participated in 2008. In IPC 2011 two preference planning tracks (which were supposed to be similar to the net-benefit planning track of IPC 2008) were announced, but were canceled because in the optimizing track as well as the satisficing track only one planner wanted to participate.

In the following, we will briefly outline the two competing planners of IPC 2008 (Section 7.4.1). Then, we will show the results achieved during the competition (Section 7.4.2) and finally discuss which of the improvements proposed Section 6.4 can be transfered to the net-benefit planner as well and compare the most recent version of GAMER with the competition version on a state-of-the-art machine (Section 7.4.3).

7.4.1 IPC 2008: Participating Planners

In the following we provide short descriptions of the two competitors, HSP$^*_p$ and MIPS XXL.

HSP$^*_p$

Haslum's HSP$^*_p$ (2008) performs a simple translation of the net-benefit problems to classical planning problems. It enumerates the set of possible soft goals and finds optimal plans for each. For this it uses regression, IDA* and a variant of the additive h^2 heuristic (Haslum and Geffner, 2000). When it is done the plan achieving the best net-benefit can be extracted.

This enumeration of the soft goals is also what prevented it from finding any plans in PEG-SOLITAIRE. Even in the most simple problem there are already 33 soft goals, resulting in $2^{33} = 8{,}589{,}934{,}592$ sets, which of course cannot all be solved in the short 30 minute time slot.

MIPS XXL

Similar to the sequential-optimal version, MIPS XXL (Edelkamp and Jabbar, 2008) performs external search. In this case, it is actually very similar to the symbolic algorithm, only that it uses files on an external storage instead of buckets of BDDs in main memory.

7.4.2 IPC 2008: Results

The competition's results are publicly available on the competition's website[20] and are also given in Table 7.1. As we can see, our planner GAMER clearly outperformed the two competing systems MIPS XXL and HSP$^*_p$, though the latter mainly failed because it was not able to solve any of the problems of the PEG-SOLITAIRE domain, as we have mentioned above.

We found that there was a small bug in the implementation of GAMER only recently, which prevented it from finding four simple solutions in the CREW PLANNING domain and two just as simple solutions in

[19] http://buddy.sourceforge.net
[20] http://ipc.informatik.uni-freiburg.de/Results

Table 7.1: Numbers of solved problems for all domains of the optimal net-benefit track of IPC 2008, competition results.

Problem	GAMER	HSP$^*_p$	MIPS XXL
CREW PLANNING (30)	4	**16**	8
ELEVATORS (30)	**11**	5	4
OPENSTACKS (30)	**7**	5	2
PEG-SOLITAIRE (30)	**24**	0	23
TRANSPORT (30)	**12**	**12**	9
WOODWORKING (30)	**13**	11	9
Total (180)	**71**	49	55

the PEG-SOLITAIRE domain, all of which can be solved within a few seconds, so that GAMER's winning margin increases even further.

7.4.3 Improvements of GAMER

In Section 6.4 we proposed four improvements for our classical planner, i. e., using a map instead of the (g, h)-matrix to handle large action costs, reordering of the variables, an incremental abstraction calculation, and an automatic choice for the amount of backward search (Kissmann and Edelkamp, 2011).

Of these four improvements the last two are not applicable in this algorithm, as it operates strictly in forward direction, so that no abstraction is necessary and no backward search is performed.

We did not yet implement the priority queue as a map instead of a vector for two reasons. The first is simply that all action costs of the entire test collection are small enough not to cause any problems, the second is that we use a one-dimensional vector for storing the layers according to their distance g to the initial state. Thus, the action costs need to be a lot larger than in classical planning in order for this structure to exceed the available memory.

Thus, all that remains is the implementation of a reordering of the variables, which is the most effective of all the improvements in our classical planner. Nevertheless, in the net-benefit planner this reordering brings some more problems. While in classical planning we only have groups of binary facts, here we also have the preference constraints, which are modeled by BDD variables. Furthermore, the net-benefit planner supports some numerical variables, which are included into the BDD structure as well. The problem with these two is that they are not part of the causal graph we construct. Thus, we reorder only the propositional variables.

As we cannot evaluate the preference constraint variables, we decided to distribute them equally before and after the propositional variables, hoping that the mean distance might decrease. Additionally, we decided to implement two versions, one (GAMER ^r_a) that performs reordering in any case and one (GAMER $^r_{\neg n}$) that performs it only in domains without numerical variables, as it might have a negative effect in such domains.

We evaluated these two versions along with the original version of GAMER on our machine, which contains an Intel $i7$ 920 CPU with 2.67 GHz and 24 GB RAM, using a timeout of 30 minutes, but no limit on the usable main memory.

In Table 7.2 we see that using the reordering we can solve some more problems of the OPENSTACKS domain, while for the other domains there is no change in the number of solved problems. In the classical planner the improvement concerning the OPENSTACKS domain was also biggest, but there the number of solved instances in the WOODWORKING domain also increased a lot, which is not the case for the net-benefit planner. While the specification of OPENSTACKS is available in several versions, one using numerical variables, others not, that for WOODWORKING is only available using numerical variables. It seems that this is what prevents us from improving over the competition results in that domain.

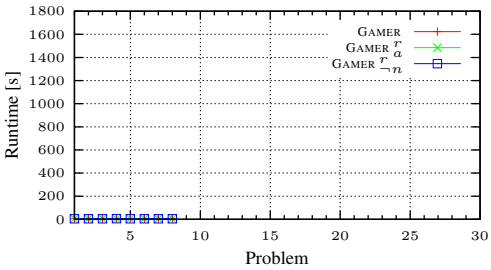
Comparing the runtimes of the algorithms (cf. Figure 7.2) we see only three domains where the reordering has a measurable impact. In two of these, namely OPENSTACKS (Figure 7.2d) and PEG-SOLITAIRE (Figure 7.2e), the runtime decreases by a factor of up to 16.5 and 2.6, respectively. The last one, ELEVATORS is an example of a domain where the reordering is counter-productive, as there the runtime increases

Table 7.2: Numbers of solved problems for all domains of the optimal net-benefit track of IPC 2008, own results.

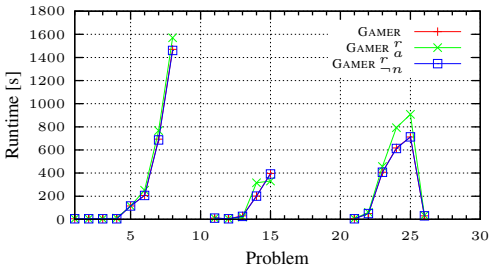
Problem	GAMER	GAMER $^{r}_{a}$	GAMER $^{r}_{\neg n}$
CREW PLANNING (30)	**8**	**8**	**8**
ELEVATORS (30)	**19**	**19**	**19**
OPENSTACKS (30)	7	**12**	**12**
PEG-SOLITAIRE (30)	**30**	**30**	**30**
TRANSPORT (30)	**15**	**15**	**15**
WOODWORKING (30)	**16**	**16**	**16**
Total (180)	95	**100**	**100**

by a factor of up to 1.5 in the formulation containing numerical variables (Figure 7.2b) and up to 1.8 in the STRIPS formulation (Figure 7.2c).

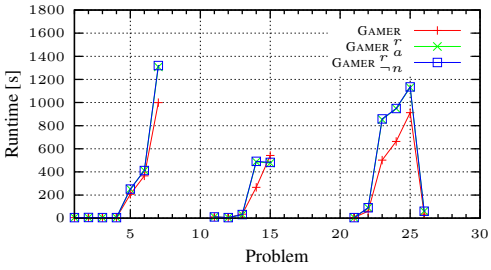
From these results and those for the classical planner it seems that we must extend the causal graph we use for the reordering, so that it can also capture the numerical variables and the preference constraints. If we cannot properly reorder the entire set of variables we often will not improve much over the original ordering.



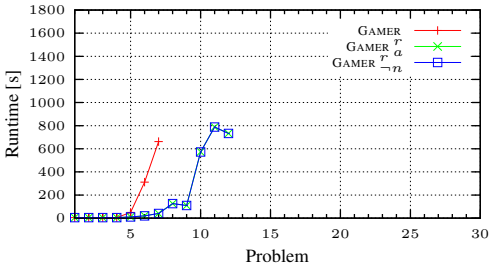
(a) CREW PLANNING (numeric).



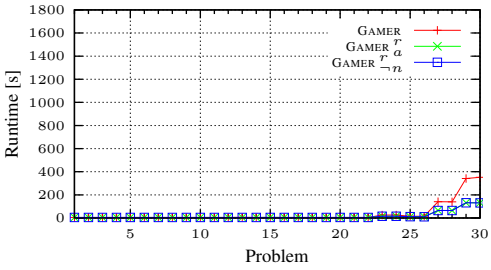
(b) ELEVATORS (numeric).



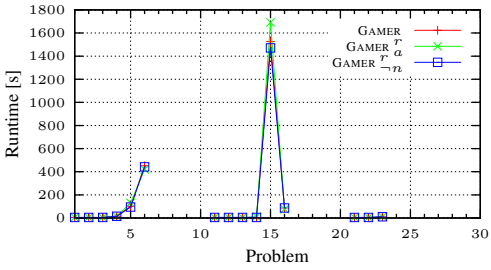
(c) ELEVATORS (STRIPS).



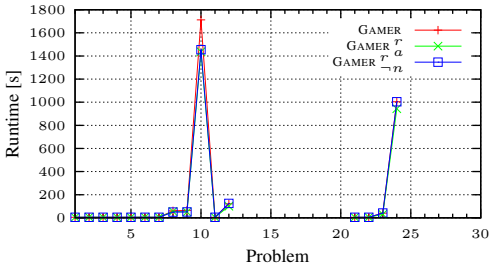
(d) OPENSTACKS (STRIPS).



(e) PEG-SOLITAIRE (STRIPS).



(f) TRANSPORT (numeric).



(g) WOODWORKING (numeric).

Figure 7.2: Runtime results for all domains of the optimal net-benefit track of IPC 2008 on our own machine. The caption denotes if it contains numerical variables (numeric) or not (STRIPS).

Part III

General Game Playing

Chapter 8

Introduction to General Game Playing

All work and no play makes Jack a dull boy.

Proverb

Game playing has been an important topic in research for a long time. On the one hand the results concerning equilibria by Nash, Jr. (1951) are still used especially in economics but also in several other domains. On the other hand, in AI games provide a good testbed as they often are not trivial and algorithms helping to speed up search for solutions or at least good moves can often be applied in other topics of AI (Russell and Norvig, 2010).

The first notable successes occurred in the 1990s, when several machines were able to defeat reigning world champions in their corresponding games. Of course, there were earlier success stories, such as the solving of CONNECT FOUR (Allen, 1989; Allis, 1988) or the NIM game, but important as they might be in the AI community, the defeat of a human by a machine in a real championship match in much more complicated games was what received much larger public interest.

The first one of these successes was the victory of the AMERICAN CHECKERS program CHINOOK (Schaeffer et al., 1992; Schaeffer, 1997) against the supposedly best human AMERICAN CHECKERS player of all time, Marion Tinsley. The first match in 1990 in the newly created *Man versus Machine World Championship* was won by Tinsley by a final outcome of four against two victories, with a total of 33 draws. Thus, Tinsley was defeated in two of the games but remained the world champion.

In 1994, a rematch was performed, but after six draws Tinsley had to retire, as he suffered from cancer. In the wake of this CHINOOK was declared world champion, though it had never actually defeated the best player in a complete match.

In an additional match in 1995 CHINOOK defeated its title against Don Lafferty, the number two after Tinsley, in a match with one victory and 31 draws. Afterward, CHINOOK's main developer, Jonathan Schaeffer, decided to retire CHINOOK from active play and concentrated on solving the game.

Another important victory for AI research is due to the CHESS player DEEP BLUE (Campbell et al., 2002). In a first match in February 1996 it defeated the reigning world champion Garry Kasparov in the first game, but lost three of the following five games; the remaining two resulted in a draw, so that the final outcome was $4 – 2$ for Kasparov (with wins counting as one point and draws as 0.5 points).

In a rematch in May 1997 DEEP BLUE was able to defeat Kasparov with a final outcome of $3.5 – 2.5$ and thus became the first machine to beat a human world champion in the game of CHESS.

In the same year, Michael Buro's LOGISTELLO (Buro, 1997) was able to beat the world champion in the game of OTHELLO, Takeshi Murakami, in a match with a final outcome of six against zero wins.

All these victories are remarkable successes, but a question one might ask is: Are the players really intelligent? Of course, the computers are used and necessarily needed to perform that good, nevertheless it is the programmer that inserts lots of domain specific knowledge such as expert knowledge and specialized heuristics into the players to enable them to perform well in the one game they were designed for. Due to the restriction to just one game the computers, are not able to play any other games. Thus, one might argue that the intelligence really comes from the programmers, not the programs.

One approach to overcome this problem is the idea of *general game playing* (GGP) (Genesereth et al., 2005). In this, the actual games to be played are unknown to the programmers; they only know the language for the description of possible games. The program must then read such a description and evaluate it in order to play the game. Furthermore, it must find sensible moves on its own to succeed. Thus, the programmer cannot insert any domain specific knowledge.

This setting is very similar to action planning. While in classical planning we are concerned with problems for only one player—the planner—that tries to reach one goal, in general game playing we are concerned with one or more players performing their moves concurrently, each with its own goals that might contradict those of the other players. Also, we typically have goals where a player achieves different rewards, so that here we prefer to speak of terminal states instead of goal states. Such a rewarding of achieving certain goals is actually similar to over-subscription planning. Another tight relationship we can identify is to adversarial or non-deterministic planning, where the environment might be interpreted as an opponent, so that we actually can use approaches for two-player games to find solutions in that setting (Bercher and Mattmüller, 2008; Kissmann and Edelkamp, 2009b).

An early approach in the direction of GGP was Barney Pell's METAGAME (Pell, 1992a). The idea of METAGAME is to have a *game generator*, which produces new games according to a pre-defined *class* of games. The best way to compare different programs is to run a tournament and let the programs play against each other—typically, the best (i. e., most intelligent) program should be able to win.

The interest in GGP has increased in recent years, mainly due to a new project initiated by the games group of Stanford University[21]. It became especially popular with the launch of an annual competition (Genesereth et al., 2005), which is held since 2005, originally only in the context of the national AAAI conference, nowadays also at the International Joint Conference on Artificial Intelligence (IJCAI).

The language that is used for these competitions is the *Game Description Language* GDL (Love et al., 2006), which is an extension of Datalog and thus a logic based language. In GDL it is possible to define single- or multiplayer games that are deterministic, discrete, and finite and where all players have full information of the game rules, the current state and the moves performed. Thus, in contrast to Pell's game classes, e. g., the symmetric CHESS-like games (Pell, 1992b) he proposed in the context of METAGAME, this approach is more general, though still quite limited.

Two of the most limiting factors, i. e., the restrictions that the games must be deterministic and that the players must have full information, have recently been lifted by Thielscher's (2010) proposal of GDL-II (for GDL with Incomplete Information). To achieve this he introduced a special role, `random`, which has no goals and always chooses a move randomly, and by the introduction of a new relation symbol, `sees`, which determines what a player actually sees of the current state. So far, research has mainly focused on the older GDL specification, though with GDL-II clearly more interesting games can be described. In the context of the international competition at AAAI or IJCAI no GDL-II track has been performed so far; the only international competition that did this was the first German Open in GGP we organized in the context of the 34th German Conference on Artificial Intelligence (KI) in 2011.[22]

In the remainder of this chapter we will introduce the Game Description Language GDL (Section 8.1), point out the main differences to the Planning Domain Definition Language PDDL (McDermott, 1998) (Section 8.2), which is the default language for describing action planning problems, and provide some more details in a short comparison to action planning (Section 8.3).

In Chapter 9 we will describe a way to instantiate GDL, i. e., to get rid of the variables in order to infer knowledge faster and to be able to use BDDs in the general game playing domain. Then, in Chapter 10 we will propose a few algorithms (using symbolic search) to actually solve general games, while Chapter 11 is concerned with playing the games and describing our approach to implement a general game player.

8.1 The Game Description Language GDL

In recent years, the Game Description Language GDL (Love et al., 2006), the language used at the annual AAAI or IJCAI GGP competition (Genesereth et al., 2005), has become the language of choice for the description of general games. In this work we are only concerned with the original GDL, so that the

[21] `http://games.stanford.edu`

[22] `http://fai.cs.uni-saarland.de/kissmann/ggp/go-ggp`

games are restricted to be deterministic, discrete and finite games of full information, but allow for single- and multiplayer settings. Nowadays, some extensions have been proposed, e. g., the Market Specification Language (MDL) (Thielscher and Zhang, 2009) that inserts support for incomplete information and is used in the Trading Agent Competition TAC, or GDL-II (Thielscher, 2010), which inserts non-determinism and incomplete information.

8.1.1 The Syntax of GDL

Though a detailed description of GDL's syntax can be found in the report by Love et al. (2006), in this section we repeat several of those definitions in the following, as we need some idea of how a game description is organized when instantiating it (Chapter 9).

Definition 8.1 (Vocabulary)**.** *The* vocabulary *of GDL consists of sets of relation symbols with corresponding arity, function symbols with corresponding arity and object constants. Furthermore, it contains variables, which are denoted by the prefix ? in GDL.*

Definition 8.2 (Term)**.** *A GDL* term *is either a variable, an object constant or a function symbol of some arity t applied to t terms.*

This *nesting* of function symbols is explicitly allowed, though it is restricted due to the requirement of the games to be finite. Nevertheless, it is used only in a small number of available games. Except for our player (cf. Chapter 11) we so far ignore games that make use of this; especially the instantiator cannot handle nested terms.

Definition 8.3 (Atomic Sentence, Literal, Expression, and Ground Expression)**.** *An* atomic sentence *(or* atom *for short) is a relation symbol of some arity t applied to t terms. A* literal *is an atomic sentence or the negation of an atomic sentence. Terms and literals are also called* expressions*. Such an expression is called* ground *iff it does not contain any variables.*

Definition 8.4 (Rule and Fact)**.** *A GDL* rule *is an implication of the form $B \Rightarrow h$ (though typically written the other way around, i. e., $h \Leftarrow B$). The body B of the implication is a conjunction $b_1 \wedge \cdots \wedge b_n$ of either literals or disjunctions of literals. The head h is an atomic sentence. A rule with only a head and no body is called a* fact*.*

Note that in a more recent specification of GDL the body of a rule may no longer contain disjunctions, so that we are actually confronted with Horn rules, and the safety property must hold.

Definition 8.5 (Horn Rule)**.** *A GDL rule is called* Horn *if its body is a conjunction of literals.*

Definition 8.6 (Safety Property)**.** *For a Horn rule the* safety property *holds if any variable appearing in the head or in any negative literal also appears in a positive literal in the body.*

We stick to the older notion because there are still many games that use disjunctions. Also, in our experience game descriptions using disjunctions tend to be shorter and easier to understand by humans, though that is of no concern in the development of a GGP agent.

In GDL, all rules are implicitly connected by disjunction, so that it is possible to split rules containing disjunctions into a number of Horn rule. Thus, we can see the older notion with disjunctions modeled explicitly and the newer one without those as equivalent in terms of expressiveness.

Lemma 8.7 (Splitting of rules)**.** *Any rule according to Definition 8.4 can be converted to a number Horn rules.*

Proof idea. The idea is to calculate the disjunctive normal form (DNF) by applying the distributive property and to split a rule if its base operator is a disjunction, i. e., a rule in DNF of the form $h \Leftarrow b'_1 \vee \cdots \vee b'_m$ with each b'_i being either a literal or a conjunction of literals can be split to m rules $h \Leftarrow b'_1, \ldots, h \Leftarrow b'_m$. □

Note that for a rule to be a valid GDL rule, after the splitting the safety property must hold in all resulting Horn rules.

In GDL negation is treated as *negation-as-failure*. To avoid any problems arising from this treatment the rules must be *stratified*. Stratification of a set of rules means that some relation r may never be dependent on the negation of r. Thus, the stratification is defined based on a *dependency graph*.

Definition 8.8 (Dependency Graph)**.** *For a set of GDL rules a* dependency graph *can be constructed. Its nodes are the relation symbols in the vocabulary and it contains an edge from r_2 to r_1 if there is a rule with head r_1 and with r_2 in the body. If r_2 is a negative literal the edge is labeled with $\neg$.*

Definition 8.9 (Stratified Rules)**.** *A set of GDL rules is* stratified *iff the corresponding dependency graph does not contain any cycles that include an edge labeled with $\neg$.*

Stratification alone is not enough. In case we are confronted with recursive rules we must make sure that we cannot run into cycles. Thus, each GDL rule must also obey the *recursion restriction*.

Definition 8.10 (Recursion Restriction)**.** *Given a set of GDL rules containing a rule of the form*

$$p\left(a_1, \ldots, a_{p_n}\right) \Leftarrow b_1 \wedge \cdots \wedge b_k\left(v_1, \ldots, v_{k_n}\right) \wedge \cdots \wedge b_n$$

where b_k is in a cycle with p in the dependency graph representation. Then the recursion restriction *holds if for all $j \in \{1, \ldots, k_n\}$ it holds that v_j is a constant, or $v_j \in \{a_1, \ldots, a_{p_n}\}$, or $\exists i \in \{1, \ldots, n\}$. $b_i = r\left(\ldots, v_j, \ldots\right)$ for some relation symbol r that does not share a cycle with p in the dependency graph representation.*

The semantics of GDL is based on first-order logic semantics (with negation being treated as negation-as-failure. Thus, we will skip the details and refer the reader to the original GDL specification (Love et al., 2006).

8.1.2 Structure of a General Game

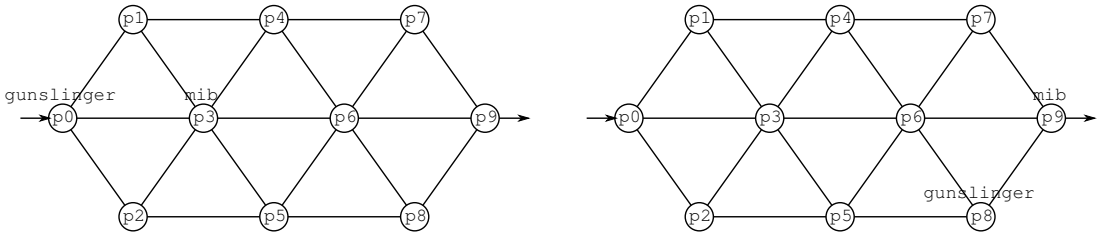
In contrast to PDDL in GDL we do not have two descriptions for the domain and a certain instance of that domain, but rather both are done together in one description. It consists of the specification of several aspects of the game, which we will describe in more detail in the following. To make things clearer, we will explain the different concepts along our running example of the man in black running through the desert followed by the gunslinger.

While in the planning formulations we restricted ourselves to a static man in black, which the gunslinger had to reach on a path as short as possible, here we can finally allow the man in black to move as well. Thus, in this formulation the gunslinger starts at position `p0` and the man in black (`mib`) has only a small head start at position `p3` (cf. Figure 8.1). The game would be awfully easy if both were equally fast, so that we decided to allow the gunslinger to move a distance of two positions in one step, while the man in black can only move as far as an immediately adjacent position. The game would still not be any challenge whatsoever if the two players moved alternatingly, so that instead they move simultaneously and the gunslinger can only win if he reaches the same position as the man in black unless it is the end of the desert at position `p9` where the man in black is able to escape and thus win the game. To keep the game finite we use a step counter, so that the game ends after ten steps if neither the gunslinger succeeds in capturing the man in black nor the man in black succeeds in escaping.

A part of the game's description is given in Figure 8.2; the full description can be found in Appendix A.

Definition 8.11 (General Game)**.** *A* general game *is a tuple $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$ with $\mathcal{P}$ being the set of available players or* roles*, $\mathcal{L}$ the set of legal rules, $\mathcal{N}$ the set of next rules, $\mathcal{A}$ the set of rules for additional relations, which we call* axioms*, $\mathcal{I}$ the initial state, $\mathcal{T}$ the set of terminal states, and $\mathcal{R}$ the set of reward rules.*

In GDL, which uses the same notation as the knowledge interchange format KIF (Genesereth and Fikes, 1992), all predicates and formulas are in prefix notation and variables are denoted by the prefix `?`. Several relations are predefined in GDL. In the following we will outline the meanings of these relations and show how they are implemented in our example game.



(a) Initial state. (b) Terminal state won by the `mib`.

Figure 8.1: GDL version of the running example.

```
(role gunslinger) (role mib)

(init (position gunslinger p0))
(init (position mib p3))
(init (step 0))

(<= (legal gunslinger (move ?l1 ?l3))
    (true (position gunslinger ?l1))
    (adjacent ?l1 ?l2) (adjacent ?l2 ?l3) (distinct ?l1 ?l3))
(<= (legal mib (move ?l1 ?l2))
    (true (position mib ?l1)) (adjacent ?l1 ?l2))

(<= (next (position ?p ?l2))
    (does ?p (move ?l1 ?l2)))
(<= (next (step ?s2))
    (true (step ?s1)) (succ ?s1 ?s2))

(<= terminal caughtMIB)
(<= terminal escapedMIB)
(<= terminal (true (step 10)))

(<= (goal mib 100) escapedMIB)
(<= (goal mib 10)
    (true (step 10)) (not caughtMIB) (not escapedMIB))
(<= (goal mib 0) caughtMIB)

(succ 0 1) ... (succ 9 10)
(adjacent p0 p1) ... (adjacent p8 p9)
(adjacent p1 p0) ... (adjacent p9 p8)

(<= caughtMIB
    (true (position gunslinger ?l)) (true (position mib ?l))
    (not escapedMIB))

(<= escapedMIB (true (position mib p9)))
```

Figure 8.2: Part of the GDL description of the running example.

Roles To specify the roles of the different players, the unary relation `role` is used. The only argument is a player's name within the game description. All roles must be defined at the start of the game and the set of players may never change in the course of one game. According to the specification,

a `role` relation may only appear as a fact, though in practice lots of games use this keyword within the bodies of other rules to simplify the description.

In the running example, we have two players, `gunslinger` and `mib`.

Fluents In GDL a state is described by functions for which the relation `true` holds in that state. These are called *fluents*.

Initial State The initial state is specified by the unary relation `init`. It might either be a head of a rule or a fact. The argument of `init` is a function and all possible instantiations of this argument denote the set of fluents true in the initial state. Any fluent that is not a possible instantiation of the argument does not hold initially.

In the example the initial state consists of two `position` fluents, specifying the position of the gunslinger and of the man in black at the start of the game. Additionally, the `step` counter is initially set to `0`. Note that we could have chosen any other object constant, as GDL does not know the concept of numbers.

Inequality In several cases it is important to know that some parameters must differ. For those cases the binary relation `distinct` is used. As this is an implicit negation, if any of the two arguments is a variable due to the safety property (cf. Definition 8.6) that variable must also appear in a positive literal within the same rule.

Legal Moves In any reachable non-terminal state all players must choose a move. GDL specifies what moves are possible in which state. The rule's head is the binary `legal` relation. The first argument of this is the player's name, the second the move that the player can perform.

By default, GDL allows only the description of simultaneous-move games. To make a game turn-taking (or, more generally, non-simultaneous), such as CONNECT FOUR, CHESS and many others, a *noop* (for *no operation*) move must be specified. This is the only move that can be chosen by the player who is not the active one. In order to make the example game turn-taking, we need an additional fluent, `control`. This specifies which player is currently the active one and may perform a move. A `legal` rule in the turn-taking setting then looks like this:

```
(<= (legal mib (move ?l1 ?l2))
    (true (control mib))
    (true (position mib ?l1)) (adjacent ?l1 ?l2))
```

Chosen Moves To denote which moves the players have chosen the `does` relation is used. Similar to the `legal` relation it is a binary relation with the first argument denoting the player who has chosen the move specified in the second argument. This relation appears only in the body of rules.

Successor States To determine the successor state once all players have chosen a move the unary `next` relation is used. Its single argument is a function that denotes a fluent true in the successor state. This relation may only appear as the head of a rule.

Note that in GDL the frame is modeled explicitly, i. e., any fluent that is not the argument of the head of a satisfied `next` rule is supposed to be false in the successor state. Due to this the game descriptions tend to be a bit more lengthy than those of PDDL, where the frame is modeled implicitly, so that only those facts that change are given in the effects of an action.

In the example game both players move simultaneously and the first `next` rule updates their positions. The `step` counter is increased by the second `next` rule. These are the only `next` rules we need in this game; the frame is no problem here. If the game was turn-taking things would look a bit different. In that case, we would need to store the position of the player whose turn it is to wait, for example:

```
(<= (next (position ?p ?l))
    (true (position ?p ?l)) (not (true (control ?p))))
```

Terminal States Once a terminal state is reached the game ends and no more moves are performed by any of the players. To determine when a terminal state is reached the `terminal` rules are used. Once one of these is satisfied the game ends.

The example game ends in one of three cases. Either if the man in black is caught, or the man in black has escaped or the step counter has reached the value of `10`.

Rewards To generate the outcome of a game, i. e., to determine the rewards for all the players, the binary `goal` relation is used with the first argument being the player for whom this relation is relevant and the second one the reward that player will get. The rewards are integral and range from 0 to 100, with higher rewards being better.

In the example game the winner gets 100 points and the loser 0, i. e., if the man in black can escape he has won; if he is caught before escaping the gunslinger has won. In case no one has won and the step counter has ended the game both players get 10 points.

Additional Relations All additional relations, the axioms, can appear anywhere, i. e., they can be part of the body of a rule, the head of a rule or also a fact. They are similar to the derived predicates from PDDL. In principle, this is a way to keep the game description shorter (and thus also more readable), as they can be removed by replacing each appearance of a relation by the body of a corresponding rule.

Examples for this concept in the example game are the relations `succ`, `adjacent`, `caughtMIB`, and `escapedMIB`, denoting a successor relation for the step counter, the adjacency of the positions, whether the man in black has been caught, and whether the man in black has escaped, respectively.

GDL constrains the description further than just to deterministic, discrete and finite games of full information following the definitions of Section 8.1.1. In order to produce *well-formed games*, which we typically expect in a competition setting, the description must satisfy three additional conditions. Once again, these definitions are from the original GDL specification (Love et al., 2006).

Definition 8.12 (Termination)**.** *A general game* terminates *if all (infinite) sequences of legal moves from the initial state reach a terminal state after a finite number of steps.*

Definition 8.13 (Playability)**.** *A general game is* playable *iff each player has at least one legal move in every reachable state that is not terminal.*

Definition 8.14 (Winnability)**.** *A general game is* strongly winnable *iff for one player there is a sequence of moves of that player that leads to a terminal state where that player receives the maximal reward of* 100 *points, i. e., if the player can find a sequence of moves that ensure victory, no matter what moves the opponents perform.*

A general game is weakly winnable *iff for every player there is a sequence of joint moves of all players that leads to a terminal state where that player receives the maximal reward of* 100 *points, i. e., if it is possible that the player can achieve the maximal reward if the other players help it to achieve this goal.*

Note that a single-player game that is weakly winnable is also strongly winnable.

Definition 8.15 (Well-Formed Game)**.** *A general game is* well-formed *if it terminates, is playable and is weakly winnable.*

8.2 Differences between GDL and PDDL

Apart from the ability to describe multiplayer games instead of just single-agent planning problems the two input languages PDDL and GDL differ in several other points as well.

- In GDL, no list of all relation symbols or object constants is provided. They can only be determined by scanning the complete game description.

- GDL does not support typing. While in PDDL most of the predicates are typed, i. e., the possible domain of the various parameters is already specified, in GDL this is not the case. There, either the agents must assume that a variable parameter can be replaced by every constant, or they must infer additional knowledge by analyzing the game description in more detail. A kind of typing can be achieved by using relations corresponding to the types and stating those at the start of the rules where variables of some type are used, but this approach of course is up to the modeler and is not mandatory and actually only rarely used in the existing games.

- In GDL, the frame is modeled explicitly. This is one of the most important differences. While in PDDL the frame is modeled implicitly by specifying only the add- and delete-effects of an action, in GDL every fluent that is not an argument of a satisfied `next` rule is assumed to be false in the successor state. On the one hand this increases the size of the game description, on the other hand it makes the analysis much more difficult, as it is hard to determine which fluents are added and which are deleted by a move. There are some approaches that try to identify potential add and delete effects (Günther et al., 2009; Zhao et al., 2009), but they do not work well in all cases.

- In PDDL, actions consist of a precondition and a list of effects. In GDL, premises of the `legal` rules are the preconditions for the corresponding move. The effects are determined by the `next` rules. These often are dependent on the chosen move, in which case a `does` relation is present, which can be arbitrarily deep in the formula specifying the body, so that a connection between the two is not immediate.

Especially the last two points prevent a simple translation from GDL input to PDDL, so that the fairly sophisticated planning technology cannot be used immediately. Rather, we had to come up with a method to instantiate the descriptions in such a way that it results in an output that the parser we used for action planning can handle (Kissmann and Edelkamp, 2009a).

8.3 Comparison to Action Planning

General game playing can be seen as an extension of action planning. The domains handled in classical planning as we introduced them in Chapter 6 can all be seen as single-player games. The planner is the player and as such tries to reach a terminal state. Nevertheless, there are also some differences. In action planning we are mainly interested in minimizing either the number of actions in a plan or the total action cost, while in general game playing the players get certain rewards depending on the terminal state reached in the end. There are no costs for the moves and also the total number of moves is irrelevant. These of course could be modeled as well by using a function that is increased in each step by the modeled action cost and influences the rewards accordingly. However, as we are restricted to integral rewards ranging from 0 to 100 we cannot model every planning problem in such a way.

A setting similar to the one in GGP can be found in net-benefit planning—or over-subscription planning, if we do not wish to handle action costs—as introduced in Chapter 7. There we have a goal that must be reached and some soft goals that the planners should reach but need not necessarily do so. The planners are awarded rewards dependent on the satisfied soft goals.

The main difference of course comes with the number of players. While classical and net-benefit planning allow only for one player, in general game playing any finite number of players is supported. A planning track we did not handle is non-deterministic planning. In this, the outcome of the actions is non-deterministic, so that a planner does not know the exact successor state that will be reached. It is required to find a plan (or in this case a *policy*) that can transform the initial state to a goal state, independent of any non-determinism (if such a policy exists). Such a problem can be modeled as a two-player game, where one player takes the role of the planner and the other the role of the environment, which decides the outcome of the non-deterministic choices (Bercher and Mattmüller, 2008; Kissmann and Edelkamp, 2009b).

Another difference comes with the competition settings. In case of action planning the planners get a certain amount of time, typically 30 minutes, to find a good plan, while in general game playing we have two times, a startup time and a playing time. The startup time is used at the beginning of a match to enable all players to start analyzing the game at hand. The playing time is used after each move to calculate the next

move. Thus, general game playing is more in the direction of real-time playing, while in action planning the plans are generated beforehand.

Furthermore, in action planning the planners are executed by the organizers of a competition, so that all run on the same hardware under the same conditions. In GGP the players are run by the teams themselves, so that the actual outcome does not necessarily depend on the quality of the player but in sometimes also on the quality of the hardware, if some teams have lots of money to run their players on a big cluster and others can only use a normal personal computer.

Nevertheless, the setting of both domains is very similar—finding good (or optimal) solutions without intervention of a human—and this setting is what makes research and also the results in these domains very interesting.

Chapter 9

Instantiating General Games

> The Veil had parted before me, as abruptly as it had occurred. I had passed beyond it and acquired something, I had gained a piece of myself.
>
> Roger Zelazny, *Nine Princes in Amber*
> (*The Chronicles of Amber* Series, Book 1)

Though they differ in several aspects, PDDL and GDL have one thing in common. They both use a logic based format incorporating variables to describe planning problems and general games, respectively. The problem with this structure is that to use it during the planning or playing and solving process either a mechanism that can handle these variables directly such as Prolog has to be used, or the variables must be removed. To do this, the descriptions are *instantiated* (often also called *grounded*).

Especially the BDD based solver we have implemented is dependent on instantiated input—using BDDs with variables appearing as arguments of the literals is not possible, all have to be grounded. The solver is described in detail in Chapter 10.

To play uninstantiated general games we have implemented a player based on an interface to SWI-Prolog[23]; a thorough description of our approach follows in Chapter 11. For a quick comparison of the performance with instantiated and uninstantiated input we implemented two simple Monte-Carlo approaches. They simply perform a number of purely random simulations, i. e., they start at the initial state and choose a random legal move for each player in each non-terminal state. Once a terminal state is reached the average reward of the first move of this simulation run is updated. In order to play a game, in the end the move with the best average reward is chosen, though here we are not interested in actually playing the games but only in the number of expansions that can be performed by the two approaches in the same time. The difference of the approaches is that one uses Prolog to infer knowledge about the legal moves and the successor states while the other one utilizes the information of the instantiated input. The results of this initial comparison are depicted in Table 9.1 and clearly show that using the instantiated input can greatly speed up the search: On the set of tested games the instantiated approach can perform 3.7 to 251 times as many expansions as the Prolog based approach.

Many of the recent successful players such as CADIAPLAYER (Björnsson and Finnsson, 2009) or ARY (Méhat and Cazenave, 2011a) use Prolog's inference mechanism. Nevertheless, after these results it seems a necessity to come up with an efficient instantiation of the games to make use of the improved number of expansions.

In the domain of action planning this is also an important approach—nearly all recent planners (e. g., GRAPHPLAN (Blum and Furst, 1997), SATPLAN (Kautz and Selman, 1996), FF (Hoffmann and Nebel, 2001), FAST DOWNWARD (Helmert, 2006, 2009), LAMA (Richter and Westphal, 2010), or our GAMER (Edelkamp and Kissmann, 2009)) instantiate the input before actually starting the planner. Unfortunately,

[23] `http://www.swi-prolog.org`

Table 9.1: Number of expanded states using pure Monte-Carlo search with Prolog and with instantiated input. The timeout was set to 10 seconds. All game descriptions are taken from Dresden's GGP server.

Game	Exp_{Prolog}	$\text{Exp}_{Inst.}$	Factor
asteroidsserial	59,364	219,575	3.70
beatmania	28,680	3,129,300	109.11
chomp	22,020	1,526,445	69.32
connectfour	44,449	2,020,006	45.45
hanoi	84,785	7,927,847	93.51
lightsout	28,800	7,230,080	251.04
pancakes6	154,219	2,092,308	13.57
peg_bugfixed	19,951	1,966,075	98.55
sheep_and_wolf	20,448	882,738	43.17
tictactoe	65,864	5,654,553	85.85

Algorithm 9.1: Instantiating General Games

Input: General Game.
Output: Instantiated Description of General Game.
1 Normalize the game description
2 Calculate the supersets of all reachable fluents, moves and axioms
3 Instantiate all rules
4 Find groups of mutually exclusive fluents
5 Remove the axioms
6 Generate the instantiated output

the input languages are too different, so that we did not find a translation from GDL to PDDL to make use of the existing grounders and thus had to come up with our own for GDL.

In the following we will describe the instantiation process (Section 9.1) and show the results for instantiating a number of the games found on Dresden's GGP server[24] (Section 9.2).

9.1 The Instantiation Process

Definition 9.1 (Instantiated General Game)**.** *An* instantiated general game *is a general game $\mathcal{G}$ with no variables, i. e., the arguments of all literals appearing in the game's description are constants.*

We have implemented two different approaches to instantiate general games. The first one uses Prolog's inference mechanism to find supersets of reachable fluents, moves and axioms (Kissmann and Edelkamp, 2009a), while the second one uses the information it can gain by evaluating the dependency graphs of the various rules, which eventually results in—typically larger—supersets that can be compressed in an additional step to the same supersets found by the first approach (Kissmann and Edelkamp, 2010a).

The outline of the instantiation process as shown in Algorithm 9.1 is the same in both cases. It is inspired by the PDDL instantiators by Edelkamp and Helmert (1999) as well as Helmert (2008), which are still used in successful action planners.

In the following, we will present all the steps of the instantiation process in more detail.

9.1.1 Normalization

In this step we want to simplify the rules as much as possible. In fact, we aim at generating rules whose bodies consist only of conjunctions of literals.

[24] http://ggpserver.general-game-playing.de

Definition 9.2 (Normalized General Game)**.** *A* normalized general game *is defined as a tuple* $\mathcal{G}_\mathcal{N} = \langle \mathcal{P}, \mathcal{L}_\mathcal{N}, \mathcal{N}_\mathcal{N}, \mathcal{A}_\mathcal{N}, \mathcal{I}_\mathcal{N}, \mathcal{G}_\mathcal{N}, \mathcal{R}_\mathcal{N} \rangle$ *with all rules, i. e., $\mathcal{L}_\mathcal{N}$, $\mathcal{N}_\mathcal{N}$, $\mathcal{A}_\mathcal{N}$, $\mathcal{I}_\mathcal{N}$, $\mathcal{G}_\mathcal{N}$, and $\mathcal{R}_\mathcal{N}$, are either facts or consist of a head and a body, where the head is a positive literal and the body the conjunction of (positive and negative) literals.*

To arrive at a normalized game we must split all the rules present in the original game description as described in Lemma 8.7. As the rules in GDL can consist only of conjunctions, disjunctions and negations this is straight-forward. We first of all apply DeMorgan's laws and the removal of double negations until all negations appear only right in front of an atomic sentence, resulting in rules in *negation normal form* (NNF). Then, we make use of the distributivity of conjunctions and disjunctions, until the bodies of all the rules are in *disjunctive normal form* (DNF), i. e., the rules consist of one disjunction of several conjunctions of (positive or negative) literals or one conjunction of several literals or only a single literal. Afterward, we split a rule if its body's outermost operator is a disjunction, i. e., for a rule

$$head \leftarrow (body_1 \vee \cdots \vee body_n)$$

with $body_i$ $(1 \leq i \leq n)$ being a conjunction of literals we create n rules with the same head, which are implicitly connected by disjunction:

$$\begin{aligned} head &\leftarrow body_1 \\ &\vdots \\ head &\leftarrow body_n \end{aligned}$$

Note that for games following the most recent GDL specification this step is no longer necessary, as in those games the rules are already Horn. However, given that this is not the case in older games and especially disjunction is used quite often this step is important in order to handle those.

9.1.2 Supersets

The next step is to find supersets of all reachable fluents, moves and axioms. This is the main step where our two approaches differ.

Prolog

In the Prolog based approach we generate a Prolog description of the game, but we omit the negative literals. Also, the terminal and goal rules are not relevant for finding the supersets.

We start by calculating the supersets of reachable fluents and moves (cf. Algorithm 9.2), making use of Prolog's knowledge base, in which we store all fluents and moves that we find during the search. We start with the fluents from the initial state. For these we find all legal moves for all players by querying Prolog for `legal(P, M)`[25] (line 11). This returns all possible instances given the current knowledge base. All new moves are added to the set of reachable moves ($reachMoves$, line 15) as well as to the knowledge base by using Prolog's assert mechanism (line 14).

In a similar fashion we find the reachable fluents ($reachFluents$) given the current knowledge base. These are initiated to be the ones from the initial state (lines 2 to 6) and updated according to the applicable `next` rules (lines 16 to 20).

As we have omitted all negations and never remove any fluents or moves from the knowledge base, we can safely add the new fluents and moves to the knowledge base, as they can only increase the number of reachable fluents and moves, never decrease it, i. e., they cannot make any rule's body false that was evaluated to true in a previous step. This way of handling the problem is actually very similar to the delete relaxation in case of action planning (cf. Section 6.3)

We repeat these steps for finding reachable fluents and moves until we cannot find any new ones. In that case we have found supersets of all reachable fluents and moves. What remains is finding the superset of reachable axioms.

[25] In Prolog, we use capital letters to denote the variables.

Algorithm 9.2: Finding supersets of reachable fluents, moves, and axioms (Prolog)

Input: Normalized general game $\mathcal{G}_\mathcal{N} = \langle \mathcal{P}, \mathcal{L}_\mathcal{N}, \mathcal{N}_\mathcal{N}, \mathcal{A}_\mathcal{N}, \mathcal{I}_\mathcal{N}, \mathcal{G}_\mathcal{N}, \mathcal{R}_\mathcal{N} \rangle$ in Prolog description.
Output: Triple: superset of reachable fluents, superset of reachable moves, superset of reachable axioms.

```
1  Prolog: load description of the game                          // The description is loaded by Prolog.
2  reachFluents ← ∅                                              // Set of reachable fluents initially empty.
3  Prolog: init(F).                                              // Query Prolog for initial fluents.
4  for all f ∈ F do                                              // For all fluents true in the initial state...
5      reachFluents ← reachFluents ∪ fluent(f)                   // Add fluent to the set of reachable fluents.
6      Prolog: assert(true(f)).                                  // Insert fluent to Prolog's knowledge base.
7  newFluents ← reachFluents                                     // All reachable fluents are newly found ones.
8  reachMoves ← newMoves ← ∅                                     // No moves yet.
9  while newFluents ≠ ∅ or newMoves ≠ ∅ do                       // While new fluents or moves are found...
10     newFluents ← newMoves ← ∅                                 // Forget new fluents and moves from previous iteration.
11     Prolog: legal(P, M).                                      // Query Prolog for legal moves.
12     for all p ∈ P, m ∈ M and move(p, m) ∉ reachMoves do       // For all players and new moves...
13         newMoves ← newMoves ∪ move(p, m)                      // Store move internally.
14         Prolog: assert(does(p, m)).                           // Insert move to Prolog's knowledge base.
15     reachMoves ← reachMoves ∪ newMoves                        // Add new moves to set of reachable moves.
16     Prolog: next(F).                                          // Query Prolog for successor fluents.
17     for all f ∈ F and fluent(f) ∉ reachFluents do              // For all new fluents...
18         newFluents ← newFluents ∪ fluent(f)                   // Store fluent internally.
19         Prolog: assert(true(f)).                              // Insert fluent to Prolog's knowledge base.
20     reachFluents ← reachFluents ∪ newFluents                  // Add new fluents to set of reachable fluents.
21 reachAxioms ← ∅                                               // Set of reachable axioms—initially empty.
22 for all ax/n being head of an axiom rule in A_N do            // For all axioms...
23     Prolog: ax(V1,...,Vn).                                    // Query Prolog for instantiations of axiom.
24     for all (v1,...,vn) ∈ (V1,...,Vn) do                      // For all instantiations...
25         reachAxioms ← reachAxioms ∪ axiom(ax(v1,...,vn))      // Store new axiom internally.
26 return (reachFluents, reachMoves, reachAxioms)                // Return the calculated supersets.
```

With the last iteration the knowledge base already contains the supersets of all reachable fluents and moves, so that finding the superset of all reachable axioms is straight forward. For each axiom ax with arity n we send a query of the form `ax(V1, ..., Vn)` to Prolog (line 23), which returns all possibly reachable instances. These we add to the superset of all reachable axioms ($reachAxioms$, line 25) and finally return the three supersets.

Dependency Graph

In the course of experimental evaluation for some general games we found out that in several cases the Prolog based approach is rather slow. This is due to a large number of duplicates that can be found. In fact, in many games there is a huge amount of possibilities to satisfy a rule, which are all returned by the queries.

To overcome this problem, we implemented our second approach, which makes use of a dependency graph structure similar to the ideas by Schiffel and Thielscher (2007b), which actually captures the dependencies of the arguments, not the relation constants as in Definition 8.8. Thus, we call the structure an *argument dependency graph*.

Definition 9.3 (Argument Dependency Graph)**.** *For a rule $h \Leftarrow b_1, \ldots, b_n$ the nodes of the* argument dependency graph *consist of the arguments of the head and all literals of the body of the rule as well as the object constants of the game's description. The ith argument of atom a is denoted as a^i. A directed edge is*

```
(init (step 0))
(<= (next (step ?s2))
    (true (step ?s1))
    (succ ?s1 ?s2))
(succ 0 1) (succ 1 2) ... (succ 9 10)
```

Figure 9.1: Rules for the fluent `step` and the relevant facts for `succ` in the example game.

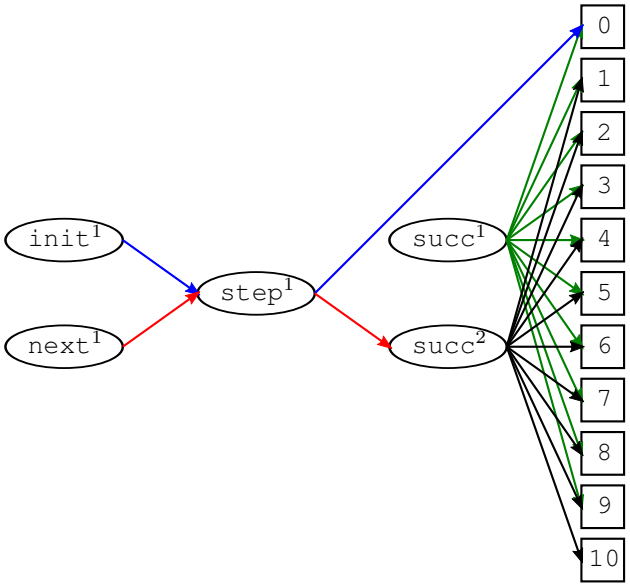


Figure 9.2: Merged argument dependency graph for the rules for `step` in the example game.

inserted from h^j to a^i if a is an atom in the rule's body and h^j and a^i represent the same variable. An edge is inserted from h^j to c if c is an object constant and h^j is c in the description.

For a set of rules the argument dependency graph is the result of merging the graphs. This is done by merging all nodes with the same labels.

For the rules for the fluent `step` of the example game (as shown in Figure 9.1) we can create an argument dependency graph (cf. Figure 9.2). According to the `init` rule, the first (and only) argument of `step` matches the constant `0` (blue in Figure 9.2). From the `next` rule we see that the argument of `step` may match the second argument of `succ` (red in Figure 9.2). Finally, from the axiom facts for `succ` we can learn that the first parameter of `succ` may take the values from `0` to `9` (green in Figure 9.2) while the second one can take values from `1` to `10` (black in Figure 9.2).

A pseudocode for the construction of an argument dependency graph is provided in Algorithm 9.3 (lines 1 to 10).

To determine possible instantiations for the arguments of the atoms we must evaluate the dependency graph. We resolve the dependencies in topological order, i. e., we first determine the possible instantiations of those arguments only dependent on constants, then those dependent on constants and those already evaluated and so on, until all arguments can be instantiated.

For an atom with more than one argument we then combine all possible instantiations of all arguments. This way we arrive at a superset of all reachable atoms, i. e., fluents, moves, and axioms (lines 11 to 14).

With the dependencies from Figure 9.2 we can deduce that the argument of `step` can take all values from `0` to `10`. Also, the first argument of `succ` can take values from `0` to `9` and the second one values from `1` to `10`. With this information, we are already able to generate a superset of all reachable heads, i. e., {`(step 0)`, `...(step 10)`, `(succ 0 1)`, `...`, `(succ 0 10)`, `(succ 1 0)`, `...`, `(succ 9 10)`}. This clearly is a superset of all reachable instances of `step` and `succ` as all intended instances are included, but it contains too many elements, as can be seen for `succ`.

Algorithm 9.3: Finding supersets of reachable fluents, moves, and axioms (dependency graph)

Input: Normalized general game $\mathcal{G}_\mathcal{N} = \langle \mathcal{P}, \mathcal{L}_\mathcal{N}, \mathcal{N}_\mathcal{N}, \mathcal{A}_\mathcal{N}, \mathcal{I}_\mathcal{N}, \mathcal{G}_\mathcal{N}, \mathcal{R}_\mathcal{N} \rangle$.
Output: Triple: superset of reachable fluents, superset of reachable moves, superset of reachable axioms.

$E \leftarrow \emptyset$ // The argument dependency graph initially contains no edges
$V \leftarrow \left\{ a^i \mid a \text{ atom of arity } m, 1 \leq i \leq m \right\} \cup \{ c \mid c \text{ is object constant} \}$ // but all nodes.
for all $h \Leftarrow b_1, \ldots, b_n \in \{\mathcal{I}_\mathcal{N}, \mathcal{N}_\mathcal{N}, \mathcal{L}_\mathcal{N}, \mathcal{A}_\mathcal{N}\}$ **do** // For all rules...
  for $0 \leq i < arity\,(h)$ **do** // and all arguments of rules' heads...
    if $isVariable\left(h^i\right)$ **then** // If argument is a variable...
      for all b_k with some $b_k^j = h^i$ **do** // Find all arguments in body being the same variable.
        $E \leftarrow E \cup \left(h^i, b_k^j\right)$ // Add edge from h^i to b_k^j in argument dependency graph.
    else // Otherwise the argument must be a constant.
      let $h^i = c$ // Let c be the constant being ith argument of the head.
      $E \leftarrow E \cup \left(h^i, c\right)$ // Add edge from h^i to c in argument dependency graph.
$resolveDependencies\,(V, E)$ // Resolve arguments' dependencies (in topological order).
result: domain $\mathcal{D}$ for each atom's arguments
apply all combinations for all arguments of atoms a according to $\mathcal{D}$ // Calculate (and store) superset // of reachable instantiated atoms.
result: $a_\mathcal{I}$; set $Atoms_\mathcal{I} = \{a_\mathcal{I} \mid a \text{ atom of } \mathcal{G}_\mathcal{N}\}$
$Atoms_\mathcal{I} \leftarrow reduceSupersetsDG\,(\mathcal{G}_\mathcal{N}, \mathcal{D}, Atoms_\mathcal{I})$ // Reduce set of instantiated atoms.
return $retrieveSupersets\,(Atoms_\mathcal{I})$ // Return found supersets.

This is a problem that often arises with the dependency graph based approach, i. e., it leads to supersets that are too large. These too large supersets are due to the use of very simple dependencies. We actually lose a lot of information on the interaction between the different arguments, e. g., for `succ` we lose the information that the second argument is greater by exactly one (if we are able to interpret the numbers correctly) and not by any amount—or even smaller as the result of the dependency graph analysis also allows—when we check the possible instantiations of each argument independently of each other. This problem does not appear with the Prolog based approach, as we evaluate the complete rules, so that typically the supersets generated by Prolog are stricter, which also means that the remaining steps of the instantiation process take a lot more time with the dependency graph based approach.

In some cases it is easy to overcome this problem. One such case is the handling of axioms that are only specified by facts, i. e., rules without a body. In that case we can use the exact instantiations as given in the game's description. In other cases it is not so easy. If we look at the rule for a legal move for the man in black in the running example, we see that he may move to any adjacent position. The dependency graph approach would result in impossible moves, e. g., `(move 1 1)` or `(move 1 10)`. For these we do not have only a fact but a rule that depends on the arguments of `adjacent`, which in turn is defined by a number of facts.

To overcome this problem for such more complex cases we must perform two post-processing steps. Only one can be performed immediately, the other only after the rules have been instantiated with the too large supersets. First of all, we generate all the possible fluents, moves, and axioms that we can deduce when evaluating the argument dependency graph. For each of the corresponding rules (i. e., `next`, `legal`, or axiom rules) we apply all supposed possibilities for the head and check for each assignment if the body is compatible with this (cf. Algorithm 9.4). If for some atom of the body the assignment to an argument is not in its domain then this atom can never be made true, so that the entire body can never be made true and thus the rule can never hold. In the pseudocode we assume that we have only one rule for each head. Note that due to the splitting of the rules this might actually not be the case. If we have more than one rule for a head we have to check the bodies of all rules with the corresponding head. If the bodies of all rules can never be made true then we can safely remove that assignment to the head atom from the set of all reachable atoms. Once such an atom is discarded, it might be possible that other rules' bodies are no

Algorithm 9.4: Reducing supersets of instantiated atoms ($reduceSupersetsDG$)

Input: Normalized general game $\mathcal{G}_\mathcal{N} = \langle \mathcal{P}, \mathcal{L}_\mathcal{N}, \mathcal{N}_\mathcal{N}, \mathcal{A}_\mathcal{N}, \mathcal{I}_\mathcal{N}, \mathcal{G}_\mathcal{N}, \mathcal{R}_\mathcal{N} \rangle$.
Input: Domains for all atoms' arguments $\mathcal{D}$.
Input: Set of all instantiated atoms $Atoms_\mathcal{I}$.
Output: Reduced set of instantiated atoms.

```
1  changed ← ⊤                                          // Change in set of reachable atoms.
2  while changed = ⊤ do                                 // While some change in set of reachable atoms...
3      changed ← ⊥                                      // In this iteration, so far no change in set of reachable atoms.
4      for all h ⇐ b_1, ..., b_n ∈ {I_N, N_N, L_N, A_N} do   // For all rules...
5          for all h_I compatible with h do             // For all instantiated heads...
6              replace variables in each b_i according to h_I   // Substitute variables in body.
7              result: b_{1_I}, ..., b_{n_I}
8              for all 1 ≤ i ≤ n, 1 ≤ j ≤ arity(b_i) do   // For each argument of the body...
9                  if ¬isVariable(b^j_{i_I}) and b^j_{i_I} ∉ D(b^j_i) then   // If body unsatisfiable...
10                     Atoms_I ← Atoms_I \ h_I          // Head no longer reachable.
11                     changed ← ⊤                      // Change in set of reachable atoms.
12 return Atoms_I                                       // Return reduced superset of instantiated atoms.
```

longer reachable as well, so that we apply a fix-point analysis, which ends once the set of reachable fluents, moves, and axioms remains unchanged for one iteration.

In the running example, when testing `(move 1 10)` we see that `(adjacent 1 10)` must also be possible, which is not the case, so that the illegal moves will be removed from the superset. However, for other rules it might very well be that we still come up with much larger supersets than in the Prolog based approach. This happens especially in rules that are dependent on themselves, i. e., if they are defined recursively. To handle these self-dependencies, we need another post-processing step, but this can only be applied once the instantiation of all the rules is done, which is the next step.

9.1.3 Instantiation

After the calculation of the supersets of all reachable fluents, moves, and axioms we can instantiate all the rules. A rather naïve idea would be to assign each possible instantiation to each atom of a rule. In the process, an immense amount of temporary partially instantiated rules would be present, so that this approach will not be fruitful in practice.

Rather than trying all possibilities and storing all intermediate results, we make use of the structure of the bodies of the rules. As we have split them in the very first step, the bodies now contain only conjunctions of literals.

For each rule we first of all determine the number of different variables. With this information we generate a matrix whose columns represent the different variables while we store their possible assignments in the rows.

For each atom in the rule (including the head) we can create a block of possible assignments for its variables by retrieving the possible instantiations of this atom from the supersets. Initially, each of these blocks is independent of the other blocks, which represent the other atoms.

To generate the full instantiations we combine the assignments of the variables. As we have only conjunctions of literals the assignments must be combined in such a way that for each block at least one row is satisfied, as this represents a possible assignment to the variables of one of the atoms. If additionally a `distinct` term is present, we also must make sure that its arguments really are distinct. Furthermore, we ignore all negated atoms, as their arguments might take any value, even one that is not reachable—in that case, the negated atom will always evaluate to true.

In the example game, for the `next` rule for `step` and the `legal` rule for the man in black's move and given the information of the superset calculations we would create the matrices given in Table 9.2.

Table 9.2: The matrices for instantiating two rules of the example game.

(a) Matrix for the `next` rule for `step`.

	?s2	?s1
(step ?s2)	0	
	⋮	
	10	
(step ?s1)		0
		⋮
		10
(succ ?s1 ?s2)	0	1
	⋮	⋮
	9	10

(b) Matrix for the `legal` rule for `mib`'s move.

	?l1	?l2
(move ?l1 ?l2)	1	2
	1	3
	1	4
	2	4
	⋮	⋮
	9	10
(position mib ?l1)	1	
	⋮	
	10	
(adjacent ?l1 ?l2)	1	2
	1	3
	1	4
	2	4
	⋮	⋮
	9	10

In the second one (cf. Table 9.2b) we see that for the case of the dependency graph based approach the removal of the unreachable heads of the `move` reduces the number of possibilities. But even if we had not done this before, at the latest in this step we would catch those unreachable states, as the conjunction with `adjacent` reduces the number of possibilities.

In case of the dependency graph based approach we must perform another post-processing step when we have instantiated all rules. This step is necessary to further decrease the number of instantiated rules. Of course, we could already use the instantiations we have generated in this step to create an output with which the game can be played or solved, but the step where the axioms are removed (Section 9.1.5) would take a lot longer, as there might be too many axioms present. Furthermore, the generation of the output would take a while longer, and finally the enlarged output would mean that the player (or solver) would be slowed down as it would have to evaluate all the rules that never can be satisfied when searching for possible moves or successor states.

This second post-processing step performs a reachability analysis using the instantiated rules. Here we temporarily remove all negated atoms, similar to the second step in the Prolog based approach. Actually, the complete reachability analysis works in the same way, only that we do not use Prolog to infer knowledge on satisfied *legal* or *next* rules, but can stick with the datastructures we generated in the course of the instantiation process, which greatly increases the inference speed. This analysis results in the same supersets of reachable state atoms, moves, and axioms as the Prolog based analysis, so that afterward we can remove all the impossible rules. Thus, after this post-processing step the results of both approaches are equal.

9.1.4 Mutex Groups

For single-player and two-player non-simultaneous games we have implemented a general game playing solver, which uses BDDs to handle large sets of states. Similar to our planners (see Chapters 6 and 7) it is important to find groups of mutually exclusive (or *mutex* for short) fluents. Finding these allows us to encode a group of n fluents together by using only $\lceil \log n \rceil$ BDD variables, so that we can decrease the space requirements of the BDDs even further.

In planning, Edelkamp and Helmert (1999) and Helmert (2008) introduced an approach to automatically identify such mutually exclusive atoms by analyzing the actions. However, due to GDL's different action concept, especially the fact that the `legal` and `next` rules are separated and the frame is modeled explicitly, so that we do not have information concerning add and delete effects, we do not yet know how to

calculate these sets in a similar way. Thus, we chose to rely on a number of simulation steps to gain some certainty in hypotheses specifying which fluents might be mutually exclusive.

This approach is similar to the ideas by Kuhlmann et al. (2006) and Schiffel and Thielscher (2007b). They partition the parameters of each fluent into two sets, one representing the input parameters, the other the output parameters. Given a fluent's input parameters, in any state only one assignment of the output parameters can result in the fluent holding in that state. Thus, all fluents sharing the same input but different output parameters are mutually exclusive. Typical examples for input and output parameters are in the fluents describing game boards. The input parameters are the position on the board, while the output parameters represent the piece occupying that position (in each state, only one piece can occupy a position, or the position can be empty, in which case it is occupied by a blank). If all figures are unique, the inverse often also holds, i. e., each figure can be placed only on one position at once. This is the case in our running example, where the man in black as well as the gunslinger can only be at one position at any given time, though here it is possible for them to share the same position (in which case the gunslinger catches the man in black, unless it is the desert's exit). Thus, in this example, for the `position` fluent the first parameter (the person) is the input and the second one (the location) the output parameter.

In previous work the input and output parameters were determined in order to gain some knowledge about the structure of a game, especially to find some fluents that represent the game's board. While Kuhlmann et al. (2006) only supported fluents with three parameters as possible boards, Schiffel and Thielscher (2007b) chose to regard all fluents as possible "boards", even if they did not actually represent a game board. In our approach we are not interested in finding possible game boards, but only in finding the input and output parameters of all fluents, so that we chose to operate similar to Schiffel and Thielscher's approach.

The approach works as follows. First of all, we create several hypotheses for each fluent, each specifying which of the parameters might be input parameters. We have to check all possibilities, i. e., only one parameter might be an input parameter, any combination of two parameters, any of three and so on. Thus, we need an exponential number of these hypotheses. However, most games contain only fluents with up to four parameters, which would result in up to six hypotheses, so that this is no big overhead in most cases.

To check these hypotheses we generate a number of states by simulating a number of random moves and evaluate the fluents. For this simulation we generate a Prolog program with which we can determine the legal moves and calculate the successor state given the chosen moves. Once a terminal state is reached we restart at the initial state. We repeat these simulation steps until we have generated a predefined number of (not necessarily different) states.

In each state we evaluate the fluents that are currently true. To determine if a hypothesis holds, we sort the corresponding fluents according to their supposed input parameters. This way, two instances with the same input parameters are sorted right after each another. If we find two such fluents we know that there is at least one state in which these parameters cannot be input parameters, as two instances with the same assignment but different supposed output parameters hold at the same time. Thus, we remove the corresponding hypothesis.

When all simulation steps are done we analyze the retained hypotheses for each fluent. If only one holds, this clearly specifies the input and output parameters. If there is more than one we check which ensures the smallest encoding size. This we retain for our BDD based solver.

Opposed to Kuhlmann et al. (2006) we found out that only checking three states is not sufficient in several games, especially if the board is not fully specified in the initial state. An example for this is CONNECT FOUR, where the initial state consists only of the fluent that determines the active player. After several steps it might be that both players placed all their pieces in the same columns (or rows) but different columns (or rows) than the opponent. Thus, for a given column (or row) all positions are occupied by the same player, so that the simulation might result in the column (or row) being a possible input parameter, while in truth both, column and row together, constitute the input parameters. Thus, if we do not evaluate enough states the resulting input parameters might be chosen wrongly and the corresponding groups of mutually exclusive fluents might become too large.

Unfortunately, due to the random nature of the simulations even with an arbitrarily large number of steps we still cannot be sure that the resulting groups are correct. However, 50 steps were enough to determine the groups correctly in all cases we analyzed. An in-depth analysis might be preferable, but so far we have

not found an algorithm to do so. Another shortcoming of the current approach is that it can only consider the possible instantiations of one fluent, while in some games even different fluents might be mutually exclusive, so that a more general approach that can combine different fluents into larger groups might be beneficial as well.

Additionally to finding the groups of mutually exclusive fluents we use the simulation steps to check another hypothesis. Initially, we assume each game to be non-simultaneous. If during the simulation steps we find any state where more than one player has the choice between more than one move the game is clearly simultaneous. If we do not find such a state, the assumption still holds. In order to help our player and solver (and also to improve the output) we additionally store the moves that correspond to the noops, i. e., the moves that players can take when it is not their turn.

9.1.5 Removal of Axioms

Before generating the output we remove the axioms, so that the rules contain only conjunctions, disjunctions and negations of fluents and chosen moves (represented by the `does` terms).

For this we proceed similar to the approach presented by Borowsky and Edelkamp (2008). They used it in the area of action planning to get rid of the derived predicates, and GDL's axioms are very similar to those.

The idea is rather simple. All it needs is to sort the axioms topologically, i. e., those dependent on no other axioms are handled first, next those dependent on those already handled and so on, until all axioms are replaced. One such replacement consists of finding all appearances of an axiom in the body of some rule and replacing it by the body of the axiom's rule. As the rules are already instantiated there is no need for the introduction of new variables as it would be in case of input still containing variables. This makes the replacement straight-forward.

Before calculating the topological order we make sure that each rule's head is unique, i. e., those rules whose bodies were split into several rules with the same head are merged again into a single rule with the corresponding head and a body consisting of a disjunction over the bodies of all the rules for that axiom.

The approach works fine because the rules in GDL are stratified (Definition 8.9) and the recursion restriction (Definition 8.10) holds. Thus, we cannot end up in circular dependencies of the instantiated axioms, but can find a topological order to handle them.

9.1.6 Output

During the implementation we have experimented with two different outputs. The first one, which we call *GDDL* (Kissmann and Edelkamp, 2009a), is largely inspired by PDDL. It consists of three parts. The domain description contains the set of actions as well as the rewards, the problem description the initial state and the terminal states, while the partitioning contains the information we have generated when calculating the mutex groups. For a full BNF description of GDDL's domain and problem descriptions consider Appendix B. The partitioning simply consists of groups of fluents. Each line of the file contains either one fluent, if that belongs to the current group, or is empty, if the current group is complete and a new one begins.

The initial state and the terminal states are straight-forward, as they exist in a similar fashion in GDL. The rewards consist of the `goal` rules, which assign the reward a player achieves in a certain state.

To handle the moves we decided to introduce the new *multi-actions*. A multi-action can be seen as the GDDL equivalent of a PDDL action, bringing together the `legal` and `next` rules of the game description. While the `legal` rules describe the precondition necessary in order to perform a move, the `next` rules generate the effects of applying a move. The effects of a move are not directly linked to the move itself in GDL, but the links can be calculated by evaluating the bodies of the `next` rules and checking which `does` terms must be satisfied for the rule to fire.

Each multi-action stands for one of the various moves of each player, so that the first information we give is a set of player–action pairs, which specify the moves of the players. Next follows a *global precondition*, which corresponds to the conjunction of the bodies of the `legal` rules corresponding to the current moves. Finally, the successor state can be calculated by evaluating a number of precondition/effect pairs, which we generate based on the `next` rules. The effect is the head of a rule, while the body becomes the precondition

(after replacing the `does` terms either by true, if they correspond to one of the moves represented by this multi-action, or false, if they do not).

A multi-action $multiAc$ with global precondition $globalPre$ and precondition/effect pairs (pre_1, eff_1), $\ldots$, (pre_n, eff_n) can be translated to the Boolean formula

$$multiAc = globalPre \wedge (pre_1 \leftrightarrow \mathit{eff}_1) \wedge (pre_n \leftrightarrow \mathit{eff}_n),$$

so that it can be handled similar to PDDL's conditional effects.

We can also translate a multi-action to a number of classical PDDL actions, at the cost of an exponential blow-up. Given a multi-action consisting of n precondition/effect pairs we can generate 2^n normal actions. This comes from the fact the we do not know which are add and which delete effects and thus we must handle each effect as an add effect if its precondition is satisfied and as a delete effect if its precondition is not satisfied. To translate the multi-action $multiAc$ we create the actions ac_0 to ac_{2^n-1}. The precondition pre_{ac_i} of the ith action is the conjunction of $globalPre$ with the binary representation of i applied to pre_1 to pre_n, i. e., if we have five precondition/effect pairs and want to represent the sixth precondition, that is

$$pre_{ac_6} = globalPre \wedge \neg pre_1 \wedge \neg pre_2 \wedge pre_3 \wedge pre_4 \wedge \neg pre_5.$$

The effects are constructed similarly, only that there is no "global effect", so that the effect eff_{ac_6} for the 6th action is

$$\mathit{eff}_{ac_6} = \neg \mathit{eff}_1 \wedge \neg \mathit{eff}_2 \wedge \mathit{eff}_3 \wedge \mathit{eff}_4 \wedge \neg \mathit{eff}_5.$$

During the generation of the multi-actions, the additional information if a game is turn-taking helps to keep the output smaller. If it is turn-taking and we know the moves that represent the noops of the different players, we do not have to output those multi-actions that would be established by combining more than one non-noop move. So, for an n-player game with m moves per player (including one noop move each), the number of multi-actions decreases from m^n for the case where we do not know anything about the turn-taking status to $(m-1) \times n + 1$ (one non-noop move combined with $(m-1)$ noop moves and once all noop moves combined), where we omit the clearly impossible multi-actions.

The other output we have used is *instantiated GDL* (Kissmann and Edelkamp, 2010a). It is very similar to GDL, only that variables are not allowed and we use arbitrary Boolean formulas in the bodies, i. e., formulas consisting of conjunctions, disjunctions, negations, and literals. These arbitrary formulas are due to the removal of the axioms, because on the one hand we have joined all axiom rules with the same head to one rule with the same head and the disjunction of the various rules' bodies as its body, on the other hand the topological order can be of arbitrary (but still finite) depth.

Just as in the uninstantiated case we can replace these rules by a number of Horn rules, so that the instantiated GDL can be handled by any player that supports rules according to the most recent GDL specification. Note that in the instantiated case the rules are often much more deeply nested and rather large, so that the calculation of the DNF brings an exponential blowup.

One possibility to overcome this problem would be to omit the removal of the axioms. In that case all instantiated rules would still be Horn. However, our player and solver depend on input without axioms, which is why we decided to remove them and to keep the arbitrary formulas in the bodies, as they are no problem for our player and solver.

The advantage of instantiated GDL over GDDL is that on the one hand it is a lot faster to generate, as we do not have to evaluate all the `next` rules for each possible move combination, and on the other hand all general game players that can handle the uninstantiated input can immediately work with this input as well—at least if we retain the axioms. The disadvantage is that we have to evaluate the `next` rules in a similar way in each step, so that the decrease in runtime of the instantiator often means an increase in the runtime of the player for each step. Thus, in practice we decided to use GDDL for the solver and the player. However, in many cases, especially in simultaneous move games, the GDDL output, which is the same for both versions of the instantiator, dominates the overall instantiation runtime. That is why we decided to use instantiated GDL in the experimental evaluation of the instantiators, in order to better see the influence of the Prolog based approach and the dependency graph based approach, respectively.

9.2 Experimental Evaluation

We performed experiments with the enabled games specified on Dresden's general game playing server[26]. At the time of the experiments (at the end of 2011) the server contained 259 general games, of which 52 were disabled, mainly due to bugs in the description, but in some cases also because the games simply are too difficult for a normal player to handle, which left a total of 207 enabled games.

We have implemented the presented approach for instantiating general games and performed the experiments on a machine consisting of an Intel Core $i7$ 920 CPU with 2.67 GHz, of which we used only one core, and 24 GB RAM. For the output we chose the faster instantiated GDL, as with GDDL often the output, especially the generation of the multi-actions and the immense number of them in case of simultaneous-move games, dominates the total runtime. We used a timeout of ten minutes and performed twenty runs for each problem—ten with the Prolog based approach and ten with the dependency graph based approach—to increase the reliability of the runtime results.

The detailed results can be found in Tables 9.3 and 9.4. Of the 207 enabled games we can instantiate 124 games if the algorithm does not matter. With the Prolog-based approach we are able to successfully instantiate 120 games, with the dependency graph based approach only 111. For the remaining 83 games the difficulties range from timeouts to too much memory consumption and further to some language features we do not yet support, such as nested functions or variables for the moves in the head of the `legal` rules, so that at the moment for 31 games even more time and memory would not help in instantiating them.

The runtimes of the two approaches are very similar in a large number of games: In 72 cases the runtimes differ by a factor of less than two. For other games, however, this factor can be a lot larger. For example, in `catcha_mouse` the Prolog based approach outperforms the dependency graph based approach by a factor of nearly 65, while in `duplicatestatelarge` it is the other way around, with a factor of 120.

Furthermore, we can see that the Prolog based approach can be up to 85 seconds faster than the dependency graph based approach (in `mimikry`), while the dependency graph based approach can be faster by up to 357 seconds (in `bidding-tictactoe_10coins`).

When considering the runtimes of the games that were instantiated by only one approach as well, the range in differences gets even bigger. For `knightstour`, the Prolog based approach found the instantiation after 0.12 seconds, while the dependency graph based approach timed out; for `ruledepthlinear` it is just the other way around and the dependency graph approach instantiates it in 0.10 seconds. This illustrates that games that are easily instantiated by one approach in a very short time cannot be instantiated by the other one in the allowed time limit. For the other games it is very similar. A large factor between the two runtimes can appear in all cases, no matter how hard the games are for one or the other approach.

Summing up the total runtime for the 107 games that were instantiated by both approaches we arrive at 1,659 seconds for the Prolog based approach and at 956 seconds for the dependency graph based approach, which suggests that the latter is faster. However, when comparing the number of instantiated games over the time it took the approaches to instantiate them (cf. Figure 9.3) we see that from roughly 0.3 seconds onwards the Prolog based approach always is slightly ahead of the dependency graph based approach.

Overall, we cannot see any clear trend. In some games the one approach is a lot better, in others the other approach. What remains is an analysis why this behavior happens. Especially, it is important to understand why one of the approaches fails while the other works well.

As we have pointed out earlier, in some domains the Prolog based approach fails due to an immense number of duplicates that Prolog has to handle. These duplicates appear whenever there are many possible ways to achieve some atom. The dependency graph based approach typically does not suffer from such duplicates. Rather, if the rules are defined in a recursive manner it often comes up with supersets that are much larger than those returned by the Prolog based approach, so that in those cases it can run out of memory or time when instantiating the various rules.

So far, we were not able to find any automatism to identify when the one or the other might happen, so that we cannot say when to prefer which approach. Thus, in practice we decided to run both instantiators in parallel and wait for the first one to finish. This way we need more CPU time and more memory, but we are able to instantiate more of the available games.

[26] `http://ggpserver.general-game-playing.de`

Table 9.3: Runtime results for the instantiation of general games (averaged over ten runs), table 1 / 2. All times in seconds. Instances that ran out of time are denoted by o.o.t., those where the memory was insufficient are denoted by o.o.m.

Game	Time (Prolog)	Time (DG)	Factor (PL/DG)
knightstour	0.120	o.o.t.	—
pancakes	1.277	o.o.t.	—
pancakes6	1.291	o.o.t.	—
wallmaze	8.012	o.o.t.	—
hanoi7_bugfix	8.611	o.o.t.	—
frogs_and_toads	35.903	o.o.m.	—
cubicup	30.376	o.o.t.	—
twisty-passages	53.394	o.o.t.	—
hallway	65.233	o.o.t.	—
golden_rectangle	72.366	o.o.m.	—
quad_5x5_8_2	281.852	o.o.m.	—
knightwar	361.084	o.o.t.	—
endgame	437.307	o.o.t.	—
catcha_mouse	0.428	27.663	0.015
hanoi_6_disks	0.805	24.932	0.032
clobber	1.763	32.409	0.054
nim1	0.133	2.379	0.056
nim2	0.178	2.695	0.066
mimikry	7.829	93.674	0.084
minichess-evilconjuncts	2.112	20.101	0.105
minichess	2.621	22.997	0.114
hanoi	0.130	1.008	0.129
nim4	0.481	2.940	0.164
nim3	0.479	2.891	0.166
grid_game	2.658	13.524	0.197
sheep_and_wolf	1.890	9.539	0.198
knightmove	1.660	7.819	0.212
racetrackcorridor	0.480	1.990	0.241
tictactoex9	3.511	13.307	0.264
chomp	0.448	1.643	0.273
peg_bugfixed	1.962	6.620	0.296
tpeg	1.968	6.640	0.296
chinesecheckers1	0.712	2.313	0.308
kitten_escapes_from_fire	0.567	1.586	0.358
eotcatcit	0.064	0.114	0.561
roshambo2	0.046	0.077	0.597
cube_2x2x2	0.165	0.275	0.600
connect4	0.263	0.416	0.632
blocksworldparallel	0.175	0.266	0.658
blocksworldserial	0.065	0.085	0.765
mummymaze2p-comp2007	10.541	13.296	0.793
pacman3p	68.799	84.048	0.819
tictactoe-init1	0.056	0.060	0.933
conn4	0.194	0.207	0.937
toetictac	0.051	0.054	0.944
numbertictactoe	0.236	0.249	0.948
connect5	1.747	1.841	0.949
logistics	20.165	21.216	0.950
troublemaker02	0.040	0.042	0.952
connectfoursuicide	0.129	0.135	0.956
tictactoe_3d_small_2player	0.130	0.136	0.956
tictactoe_orthogonal	0.173	0.181	0.956
tictactoe_3d_2player	0.416	0.435	0.956
tictactoelargesuicide	0.071	0.074	0.959
tictactoelarge	0.072	0.074	0.973
connectfour	0.131	0.133	0.985
pentago_2008	0.772	0.774	0.997
blocks	0.050	0.050	1.000
ghostmaze2p	2.919	2.916	1.001
chickentoetictac	0.054	0.053	1.019
buttons	0.041	0.040	1.025
tictactoeserial	0.205	0.200	1.025

Table 9.4: Runtime results for the instantiation of general games (averaged over ten runs), table 2 / 2. All times in seconds. Instances that ran out of time are denoted by o.o.t., those where the memory was insufficient are denoted by o.o.m.

Game	Time (Prolog)	Time (DG)	Factor (PL/DG)
doubletoetictac	0.105	0.102	1.029
coopconn4	0.245	0.238	1.029
gt_centipede	0.061	0.059	1.034
gt_chicken	3.244	3.136	1.034
circlesolitaire	0.058	0.056	1.036
tictactoeparallel	0.552	0.531	1.040
blocker	0.071	0.068	1.044
breakthroughsuicide_v2	72.906	69.727	1.046
knightthrough	78.281	74.749	1.047
tictactoe	0.065	0.062	1.048
doubletictactoe	0.105	0.100	1.050
blockerserial	4.093	3.898	1.050
3conn3	0.316	0.300	1.053
bomberman2p	3.521	3.333	1.056
breakthrough	72.909	69.042	1.056
chickentictactoe	0.068	0.064	1.063
sum15	0.066	0.062	1.065
statespacesmall	0.046	0.043	1.070
maze	0.045	0.042	1.071
gt_prisoner	2.526	2.358	1.071
tictactoe_3player	0.341	0.318	1.072
double_tictactoe_dengji	0.149	0.138	1.078
gt_attrition	0.085	0.075	1.133
sat_test_20v_91c	0.152	0.134	1.134
oisters_farm	0.074	0.065	1.138
troublemaker01	0.045	0.039	1.154
statespacemedium	0.095	0.082	1.159
statespacelarge	0.271	0.232	1.168
firefighter	0.227	0.193	1.176
asteroids	0.194	0.158	1.228
tictictoe	0.151	0.121	1.248
blocks2player	0.100	0.080	1.250
eotcitcit	0.149	0.118	1.263
rendezvous_asteroids	0.339	0.263	1.289
grid_game2	0.351	0.250	1.404
asteroidsparallel	0.382	0.272	1.404
brain_teaser_extended	0.125	0.086	1.453
lightsout2	0.125	0.085	1.471
beatmania	0.137	0.092	1.489
incredible	0.269	0.173	1.555
3qbf-5cnf-20var-40cl.0.qdimacs	0.175	0.111	1.577
lightsout	0.132	0.081	1.630
point_grab	0.103	0.061	1.689
asteroidsserial	0.564	0.298	1.893
duplicatestatesmall	0.130	0.056	2.321
bidding-tictactoe	9.986	4.275	2.336
max_knights	319.558	124.069	2.576
3pttc	7.917	2.351	3.368
bidding-tictactoe_10coins	484.585	127.341	3.805
8puzzle	3.545	0.758	4.677
Qyshinsu	61.634	11.404	5.405
3pffa	20.773	3.572	5.816
tictactoe_3d_small_6player	2.945	0.459	6.416
4pttc	131.036	17.550	7.466
tictactoe_3d_6player	10.933	1.212	9.021
duplicatestatemedium	4.844	0.159	30.465
smallest_4player	152.915	4.843	31.574
duplicatestatelarge	63.979	0.533	120.036
CephalopodMicro	o.o.m.	258.138	—
Zhadu	o.o.t.	37.968	—
ticblock	o.o.m.	0.548	—
ruledepthlinear	o.o.t.	0.103	—

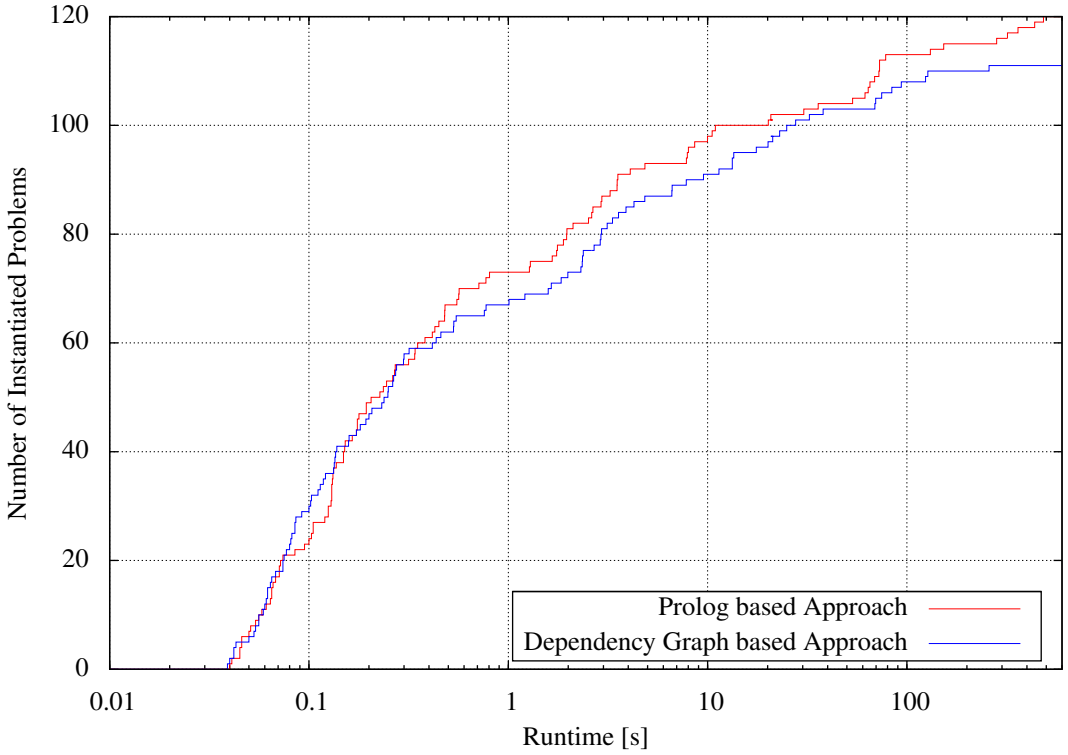


Figure 9.3: Number of instantiated general games versus runtime.

Chapter 10

Solving General Games

> When you have eliminated the impossible, whatever remains, however improbable, must be the truth.
>
> Sherlock Holmes

This chapter is concerned with finding solutions for general single-player and non-simultaneous two-player games. In the past especially two-player *zero-sum* games (see, e. g., Rapoport, 1966) have been studied quite extensively.

Definition 10.1 (Two-Player Zero-Sum Game)**.** *A* two-player zero-sum game *is a game where two players take turns choosing a move. In the end the reward of the first player (which might also be negative) equals the loss of the second player, i. e., the second player's reward is the negative of the first player's, so that the sum of their rewards always equals zero.*

In case of GGP a zero-sum game cannot be defined in the same way, as here only non-negative rewards ranging from 0 to 100 are allowed.

Definition 10.2 (Zero-Sum Game in General Game Playing)**.** *In general game playing, a* zero-sum game *is a game where the rewards of all players sum up to* 100 *in every terminal state. It is a* two-player zero-sum game *if two players are involved.*

For two-player zero-sum games solutions can be anything from the information who will win the game to a full strategy. Thus, we distinguish three kinds of solutions, ranging from ultra-weak to strong (Allis, 1994).

Definition 10.3 (Ultra-Weak Solution)**.** *A* solution *or* ultra-weak solution *determines whether the starting player will win or lose the game or if it will end in a draw (if started in the initial state).*

Definition 10.4 (Weak Solution)**.** *A* weak solution *specifies a strategy so that the players can achieve their optimal rewards (if the game starts in the initial state) if they play according to this strategy, no matter what the opponent does.*

Definition 10.5 (Strong Solution)**.** *A* strong solution *determines the optimal move to take for any reachable state of the game. When following the strong solution at least the optimal outcome calculated for the current state can be assured even after having performed a sub-optimal move, no matter what moves the opponent takes.*

For two-player zero-sum games, strong solutions can be found by using the *minimax* algorithm (von Neumann and Morgenstern, 1944), where we want to maximize our own reward while we expect the opponent to try to minimize it. To find weak solutions we can cut off some branches of the search tree by extending minimax with $\alpha\beta$ *pruning* (Knuth and Moore, 1975).

For the case of single-player games the three kinds of solutions are also applicable. An (ultra-weak) solution tells us only if a path can be found to reach a terminal state with highest reward. A weak solution corresponds to a plan from action planning, i. e., a sequence of moves leading from the initial state to a terminal state achieving the highest possible reward. Such a plan suffices as we are not concerned with any opponent that might take moves leading to states not reachable by following the plan. Finally, a strong solution corresponds to a policy, where the optimal move to take is specified for every reachable state.

In this work we are only interested in finding strong solutions. This of course is the most difficult kind of solution, but it enables us to decide which move to take at any time once the solution has been constructed, so that we can start the solver, wait for it to finish and then use its result in order to play optimally from that point onwards, even if the player performed some sub-optimal moves before the solution was calculated. To achieve the solver must find the optimal outcome for each reachable state. The strong solution then consists of choosing a move that results in a successor state achieving the same result as the current one.

There are several reasons why solutions of (general) games are of interest, for example:

- Using the strong solution, a perfect player can be designed.
- Existing players can be checked, i. e., when analyzing games played by certain players, it is possible to detect when a sub-optimal move has been chosen, which might lead to the discovery of certain weaknesses of these players. On the one hand this might help in designing a player that can use these flaws to defeat them, on the other hand this is important information for the designer of the player and might be used to improve the performance of such a faulty player.
- The result, even if not completely finished, can still be used as an *endgame database*, i. e., once a state that is solved is reached, the precise outcome of the game is known. Thus, players performing some kind of forward search (such as minimax (von Neumann and Morgenstern, 1944) or UCT (Kocsis and Szepesvári, 2006)) can be enhanced, as they do not need to search to a terminal state but can stop once a solved state in the endgame database is found.

In the following Section 10.1 we will describe some games that have been solved by specialized solvers. Then, in Sections 10.2 and 10.3, we will provide symbolic algorithms to strongly solve general single- and non-simultaneous two-player games. At the end, in Section 10.4, we will evaluate our solver in a number of (instantiated) games.

10.1 Solved Games

There are a number of games that have been solved by specialized algorithms or mathematical analysis. Two examples are TIC-TAC-TOE and NIM, both of which have been strongly solved, but both are rather trivial. The former is too small to be of any interest, as its state space contains only $5{,}478$ distinct states, if symmetries are ignored—otherwise the number can be decreased even further. Thus, it can be enumerated and thus fully analyzed by any computer. For the latter Bouton (1901–1902) mathematically determined a general rule with which a NIM game of any size can be played optimally.

In this section we summarize the results for some of the more complex games that have been solved by algorithmic and / or mathematical approaches, ordered chronologically, and briefly explain about the way the solution was obtained.

QUBIC The game QUBIC is similar to TIC-TAC-TOE, only that it is played on a three-dimensional $4\times4\times4$ board and the goal is to establish a line of four own tokens. It was originally solved by Patashnik (1980), who proved it to be a first player win.

He used a simple brute-force approach, which he improved in certain aspects. First, as he was sure the game is a first player win, he did not generate all possible first player and second player moves, but rather for a chosen first player move he generated all second player moves. Thus, a tree with all terminal states being first player wins is a proof for his assumption and also gives the strategy for the first player in terms of a weak solution.

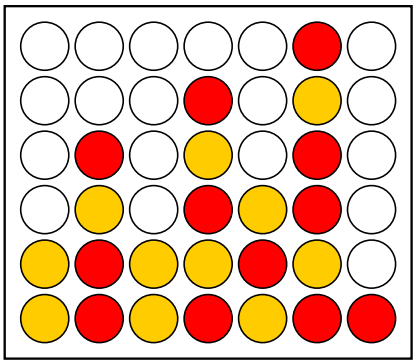


Figure 10.1: A possible state in CONNECT FOUR.

Another improvement was the removal of redundant states, i. e., states that are equivalent to other states. In other words, he used identified symmetries and other automorphisms to reduce the set of states.

For generating the first player moves he did not solely reside to the computer's choices, but applied some strategically important moves himself, while the computer did the more tactical moves, such as forcing moves (*zugzwang*). In other words, for every possible second player move Patashnik led the computer to a state that it had to prove was a sure win for the first player.

The whole process took ten and a half months (and about $1{,}500$ hours of CPU time), but in the end yielded the desired result.

CONNECT FOUR In essence, CONNECT FOUR can be seen as another extension of TIC-TAC-TOE. In this case, it is played on a 7×6 board and each player tries to establish a line of four pieces of the own color. In contrast to TIC-TAC-TOE and QUBIC in CONNECT FOUR gravity plays a role in that it pulls a played piece as far to the bottom as possible, so that in each state at most seven moves are possible (placing a token in one of the columns). Figure 10.1 shows a possible CONNECT FOUR state.

In 1988 the game was independently (weakly) solved by Allen (1989, 2011)[27] and Allis (1988; Uiterwijk et al. 1989)[28] only a few days apart. The game is won by the first player. Allis determined that the optimal first move is to drop the piece in the middle column. If dropped in one of the two adjacent columns, the optimal outcome is a draw. If the first player chooses to drop the first piece in one of the four outermost columns the second player can assure victory.

Allis implemented an approach that used nine strategic rules specific to CONNECT FOUR to find solutions for certain states as well as of the control of *zugzwang*, which is of high importance in this game. The first solutions his program could find were actually for other board sizes, e. g., for any board with an even number of rows and at most six columns the second player can ensure at least a draw. For other board sizes the solving process was not fully automatic, i. e., Allis provided moves for the second player if the computer could not calculate a solution. This he repeated until it could find solutions for all situations arising after any first player move. This led to the result that on a 7×4 board, the second player can ensure at least a draw.

On the default 7×6 board no such semi-automated process worked, as the search space is too large. Thus, he implemented another fully automatic solver, which in addition to the rules used before made use of *conspiracy number search* (Schaeffer, 1990; McAllester, 1988) in a minimax tree. With the additional use of transpositions to capture states already generated on other paths and symmetries to reduce the number of states to be searched even further, his program was able to calculate a (weak) solution.

[27] Reported in rec.games.programmer on October 1st 1988.
[28] Reported in rec.games.programmer on October 16th 1988.

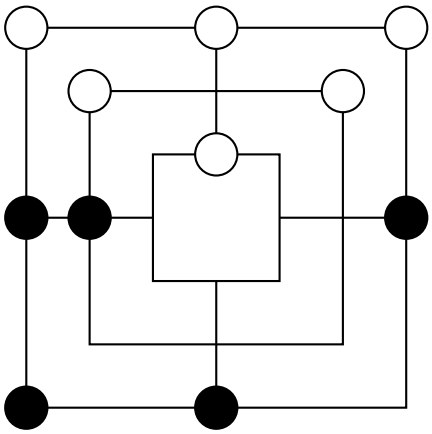


Figure 10.2: A possible state in NINE MEN'S MORRIS.

In recent years, Tromp (2008) found (strong) solutions for all game boards with a sum of height and width of up to 15.

An issue still open after the solutions provided by Allen and Allis was the number of reachable states. In his master's thesis, Allis (1988) provided an upper bound for the number of reachable states after each step, as well as an upper bound on the total number, 70,728,639,995,483. In the course of our work on solving general games, we found the exact number of states reachable on the default board of size 7×6 (Edelkamp and Kissmann, 2008e), 4,531,985,219,092, which was later proved independently by John Tromp[29].

NINE MEN'S MORRIS In 1993, Gasser (1996)[30] found a weak solution of the game NINE MEN'S MORRIS. The game is played on a board containing 24 cells that are located on the corners and the centers of three concentric squares, which are also linked along their centers (cf. Figure 10.2).

The game is divided into three phases. In the opening phase the players take turns placing one piece at a time on the board. Whenever one player establishes a (horizontal or vertical) line of three pieces, i. e., a mill is closed, a stone of the opponent that is not part of a mill is removed from the board. Once both players have placed nine pieces the midgame phase starts, where they take turns moving the pieces along the connecting edges. Similar to the opening phase, when a mill is closed an opponent's piece that is not part of a mill is removed from the board—unless all pieces are part of mills; in that case one of those is removed anyway. Finally, when a player has only three pieces left, the endgame phase starts and the player may move one of its pieces to any empty location on the board and still close a mill to reduce the number of pieces of the opponent.

The game ends if one player has only two pieces left, in which case that player loses, or if there is no move the current player can take, in which case that player loses as well. The game also ends once a state of mid- or endgame is repeated, which results in a draw.

The optimal outcome of the game is a drawn state. To solve it, Gasser used retrograde analysis for all mid- and endgame states. For these he created a total of 28 databases, one for each possible combination of pieces—i. e., one for three pieces for each player, one for three for the one and four for the other player etc.—containing the values for each state (won for the first player / lost for the first player / draw). To classify a state as lost for the active player, all successor states must be lost as well; to classify a state as won for the active player, it is sufficient to find one successor that is won for that player. To handle the large number of possible states, first of all he observed that some situations are not reachable, e. g., one player having a closed mill and the other nine pieces on the board, as well as that the game board contains five symmetry axes. Using these observations leaves

[29] `http://homepages.cwi.nl/~tromp/c4/c4.html`
[30] Reported in rec.games.abstract on November 23rd 1993.

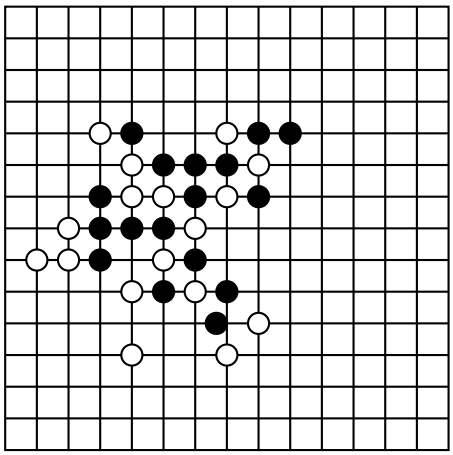


Figure 10.3: A terminal state in GOMOKU, won for the white player.

7,673,759,269 states in the mid- and endgame. In order to store the values of all these states he used a perfect hash function.

For the opening phase he performed $\alpha\beta$ search (Knuth and Moore, 1975) to find the optimal outcome of the game. As the available memory was too small, he reduced the sizes of the databases and also reduced the number of states to be evaluated assuming that in reasonable play each player closes at most one mill. In this scenario he proved that the first player can at least score a draw and that the first player can at most score a draw—resulting in the final outcome.

Recently, Yücel (2009; Edelkamp et al. 2010b) reimplemented Gasser's algorithm and ported it to work on the graphics processing unit (GPU). In contrast to Gasser he also solved all reachable states of the opening phase, so that now the optimal outcome for every reachable state is known and the optimal move can be calculated, i. e., this result is a strong solution for NINE MEN'S MORRIS. With this he also found out that with the first move of each player, nothing is lost; only from the second move of the first player onward it is possible to choose suboptimal moves and thus lose the game nevertheless.

GOMOKU (Five-in-a-Row) The game GOMOKU is a larger form of TIC-TAC-TOE. It is typically played on a 15×15 board, where the two players take turns placing their pieces. The goal is to establish a (horizontal, vertical or diagonal) line of five pieces of the own color. If the board is filled with no such line constructed the game ends in a draw. A possible terminal state is depicted in Figure 10.3.

Some additional rules were devised for GOMOKU, because the professional players believed that it was a clear win for the beginning (black) player (Sakata and Ikawa, 1981). Some of these rules are

- No *overlines*, i. e., a line of more than five pieces does not win the game.
- No move is allowed that establishes two lines of three pieces that are not blocked at one end by a piece of the opponent (*double three*).
- No move is allowed that establishes two lines of four pieces (*double four*).
- The first black piece may be placed only in the middle of the board, the first white piece only in one of the eight cells adjacent to the first piece, and the second black piece somewhere outside the 5×5 area around the first black piece.

The standard version now uses the no overlines rule (for both players). The game RENJU is similar to GOMOKU, but it applies the three former rules only to the black player and also contains special opening rules.

Allis (1994) was the first to solve the so-called *free-style* version of GOMOKU, i. e., the one without any restrictions, as well as the standard version. To achieve this he used proof number search (Allis

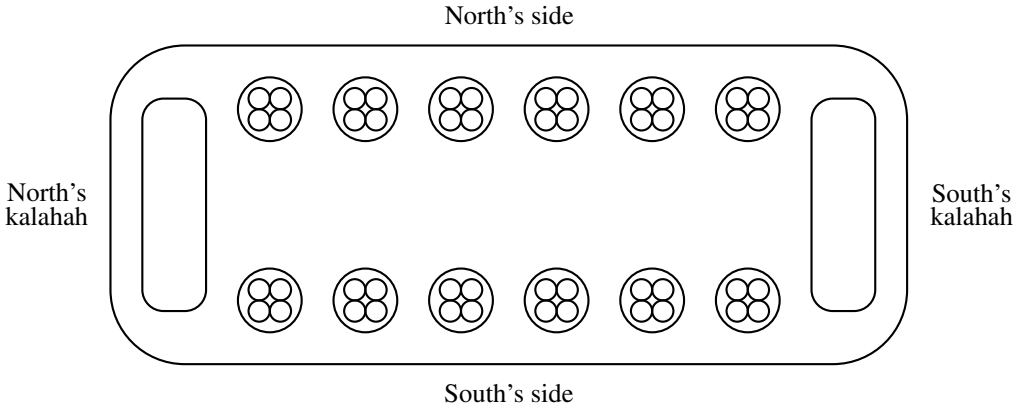


Figure 10.4: The initial state of KALAH.

et al., 1994) along with some improvements such as transpositions or restrictions for the black player's moves. The goal was to prove that black can win the game, so that when a solution is found that proves a black player win in the restricted case, black can also win in the full game. Among others, he restricted the black player to only ten moves to be checked in each layer, ordered heuristically. Also, he used *threat trees* to evaluate a state. In a threat tree, if the attacker makes a threat, the defender may occupy all possible defensive states. If the attacker can continue constructing threats that in the end result in a win, it should also be possible to win in the normal game play.

In the end, after 11.9 days of CPU time for the free-style version (respectively 15.1 days for the standard version) the algorithm finished with the proof that the game is indeed won by the starting black player. For other versions of the game, however, we are not aware of any solution.

KALAH The game KALAH is a special case from the wider class of MANCALA games. MANCALA games are typically played on a board consisting of a number of holes and some playing pieces, e. g., seeds. The seeds of one hole can be taken and *sown* on the subsequent holes, thereby possibly capturing some of the seeds. The goal is to capture as many of the seeds as possible.

In the default version of KALAH, the board consists of six normal holes on each player's side—*South* and *North*—, each initially filled with four seeds, and two special holes, called *kalahah*, one on each end of the board and owned by one of the players—the one on the eastern side of the board by the South player, the one on the western side by the North player (cf. Figure 10.4). The active player may choose one of the non-empty holes on its side and take all the seeds from it. Then the seeds are sown in the subsequent holes in counter-clockwise direction. A seed is also inserted into the player's kalahah, but not into the opponent's kalahah. Seeds in a kalahah are captured by the corresponding player and are never removed again.

After the sowing is done, there are three possibilities.

1. The last seed was placed in the player's kalahah. In that case the player may choose another move.
2. The last seed was placed in an empty hole on the player's side. In that case all seeds from the opposite opponent's hole are captured and placed in the own kalahah and the opponent's turn starts.
3. The last seed was placed anywhere else. In that case, the current player's turn ends and the opponent may perform the next move.

The game ends when all holes on a player's side are empty. In that case the other player captures all the remaining seeds. The game is won by the player that has captured more seeds.

Irving et al. (2000) solved several smaller instances of the game (smaller meaning fewer holes per player or fewer initial seeds per hole), denoted by KALAH (m, n) for a version with m holes per

player and n initial seeds per hole. They used simple forward search to find all reachable states followed by backward search to find the game-theoretic value for these. Thus, the results are strong solutions for these smaller version.

For KALAH $(6, 4)$, i. e., the original version, their machine was not powerful enough to find a strong solution. Thus they made use of $MTD\,(f)$ (or memory-enhanced test driver), which is roughly an $\alpha\beta$ search using only windows of size zero (Plaat et al., 1996a), in order to solve it weakly. The evaluation function they used calculates the difference in the number of captured seeds. Up to search depth 30 they used an iterative deepening approach with a step size of three, after that an exhaustive search. The advantage was that a better move-ordering for the later iterations could be calculated, resulting in better pruning, and that the guess of the true minimax value for $MTD\,(f)$ was improved.

Apart from this, they used several improvements to their search algorithm. For example, they used transposition tables with enhanced transposition cut-offs (Plaat et al., 1996b) and futility pruning (Schaeffer, 1986). The latter calculates the maximal possible ranges for the number of captured seeds for both players based on the non-captured ones. This helps increasing the degree of pruning as a node may be pruned if this range does not overlap with the $\alpha\beta$ window.

Another enhancement they tried is the history heuristic (Schaeffer, 1989), another extension to $\alpha\beta$ pruning that often helps increasing the degree of pruning by calculating a better expansion order of the moves. In KALAH this brought no advantage, as the decrease in the size of the tree was compensated by an increase in the runtime.

Finally, they also used endgame databases, which they generated by using retrograde analysis. These endgame databases are only based on the non-captured seeds. They generated databases for up to twenty non-captured seeds; larger databases no longer would have fit in their limited main memory of 256 MB.

With all these improvements they were able to solve instances up to KALAH $(6, 5)$. The most complicated one, KALAH $(6, 5)$ took about 4.7 hours with the result that the game is won by the starting player by a winning margin of twelve captured seeds. Due to the solved smaller instances the default instance KALAH $(6, 4)$ could be solved in only 35 seconds and they proved it to be a win for the starting player as well, by a winning margin of ten captured seeds.

AWARI Another game of the MANCALA family, AWARI, is played on a board with six holes on each player's side (and possibly one larger location to store the captured seeds). Each of these holes initially contains four seeds. The current player may take all the seeds from one of its holes and sow them counter-clockwise in the subsequent holes (one seed each). If the last hole sowed is on the opponent's side and now contains two or three seeds, the seeds from that hole are captured. If the same holds for the second to last hole, those seeds are captured as well. This continues until either a hole is reached that is not on the opponent's side or a hole is reached that contains neither two nor three seeds. The game ends when a player has captured 25 seeds. That player then has won the game. It also ends when both players have captured 24 seeds, in which case the result is a draw. A possible state is depicted in Figure 10.5.

The rules for a so-called *grand slam*, i. e., a situation where seeds from all six opponent's holes could be captured, are not unambiguous. It might be that in that case no seeds are captured, or that the seeds from all but the first (or the last) opponent's hole are captured, or that the seeds are captured by the opponent, or that such a move is just not allowed.

For computers AWARI is more difficult than KALAH for a number of reasons. First, in KALAH typically more stones are captured in one move, so that the game ends after fewer steps. Secondly, the final phase in AWARI is more difficult due to the possible grand slam rules. Furthermore, a player normally must leave the opponent a move if at all possible. Finally, AWARI can contain cycles, which cannot happen in KALAH as there a seed is placed in the own kalahah as well if the sowing process extends further than the holes on the own side.

AWARI was first strongly solved by Romein and Bal (2003). Unfortunately, from the paper it is not clear what rule for a grand slam they used. They implemented a parallel retrograde analysis and made

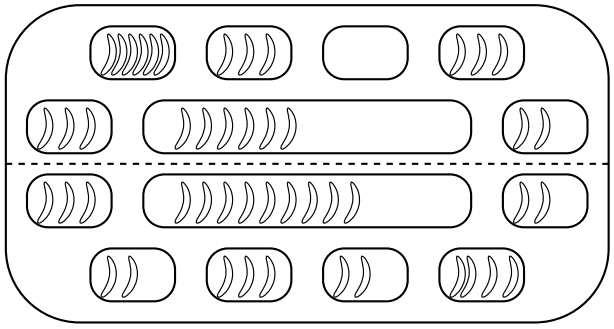


Figure 10.5: A possible state in AWARI.

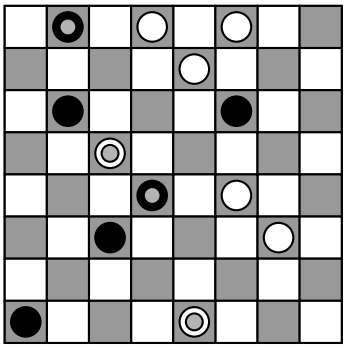


Figure 10.6: A possible state in AMERICAN CHECKERS. The kings are denoted by a gray concentric circle in the middle of the piece.

sure that no processor needed to wait for another processor by using asynchronous communication. Especially, when a processor needed data from another processor, instead of waiting for that data it simply sent the work to be done to that processor and continued elsewhere.

They used the algorithm to construct databases for all possible numbers of seeds on the board. In the end they analyzed a total of $889{,}063{,}398{,}406$ states, more than 200 billion alone for the 48 seeds database. Using their machine consisting of 144 processors they were able to determine the optimal outcome after 51 hours. The result—using their grand slam rule—is a draw.

AMERICAN CHECKERS The latest significant game to be solved is AMERICAN CHECKERS. It is played on an 8×8 chess board, with only half the cells used (cf. Figure 10.6; there, only the dark cells can be occupied). Each player starts with twelve pieces (*checkers*) and can move them one cell forward diagonally. Additionally, whenever possible a player must jump over an opponent's piece, which is then removed. Multiple jumps are also possible. Once a checker has reached the opposite side of the board it is promoted to a *king*. Kings can move one cell forward or backward diagonally and perform jumps in all four directions. The game ends when one player has no more pieces on the board or when the active one has no more possible moves. In that case that player has lost the game. If both players agree that there is no way for any of them to win the game any more it is declared a draw.

The game was solved by Schaeffer et al. (2007). They started back in 1989 by constructing endgame databases, which they generated by means of retrograde analysis. By 1996 databases for all possible states with up to eight pieces on the board were created. At that time they stopped their process and only resumed in 2001, when computational hardware was more sophisticated. In that year they started the database generation anew. Now it took only one month to generate the databases for all states with up to eight pieces on the board. In 2005 the final database for all states with up to ten pieces on the board was finished. These databases contained more than 39 trillion states (precisely, $39{,}271{,}258{,}813{,}439$).

All the states in the generated databases were strongly solved, as all possible playouts to a terminal state had been calculated (in backward direction). For the remainder of the game they chose to perform forward search. This used two parts, a proof-tree manager and proof solvers.

The proof-tree manager used proof number search (Allis et al., 1994) to find states that still had to be examined further, prioritized according to some heuristic. The proof solvers were provided these states and tried to solve them. If a state was proved (i. e., a draw or a win for one of the players), it did not require further investigation. If it was partly proved (i. e., at least a draw for one of the players) it was used by the proof-tree manager to update the proof-tree and then the manager decided which states to examine in more detail. Additionally, a state might not be proved or partly proved, but might have gotten only a heuristic estimate, which was used to prioritize the manager's list of interesting states.

In a first step, the proof solvers used $\alpha\beta$ search (Knuth and Moore, 1975). If that proved a win or loss, the state was solved. Otherwise, a depth-first proof-number search (Nagai, 2002) was started, which is a more space-efficient version of proof-number search. This resulted in the desired outcome (proved, partly proved or unknown / heuristic value).

Finally, in 2007 the program finished and returned the optimal outcome, i. e., a draw. Additionally, they found the number of states in AMERICAN CHECKERS, which is about 5×10^{20} (precisely, 500,995,484,682,338,672,639).

Most of these games were solved by use of expert knowledge and specialized algorithms, specifically designed for the game to be solved. While some of the approaches are rather general for board games, they are not yet general enough for use in general game playing without some additional analysis of the current game. As an example, the retrograde analysis used in NINE MEN'S MORRIS, AWARI and AMERICAN CHECKERS generates endgame databases for the cases of different numbers of pieces on the board, but to use this it is necessary to identify features such as game boards and pieces. Additionally, this works great as each move from one layer to another is irreversible, i. e., once a piece has been captured it will not reenter the board. This is another feature that must be identified in order to use the algorithms in the way described.

While nowadays it is possible to identify these and similar features with a certain degree of reliability (Clune, 2007; Kuhlmann et al., 2006; Schiffel and Thielscher, 2007b, 2009), we use a kind of retrograde analysis that does not rely on any of this. Recall that we want to find strong solutions specifying for each possibly reachable state which move to take. Therefore, we must solve every single state, so that the more time- and memory-efficient techniques such as $\alpha\beta$ pruning that allow us to skip over parts of the game tree cannot be applied here. Thus, the main idea of our approaches for single- and non-simultaneous two-player games is to start at the terminal states and calculate the predecessors according to the game rules. In most cases the terminal states are sets of states, so that here BDDs come in handy, as they natively handle such sets.

10.2 Single-Player Games

We have implemented a symbolic algorithm for solving general single-player games (Edelkamp and Kissmann, 2007) which is comparatively straight-forward. First of all, we find all the reachable states by performing symbolic breadth-first search (BFS) as described in the introduction to symbolic search (Chapter 3, Algorithm 3.1 on page 21). Then we perform a number of backward searches (cf. Algorithm 10.1), each time starting at the reachable terminal states (line 5). The first of these searches starts at those terminal states where we get the maximum reward of 100 points. From these we perform a BFS in backward direction (lines 6 to 8) and remove all the solved states $solved_{100}$ from the set of reachable states $reach$ (line 9). Then we switch over to the next smaller possible reward and repeat these steps. The algorithm stops when the states achieving a reward of 0 are solved. At that time all states are solved.

A property of well-formed general games we are going to need in the soundness proofs is that the set of states reachable in backward direction starting at the terminal states is a superset of the states reachable in forward direction starting at the initial state.

Algorithm 10.1: Symbolic solution of general single-player games

Input: General single-player game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$.
Output: Strong solution (vector of BDDs, representing solutions for each reward from 0 to 100).

```
1  reach ← BFS(G)                                   // BFS for finding all reachable states.
2  terminals ← reach ∧ T                            // Find all reachable terminal states.
3  reach ← reach ∧ ¬terminals                       // Remove reachable terminal states from reachable states.
4  for r ← 100 to 0 do                              // For all possible rewards, starting at 100...
5  |  solved_r ← newSolved ← terminals ∧ R(0, r)   // Start at terminals with reward r for first player.
6  |  while newSolved ≠ ⊥ do                       // While new states have been solved...
7  |  |  newSolved ← pre-image(newSolved) ∧ reach ∧ ¬solved_r   // Find new preceding states.
8  |  |  solved_r ← solved_r ∨ newSolved          // Add new states to set of solved states.
9  |  reach ← reach ∧ ¬solved_r                    // Remove solved states from reachable states.
10 return solved                                    // Return vector of solved states.
```

Lemma 10.6 (Comparison of sets of reachable states)**.** *For a well-formed general game the set of states reachable in backward direction starting at the terminal states $\mathcal{T}$ is a supersets of the states reachable in forward direction starting at the initial state.*

Proof. Let $\mathcal{S}$ be the set of all states reachable in forward direction and $\mathcal{S}'$ the set of states reachable in backward direction. Due to the playability (Definition 8.13) and termination (Definition 8.12) properties, which are necessary in each well-formed game (Definition 8.15), we know that for any state $s \in \mathcal{S}$ there is a sequence of moves of all participating players that finally reaches a terminal state $t \in \mathcal{S} \cap \mathcal{T}$. Therefore, a backward search starting at the same terminal state t must also reach s. Thus, it holds that $\mathcal{S}' \cap \mathcal{S} = \mathcal{S}$. □

Theorem 10.7 (Soundness of Algorithm 10.1)**.** *The algorithm for symbolically solving well-formed general single-player games (Algorithm 10.1) is sound and finds a strong solution.*

Proof. The BFS in forward direction finds all reachable states and the rewards of all reachable terminal states can be determined.

The *pre-image* operator returns the predecessor states of a given set of states. The restriction to the reachable states is only used to omit states unreachable in forward direction starting at the initial state from being solved. Only terminal states that are reachable in forward direction are considered, so that the backward searches essentially reach exactly the same states, as we start backward searches from all reachable terminal states, and these are a superset of the ones reachable in forward direction (cf. Lemma 10.6).

By removing solved states from the set of reachable states we ensure that they are solved only once. By traversing the terminal states in descending order of the corresponding rewards we ensure that we get the maximal reward for all states that can be achieved starting from them.

Finally, the removal of the solved states from the set of reachable states does not prevent reachable states from being solved. The states not removed after one iteration must lead to a terminal state with smaller reward, as otherwise they would have been found in an earlier iteration. Thus, in the end all states are solved, resulting in a strong solution. □

Actually, it is not necessary to perform the reachability analysis of line 1. This merely helps in reducing the number of states to be solved, as in many cases starting at the terminal states of a game in backward direction yields more states than in forward direction (i. e., often illegal states are generated, or not all terminal states are actually reachable). To omit the reachability analysis it suffices to set the BDD representing set of reachable states to $\top$ (true), so that it contains every state describable by the specified fluents, and perform the rest of the algorithm as before.

To play a game perfectly given the information generated by Algorithm 10.1 it is only necessary to look up the optimal reward for the current state and choose a move that results in the same optimal reward. As we are not confronted with an opponent we can be sure that this way we actually can achieve the calculated reward.

Algorithm 10.2: Symbolic solution of two-player zero-sum games

Input: General two-player zero-sum game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$.
Output: Strong solution represented by a triple of BDDs.

1 $reach \leftarrow BFS\,(\mathcal{G})$ // BFS for finding all reachable states.
2 **for all** $p \in \{0, 1\}$ **do** // For both players...
3 $lose_p \leftarrow reach \wedge \mathcal{T} \wedge \mathcal{R}\,(p, 0) \wedge move_p$ // Find terminal states where p has lost and has control.
4 $newlose_player \leftarrow lose_p$
5 $win_{1-p} \leftarrow\leftarrow reach \wedge \mathcal{T} \wedge \mathcal{R}\,(1-p, 100) \wedge move_{1-p}$ // Find reachable terminal states where
6 $newwin_{1-p} \leftarrow win_{1-p}$ // opponent has won and has control.
7 **while** $newlose_p \neq \bot$ **or** $newwin_{1-p} \neq \bot$ **do** // While new states are found...
8 $newwin_{1-p} \leftarrow pre\text{-}image\,(newlose_p) \wedge reach \wedge \neg win_{1-p}$ // New states won by opponent.
9 $win_{1-p} \leftarrow win_{1-p} \vee newwin_{1-p}$ // Add states to set of all opponent's won states.
10 $pred \leftarrow strong\ pre\text{-}image\,(win_{1-p})$ // States having all successors won by opponent
11 $newlose_p \leftarrow pred \wedge reach \wedge \neg lose_p$ // that are reachable and not found before.
12 $lose_p \leftarrow lose_p \vee newlose_p$ // Add newly found lost states to set of all lost states.
13 $win_0 \leftarrow win_0 \vee lose_1$ // All states won by player 0 or lost by player 1 are won by player 0.
14 $win_1 \leftarrow win_1 \vee lose_0$ // All states lost by player 0 or won by player 1 are won by player 1.
15 $draw \leftarrow reach \wedge \neg win_0 \wedge \neg win_1$ // All reachable states won by neither player are draw states.
16 **return** $(win_0, draw, win_1)$ // Return the triple of solved states.

10.3 Non-Simultaneous Two-Player Games

For non-simultaneous two-player games we came up with several different approaches. Here we start with an existing approach for two-player zero-sum games (containing only won, loss and draw states from the first player's point of view, denoted by the reward combinations 100–0, 0–100 and 50–50, respectively) in Section 10.3.1. Afterward, in Sections 10.3.2 and 10.3.3 we will explain about our approaches to handle arbitrary rewards for both players, which need not necessarily be zero-sum.

10.3.1 Two-Player Zero-Sum Games

As we have mentioned earlier, in case of GGP we speak of a zero-sum game if the rewards of all players sum up to 100 in all terminal states. Many of the existing zero-sum games are designed in such a way that there are states won by one or the other player (100–0 or 0–100) or that are drawn (50–50), so that here we restrict ourselves to these games.

For such two-player zero-sum games with only win, loss and draw states Edelkamp (2002a) proposed an algorithm to calculate the winning sets (cf. Algorithm 10.2). If a player performs only moves that lead to successor states with the same status (win, draw, or loss) as that of the current state it will play optimally, so that the winning sets suffice to represent a strong solution if they contain all possibly reachable states.

After the initial reachability analysis a backward search is started once for each player. Each backward search starts at the states lost for the current player p, if it is the active one (denoted by the BDD $move_p$), or won for the opponent $1 - p$, if that one is active (denoted by the BDD $move_{1-p}$). The backward search performs double steps (lines 7 to 12) until no new states lost for the current player can be found. A double step consists of a pre-image (recall Definition 3.6 on page 21) starting at the last states found to be lost for the current player followed by a strong pre-image (recall Definition 3.8 on page 22) starting at the states won for the opponent. The first step results in (possibly) new states won by the opponent, in all of which the opponent is the active player. The second step results in some new states lost for the current player, in all of which the current player is the active one. The intuition behind this double step is that we emulate the minimax procedure (von Neumann and Morgenstern, 1944) by calculating all those states where the current player can choose any move so that the opponent can find at least one move leading to a state that ensures that the current player will eventually lose the game.

Once the sets of states won for both player are calculated the remaining unsolved states necessarily are the draw states.

Theorem 10.8 (Soundness of Algorithm 10.2)**.** *The algorithm for symbolically solving well-formed two-player zero-sum games with only win, loss and draw states (Algorithm 10.2) is sound and finds a strong solution.*

Proof. Given the set of reachable states, the algorithm finds all states lost by one of the players. A terminal state is lost for a player p if it is lost for p and p is active or won for the opponent $1-p$ and that one is active (rewards 0 and 100, respectively).

A non-terminal state where player $1-p$, i. e., the opponent of player p, $p \in \{0,1\}$, is to move is lost for player p if player $1-p$ can find a move to a state that is lost for player p.

A non-terminal state where player p is to move is lost for player p if all possible moves lead to a state that is lost for player p. In the algorithm this is achieved by the use of the strong pre-image operator.

Finally, all states neither won by player 0 nor won by player 1 can only take the value of a draw state. □

Again, the calculation of the reachable states is not mandatory but often actually speeds up the search time and reduces the required memory. To play optimally it is sufficient to choose a move resulting in the same status as the current state, because the game necessarily ends after a finite number of steps and the status corresponds to the optimal reward if both players play optimally.

10.3.2 General Non-Simultaneous Two-Player Games

In general game playing the games are not necessarily zero-sum with only rewards of 0, 50, and 100, but the players can get any reward within $\{0,\ldots,100\}$, no matter what the opponent gets. Also, it is not trivial to find out if a game is actually zero-sum. One approach to determine this would be to calculate all reachable terminal states and determine their rewards. A different approach that uses answer-set programming to prove certain properties (among which is the zero-sum assumption) is due to Schiffel and Thielscher (2009). We did not yet implemented such an approach, so that usually the algorithm of the previous section is not enough to solve such games.

A first approach we implemented (Edelkamp and Kissmann, 2007) relies on restarts once a state is assigned different rewards. This resulted in an algorithm with exponential runtime. After some analysis we found that all the approach does is making sure that a state is finally solved only once all the successor states are solved. Having found this we were able to implement a much more efficient algorithm (cf. Algorithm 10.3) (Edelkamp and Kissmann, 2008e).

While Algorithm 10.2 worked only for turn-taking games, i. e., games in which the player to choose the next move changes after each step, this algorithm also works for any non-simultaneous two-player game, so that it is possible to solve games where a player is active for several subsequent states.

We use a 101×101 matrix of BDDs, $solution$, to store the solved states. The BDD in position (i,j) represents those states where player 0 can achieve a reward of i and player 1 a reward of j. This matrix is initialized with the reachable terminal states according to their calculated rewards (line 3). To save some time later on we manage two additional BDDs, $solved$ and $unsolved$, that store the states that are already solved, i. e., the reachable states that are inserted into the matrix, and those that are not, respectively.

To extend the solution we perform a backward search. In each step we call the strong pre-image to find those predecessors of the solved states whose successors are all solved and where the currently considered player has to choose the next move (line 8). If such states exist they are added to the set of solved states, removed from the set of unsolved states, and finally solved by the function $solveStates$ (cf. Algorithm 10.4), where they are inserted into the matrix. The update of the sets of solved and unsolved states results in the fact that it is not necessary to evaluate the entire matrix to find the states already solved after every step. Once all states are solved the algorithm stops and returns the solution matrix.

The insertion into the matrix works as follows. For the current player the algorithm checks all positions of the matrix in a predefined order (see next paragraph). If the current position is not empty, i. e., if at least one state resides there, we need to check if one of the solvable states can generate a successor that equals one of those states. As the transition relation operates in both directions, we can instead check if some

Algorithm 10.3: Symbolic solution of general non-simultaneous two-player games

Input: General non-simultaneous two-player game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$.
Output: Strong solution represented as a 101×101 matrix of BDDs.

```
1  reach ← BFS (G)                                          // BFS for finding all reachable states.
2  for all i, j ∈ {0, ..., 100} do                          // For all possible combinations of rewards...
3      solution_{i,j} ← reach ∧ T ∧ R (0, i) ∧ R (1, j)     // Initialize bucket of solution matrix.
4  solved ← ⋁_{0≤i,j≤100} solution_{i,j}                    // All states in solution matrix are solved.
5  unsolved ← reach ∧ ¬solved                               // All reachable states not solved are unsolved.
6  while unsolved ≠ ⊥ do                                    // While there are unsolved states left...
7      for all p ∈ {0, 1} do                                // For both players...
8          solvable ← strong pre-image (solved) ∧ unsolved ∧ move_p   // Determine solvable states
                                                                      // where p has control.
9          if solvable ≠ ⊥ then                             // If there is a solvable state...
10             solved ← solved ∨ solvable                   // Add solvable states to set of solved states.
11             unsolved ← unsolved ∧ ¬solvable              // Remove solvable states from set of unsolved states.
12             solution ← solveStates (G, solution, p, solvable)   // Solve the solvable states.
13 return solution                                          // Return solution matrix.
```

Algorithm 10.4: Symbolic solution of a set of states for a general non-simultaneous two-player game ($solveStates$)

Input: General non-simultaneous two-player game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$.
Input: Matrix $solution$ containing the solution so far.
Input: Index p denoting the current player.
Input: BDD $solvable$ representing the states to be solved.
Output: Extended solution.

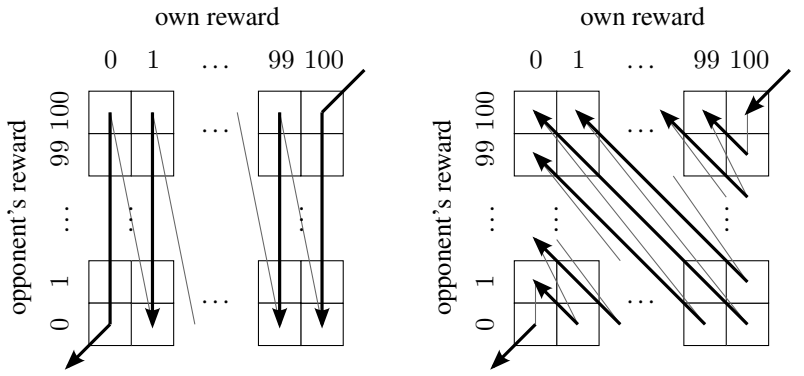
```
1  for all i, j ∈ {0, ..., 100} do in order   // Evaluate all reward combinations in predefined order.
2      if solution_{i,j} ≠ ⊥ then              // If the reward combination is reachable...
3          newSolved ← pre-image (solution_{i,j}) ∧ solvable   // Determine which solvable states are a
                                                               // predecessor of a state in the current position.
4          solution_{i,j} ← solution_{i,j} ∨ newSolved         // Add those states to the solution.
5          solvable ← solvable ∧ ¬newSolved                   // Remove those states from the set of solvable states.
6          if solvable = ⊥ then return solution               // If all solvable states are solved we are done.
```

predecessors of one of the stored states is present in the solvable states (line 3). If that is the case, those predecessors are added to the BDD in the current position and removed from the set of solvable states.

Orders to Process the Rewards For general rewards it is important to find a reasonable order to process the rewards. For two-player games, the two most reasonable assumptions are that the players try to maximize their own reward (cf. Figure 10.7a) or to maximize the difference to the opponent's reward (cf. Figure 10.7b).

The first of these is the more general assumption—in a competition scenario with more than only two players and a number of games to be played it might be more important to gain a large number of points in total than trying to prevent one of the opponents from gaining a larger reward. In a scenario with only two players this might not be the best strategy. In fact, there it would appear more reasonable to try to maximize the difference. This way, the player might get somewhat fewer points but achieve an advantage over the opponent.

As an example, take the two possible outcomes 75–100 and 50–0. In the case of maximizing the own reward the first player would prefer the first outcome, achieving 75 points and totally ignoring how many points the opponent might gain. In this case the opponent would get even more points (100). In the case of



(a) Maximizing the own reward. (b) Maximizing the difference.

Figure 10.7: The different orders to traverse the matrix.

maximizing the difference the first player would prefer the second outcome, thereby forswearing 25 points but gaining an advantage of 50 points over the opponent.

In case of a zero-sum game, these two orders actually are the same. In both cases the players prefer to win over a draw, which in turn they prefer over a loss. Thus, the algorithms using one of these orders generate the same results as the zero-sum algorithm.

Soundness of the Algorithm

Theorem 10.9 (Soundness of Algorithm 10.3)**.** *The algorithm for symbolically solving well-formed general non-simultaneous two-player games (Algorithm 10.3) is sound and calculates a strong solution corresponding to the specified traversal order.*

Proof. We will proof the soundness by induction.

When starting backward search, all states reachable in forward direction have been determined; especially, all reachable terminal states have been calculated (which are a subset of all states compatible with the termination criterion). For these states the reward for both players is unambiguous and the terminal states are inserted into the corresponding positions of the matrix.

Now, calculating the strong pre-image in line 8 results in all states whose successors are already in the matrix and thus solved according to the specified traversal order, so that these states are solvable as well. The additional condition that the current player must be the one to choose a move does not hamper the argumentation, as we immediately apply this step for both players and stop the algorithm only when no unsolved states are left.

In the end, the strong pre-image captures all states reached in forward direction. This is because on the one hand the normal backward search results in a superset of the states reachable in forward direction (cf. Lemma 10.6), on the other hand the strong pre-image finds all predecessors whose successors are already solved. As we start with all terminal states as solved states and we know that a well-formed game ends in a terminal state after a finite number of steps, so that no loops are possible, at some point during the backward search all successors of a state are solved and the solving of the new states might give rise to further solvable states.

The states are inserted as described in Algorithm 10.4. Here the matrix is traversed in the order specified before. If a predecessor of a non-empty position in the matrix is found in the set of solvable states it is inserted to this position and removed from the set of solvable states. This ensures that each state is inserted only once. Furthermore, as the traversal is according to the specified order, the state is inserted into the position that is preferred by the player to choose the move over all other possible rewards reachable from that state. Thus, all solvable states are inserted into the correct positions within the matrix. □

In this algorithm it is also possible to omit the forward search to find all the reachable states. In that case the BDD representing the unsolved states becomes redundant, as we do not know which states can yet

Algorithm 10.5: Symbolic calculation of the reachable states in a layered approach ($layeredBFS$).

Input: General non-simultaneous two-player game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$.
Output: Index of last non-empty layer (plus layers of reachable states on hard disk).

```
1 current ← I                                   // Start at the initial state.
2 layer ← 0                                     // Store index of the current layer, starting with 0.
3 while current ≠ ⊥ do                          // While the current layer is not empty...
4 |  store current as reach_layer on hard disk  // Store the current layer.
5 |  current ← image(current ∧ ¬T)              // Calculate the successor layer.
6 |  layer ← layer + 1                          // Increase the layer index.
7 return layer − 1                              // Return the index of the last non-empty layer.
```

be solved. Thus, the loop needs to be adapted to stop once no new solvable states have been found for both players. As the set of states reachable in backward direction from terminal states is a superset of the set of states reachable in forward direction from the initial state this still results in a sound algorithm.

In order to use this solution for optimal play (according to the specified traversal order) we must choose a move that results in a successor state within the same bucket as the current state. This way we make sure that we do not lose any point, but improve the result whenever the opponent makes a sub-optimal move.

10.3.3 Layered Approach to General Non-Simultaneous Two-Player Games

Finally, we came up with another new approach to solve general non-simultaneous two-player games (Kissmann and Edelkamp, 2010b). To further decrease memory consumption and thus to enable us to handle larger games this one makes use of the hard disk to store parts of the calculated information, especially the reachable states and the calculated solution. Having these results on disk brings several advantages. On the one hand it is possible to stop the search and resume later on, while on the other hand we can use the calculated results in any solver or player that can handle BDDs. Thus, using partial results as an endgame database is straight-forward.

The main idea of this approach is to partition the BDDs. For some games the BDD representing the set of reachable states is rather large, whereas the BDDs representing the BFS layers are a lot smaller. Thus, we decided to store only the BFS layers and no longer the complete set of reachable states (cf. Algorithm 10.5). This way, the BDDs typically stay smaller, but we can no longer perform full duplicate elimination in reasonable time, as this would involve loading all the previous layers and removing those states from the current layer. Under certain circumstances this leads to an increased number of layers and an increased number of states to be solved. Nevertheless, the reachability analysis ends after a finite number of steps as we are concerned with well-formed general games that are designed to be finite on any possible path.

To get an idea when we will be confronted with duplicate states in different layers we need to define a progress measure.

Definition 10.10 ((Incremental) Progress Measure)**.** *Let $\mathcal{G}$ be a general non-simultaneous two-player game, $\mathcal{S}$ the set of all reachable states and $\psi : \mathcal{S} \mapsto \mathbb{N}$ be a mapping from states to numbers.*

The function ψ is a progress measure *if $\psi(s') > \psi(s)$ holds for all $s \in \mathcal{S}$ and $s' \in succ(s)$ with $succ : \mathcal{S} \setminus \mathcal{T} \mapsto 2^{\mathcal{S}}$ determining the set of successors of state s. It is an* incremental progress measure*, if $\psi(s') = \psi(s) + 1$ holds.*

If $\mathcal{G}$ is strictly turn-taking we can use an additional definition. Let $\mathcal{S}_i$ be the states where player i is the active one. Then ψ also is a progress measure *if $\psi(s'') > \psi(s') = \psi(s)$ holds for all $s, s'' \in \mathcal{S}_0$ and $s' \in \mathcal{S}_1$ with $s' \in succ(s)$ and $s'' \in succ(s')$. It is an* incremental progress measure*, if $\psi(s'') = \psi(s') + 1$ holds.*

In all cases it holds that the progress measure of the initial state $\mathcal{I}$ is set to zero: $\psi(\mathcal{I}) = 0$.

During the experiments we found that most games contain such an incremental progress measure. Many games contain a step counter (often to ensure termination), which is a trivial incremental progress measure. In others, such as TIC-TAC-TOE or CONNECT FOUR, the players take turns placing exactly one new token

on the board, so that the number of placed tokens is an incremental progress measure. In SHEEP AND WOLF, where the wolf can move in any direction while the sheep can only move closer to the opposite side, we need the second part of the definition. Moving the wolf does not provide any progress, while moving a sheep progresses the game. Thus, the sum of the indices of rows of the sheep is an incremental progress measure.

For games such as NIM we can find a progress measure but no incremental progress measure. In NIM there are several piles of matches and the players take turns removing matches from one pile of their choice. They can remove any number of matches (from one up to all that are left on the corresponding pile), so that the progress increases by the number of removed matches. This then can lead to duplicate states in different layers. If the first player removes three matches from the first pile or the players take turns removing only one match from that pile, in both cases three matches will be removed from it—either after one or after three moves—and the second player will be active. The states are exactly the same in both cases.

In the following we will prove that general games containing an incremental progress measure do not contain duplicate states in different (BFS) layers.

Theorem 10.11 (Duplicate avoidance)**.** *Whenever there is an incremental progress measure ψ for a general non-simultaneous two-player game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$, no duplicates arise across the layers found by Algorithm 10.5.*

Proof. We need to proof this for the two cases:

1. If ψ is defined according to the first part of Definition 10.10, we claim that all states within one layer have the same progress measurement but a different one from any state within another layer, which implies the theorem. This can be shown by induction.

   The first layer consists only of $\mathcal{I}$, with $\psi(\mathcal{I}) = 0$. Let $\mathcal{S}$ be the full set of reachable states and let $succ(s)$ be the set of successor states for all $s \in \mathcal{S} \setminus \mathcal{T}$. According to the induction hypothesis all states in layer l have the same progress measurement. For all states s in layer l and successors $s' \in succ(s)$ it holds that $\psi(s') = \psi(s) + 1$. These are all inserted into layer $l+1$, so that all states within layer $l+1$ have the same progress measurement, which is greater than that of any of the states in previous layers, as it increases with each step. Thus, it differs from the progress measurement of any state within another layer.

2. If $\mathcal{G}$ is turn-taking and ψ is defined according to the second part of the definition, the states within any successive layers differ, as the fluent denoting the active player has changed. Let $\mathcal{S}_0$ ($\mathcal{S}_1$) be the set of states where player 0 (player 1) is the active one. Then, it remains to show that for all $s, s' \in \mathcal{S}$, $s_0 \in \mathcal{S}_0$ and $s_1 \in \mathcal{S}_1$ it holds that $\psi(s) = \psi(s')$ if s and s' reside in the same layer and $\psi(s_0) = \psi(s_1)$ if s_0 resides in layer l and s_1 in layer $l+1$. For all other cases, we claim that the progress measurement of any two states does not match, which proves the theorem.

   The first layer consists only of $\mathcal{I}$, with $\psi(\mathcal{I}) = 0$. All successors of this state reside in the next layer and their progress measure equals, according to the definition of ψ.

   Let l be a layer that contains only states from $\mathcal{S}_0$. According to the induction hypothesis, all states in this layer have the same progress measurement. For all states s in layer l and successors $s' \in succ(s)$ it holds that $\psi(s') = \psi(s)$. All these s' are inserted into layer $l+1$. For all states s' in layer $l+1$ and $s'' \in succ(s')$ it holds that $\psi(s'') = \psi(s') + 1$. All these s'' are inserted into layer $l+2$, so that all states within layer $l+2$ share the same progress measurement, which is greater than that of any of the states in previous layers, as it never decreases.

□

A simple conclusion is that the incremental progress measure equals the layer a state resides in.

Corollary 10.12 (Equality of incremental progress measure and layer)**.** *Let $\mathcal{G}$ be a general non-simultaneous two-player game, $\mathcal{S}$ the set of all reachable states, and ψ an incremental progress measure.*

If ψ is defined according to the first definition, then $\psi(s) = \lambda(s)$ for any state $s \in \mathcal{S}$, with $\lambda : \mathcal{S} \mapsto \mathbb{N}$ being the layer in which s resides.

If ψ is defined according to the second definition, then $\psi(s) = \lfloor \frac{\lambda(s)}{2} \rfloor$ for all states $s \in \mathcal{S}$.

Algorithm 10.6: Symbolic solution of general non-simultaneous two-player turn-taking games with a layered approach.

Input: General two-player turn-taking game $\mathcal{G} = \langle \mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{A}, \mathcal{I}, \mathcal{T}, \mathcal{R} \rangle$.
Output: Strong solution (on hard disk).

```
1  layer ← layeredBFS (G)        // Find index of last non-empty layer and generate layers on hard disk.
2  while layer ≥ 0 do            // Iterate through the layers in backward direction.
3      current ← load reach_layer from disk                    // Load the current layer.
4      currentTerminals ← current ∧ T                          // Find terminal states in current layer.
5      current ← current ∧ ¬T                                  // Remove terminal states from current layer.
6      for all i, j ∈ {0, ..., 100} do                         // Check all possible reward combinations.
7          terminals_{layer,i,j} ← currentTerminals ∧ R(0, i) ∧ R(1, j)   // Find terminal states from the
                                                               // current layer achieving the specified reward combination.
8          store terminals_{layer,i,j} on disk                 // Write the corresponding BDD to the hard disk.
9      if current ≠ ⊥ then                                     // If non-terminal states in current layer....
10         for all i, j ∈ {0, ..., 100} do in order            // Check all reward combinations in predefined order.
11             succ_1 ← load terminals_{layer+1,i,j} from disk // Retrieve terminal states achieving this
                                                               // combination in the successor layer from the hard disk.
12             succ_2 ← load rewards_{layer+1,i,j} from disk   // Retrieve non-terminal states achieving this
                                                               // combination in the successor layer from the hard disk.
13             succ ← succ_1 ∨ succ_2                          // Join these two sets of states.
14             rewards_{layer,i,j} ← current ∧ pre-image (succ)   // Find those current states being
                                                               // predecessors of the calculated set.
15             store rewards_{layer,i,j} on disk               // Write the corresponding BDD to the hard disk.
16             current ← current ∧ ¬rewards_{layer,i,j}        // Remove these states from the set of current states.
17     layer ← layer − 1                                       // Iterate to the predecessor layer.
```

A second observation is that the number of times a state is expanded is bounded by the number of layers that can exist.

Corollary 10.13 (Number of additional expansions)**.** *For a general non-simultaneous two-player game $\mathcal{G}$ that does not contain an incremental progress measure, each state can be expanded at most d_{max} times, with d_{max} being the maximal distance from $\mathcal{I}$ to one of the terminal states. This is due to the fact that in such a game each state might reside in every reachable layer, while duplicates within the same layer are captured by the BDDs we use.*

The fact that we do not remove the duplicates in the layered BFS in Algorithm 10.5 enables us to use the calculated layers in the backward search as well (cf. Algorithm 10.6). Using the layered BFS actually often helps in solving games, as now it is no longer necessary to use the strong pre-image operator any more if we solve the states layer-wise because all successors are in the next layer and thus already solved once we reach the layer the state resides in.

The solving starts in the last reached layer and performs regression search towards $\mathcal{I}$, which resides in layer 0 (line 2). This final layer contains only terminal states (otherwise the forward search would have progressed farther), which can be solved immediately by calculating the conjunction with the BDDs representing the rewards for the two players (lines 6 to 8). Once this is done, the search continues in the preceding layer (line 17), because the remainder of the layer is empty.

If another layer contains terminal states as well, these are solved in the same manner before continuing with the remaining states of that layer. Similar to the previous approach, the rewards are handled in a certain order. All the solved states of the successor layer are loaded in this order (lines 10 to 16) and the pre-image is calculated, which results in those states of the current layer that will achieve the same rewards, so that they can be stored on the disk as well (line 15).

Once $\mathcal{I}$ is solved and stored the strong solution resides completely on the hard disk.

Theorem 10.14 (Soundness of Algorithm 10.6)**.** *The algorithm for symbolically solving well-formed general non-simultaneous two-player games with a layered approach (Algorithm 10.6) is sound and calculates a strong solution corresponding to the specified traversal order.*

Proof. The forward search generates all the reachable layers. Any state might appear in more than one layer, so that all its successors are surely in the next layer. A well-formed game must end in a terminal state after a finite number of steps, so that the number of layers is finite and the last found layer can only contain terminal states.

For the backward search we will prove the soundness inductively.

The terminal states of the final layer can be solved immediately using the reward rules.

As the algorithm operates in backward direction, when solving a layer all subsequent layers are already solved. The terminal states of a layer can be solved immediately without evaluating any successors. For the remaining states it holds that—due to the layered BFS—all successors are present in the next layer and thus already solved. These successors are evaluated in the predefined order, so that the states the current player prefers (those having a higher reward or a greater difference to the opponent's reward) are evaluated before the others. Thus, when the first states $\mathcal{S}'_{i+1}$ are found for which some states $\mathcal{S}'_i$ are the predecessors it is ensured that the states $\mathcal{S}'_{i+1}$ are the optimal successors for the given traversal order. Removing the states $\mathcal{S}'_i$ from the set of remaining states of the current layer ensures that they are solved only once and are assigned a unique reward combination in this layer, which corresponds to the optimal reward combination according to the traversal order. □

Note that this algorithm only works for strictly turn-taking games. For games such as CUBICUP, where one player might be active for several turns in a row, we need to adapt the approach, as states where player 0 and other states where player 1 is active might reside in the same layer. In such a case we need to split up the loop for solving the non-terminal states (lines 10 to 16) and perform it twice, once for each player. Unfortunately, both players typically prefer a different order through the rewards, so that we cannot handle both together, but need to store the results of one player on the disk, calculate the results for the other player, load the result of the first one, join them, and store the complete results again on the disk.

In contrast to all the previous algorithms, here the forward search is mandatory as otherwise it is not clear what layer we currently are in and if all successor states are already solved.

An advantage of this algorithm is that it can easily be generalized to any number of players. All we need is the traversal order for each player (which actually corresponds to an opponent model). Unfortunately, except for single-player and two-player zero-sum games we do not know this order beforehand. In a competition setting the players have no information about the current status (i. e., how many points each player already has earned or they are matched against only one or a number of opponents) it is impossible to determine a perfect order on-the-fly. It remains future work to identify reasonable ways to traverse the rewards for multiplayer games.

10.4 Experimental Evaluation

We implemented all of the proposed algorithms using Java and the native interface JavaBDD[31] to use the efficient BDD library CUDD[32] and evaluated them on our machine (Intel Core $i7$ 920 with 2.67 GHz and 24 GB RAM). In all cases we restricted the runtime to two hours.

In the following we will show the results for solving single-player games (Section 10.4.1) as well as non-simultaneous two-player games (Section 10.4.2).

10.4.1 Single-Player Games

In the case of single-player games we compare the layered approach (Algorithm 10.6) against the older sequential approach (Algorithm 10.1), i. e., the one where we solve the different achievable rewards sequentially. For the latter we also check the effect of omitting the forward search. As we have mentioned

[31] `http://javabdd.sourceforge.net`
[32] `http://vlsi.colorado.edu/~fabio/CUDD`

Table 10.1: Runtimes for the solved single-player games. All times in [m:ss], averaged over ten runs. The maximal distance of a state to the initial state is denoted by d_{max} and the number of reachable states by $|\mathcal{S}|$. For games without incremental progress measure we give two values for for d_{max} and $|\mathcal{S}|$, the first one, without parentheses, being the number of layers and states if we remove duplicates, the second one, in parentheses, if we do not.

Game	Sequential Approach	Sequential Approach (no Forw.)	Layered Approach	Factor Sequential / Layered	d_{max}	$\|\mathcal{S}\|$
troublemaker01	0:00.242	0:00.240	0:00.242	1.000	4	9
troublemaker02	0:00.249	0:00.246	0:00.245	1.016	4	16
buttons	0:00.247	0:00.243	0:00.271	0.911	6	32
blocks	0:00.284	0:00.277	0:00.282	1.007	3	16
maze	0:00.269	0:00.266	0:00.300	0.897	9	42
statespacesmall	0:00.386	0:00.396	0:00.410	0.941	4	341
duplicatestatesmall	0:00.405	0:00.395	0:00.424	0.955	4	27
circlesolitaire	0:00.316	0:00.314	0:00.447	0.707	10 (24)	250 (2,014)
oisters_farm	0:00.374	0:00.370	0:00.556	0.673	49	388
blocksworldserial	0:00.561	0:00.726	0:00.563	0.996	6	121
ruledepthlinear	0:00.502	0:00.498	0:00.572	0.878	49	99
hanoi	0:00.547	0:03.866	0:00.616	0.888	31	2,753
blocksworldparallel	0:00.765	0:00.896	0:00.768	0.996	3	90
kitten_escapes_from_fire	0:00.766	0:00.782	0:00.850	0.901	16	143
pancakes	0:00.858	0:00.821	0:00.986	0.870	40	26,225
pancakes6	0:00.851	0:00.807	0:01.003	0.848	40	26,225
duplicatestatemedium	0:01.127	0:01.457	0:01.191	0.946	9	225
asteroids	0:01.658	0:00.872	0:01.278	1.297	49	2,766,444
brain_teaser_extended	0:01.285	0:02.002	0:01.341	0.958	15	15,502
statespacemedium	0:01.487	0:02.404	0:01.549	0.960	9	349,525
chinesecheckers1	0:01.963	21:38.922	0:01.838	1.068	39	75,495
hanoi_6_disks	0:01.499	1:13.047	0:01.975	0.759	63	31,345
hanoi7_bugfix	0:03.050	7:27.921	0:02.832	1.077	127	94,769
twisty-passages	0:03.306	0:03.288	0:03.713	0.890	199	399
incredible	0:06.232	3:41.231	0:04.845	1.286	19	566,007
duplicatestatelarge	0:06.610	0:36.206	0:06.744	0.980	14	824
8puzzle	0:22.957	0:34.410	0:16.104	1.425	60	3,590,984
lightsout2	5:24.160	o.o.m.	3:24.588	1.584	20	48,641,856
lightsout	5:20.579	o.o.m.	3:25.121	1.563	20	48,641,856
knightstour	8:04.800	o.o.m.	5:43.410	1.412	30	25,663,711
statespacelarge	9:52.523	10:43.990	9:53.637	0.998	14	357,913,941
asteroidsserial	17:18.566	o.o.m.	16:06.352	1.075	98	266,695,101,993
peg_bugfixed	105:46.874	o.o.m.	84:12.264	1.256	31	187,636,299
tpeg	o.o.t.	o.o.m.	105:10.542	—	31 (31)	264,273,046 (264,273,072)

before, it is possible to run the algorithm without forward search, so that the time for that can be saved and thus the overall runtime might decrease.

The results can be found in Table 10.1. Concerning the comparison to the case without forward search the results show clearly that this is not a good idea, as the runtime often increases a lot, sometimes from a few seconds to several minutes, and in some cases no solution can be found at all because it runs out of memory. This might happen because if the backward search is not restricted to the reachable states too many states will be solved and the BDDs become a lot larger.

When comparing the sequential and the layered approach we can see that for short runtimes the sequential approach is quite often faster. The highest factor arises in `oisters_farm`, where the sequential approach is nearly $50\,\%$ faster. This behavior is due to the loading and storing of the BDDs in the layered approach.

For the more complex games, in many cases the behavior turns around and the layered approach is faster, in case of the `lightsout` games by more than $50\,\%$. Furthermore, one game, `tpeg`, can only be solved by the layered approach. In only one of the games where the layered approach takes more than ten

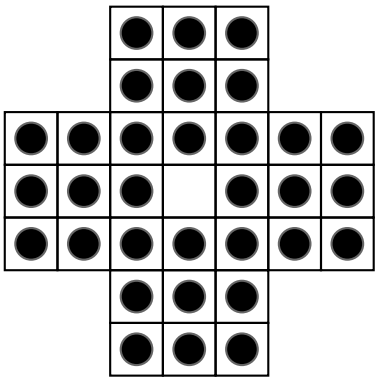


Figure 10.8: The game of PEG-SOLITAIRE.

seconds to find the solution, `statespacelarge`, is the sequential approach faster than the layered one, and in that case actually by less than a second.

However, comparing the differences in the runtimes the pictures gets even clearer. The game where the layered approach loses most time is `hanoi_6_disks`, where it is slower by 0.476 seconds, while in `peg_bugfixed` it wins most, the difference being more than 21.5 minutes. Also, comparing the sum of the runtimes for the games that were solved by both approaches we arrive at a similar result. The sequential approach takes more than 150 minutes, while the layered approach takes less than 125 minutes. An explanation for this behavior in the more complex games might be that often the BDDs for storing only the BFS layers and not the complete set of reachable states are a lot smaller and thus the image and pre-image operations take less time.

In practice, a difference of less than a second can often be ignored, as this corresponds to less than a step in a general game playing competition, while several minutes might very well mean that only the faster approach is able to find a solution, at least after some steps have already been performed. Thus, we decided to only use the layered approach in the player we implemented (cf. Chapter 11).

PEG-SOLITAIRE The game PEG-SOLITAIRE (in the table denoted by `peg_bugfixed`) is not completely trivial with nearly 200 million reachable states and is a lot more interesting than the serial version of two ASTEROIDS games, the game with the highest number of states. Furthermore, in PEG-SOLITAIRE several different rewards are specified, so that we will look slightly deeper into its solution.

PEG-SOLITAIRE is played on a cross-like grid with 33 cells as shown in Figure 10.8. Initially, all but the middle cell are filled with pegs. The only move the player can perform is to jump with one peg over another onto an empty cell. The peg that was jumped over is removed from the board. As with each move one peg is removed the game ends after at most 31 moves, i. e., when only one peg remains on the board, or when no more jumps can be performed. Thus, the number of removed pegs serves as an incremental progress measure.

The goal is to remove all but one peg from the board—and the remaining one should be positioned in the very middle, i. e., the initial positioning is to be inverted.

The game description also allows for other outcomes. The maximum of 100 points is awarded for achieving the inverted initial positioning; 99 points for reaching a state with only one peg left on the board that is not located in the middle. $90, \ldots, 10$ points will be given if $2, \ldots, 10$ pegs remain on the board without the ability to perform any more jumps. Finally, 0 points represent the remaining states, i. e., eleven or more pegs left on the board.

The detailed results are depicted in Table 10.2. An interesting observation is the fact that only four states leading to a position with one peg that is not in the middle are reachable. For the last move to achieve the full 100 points there are four possibilities: A jump from the left, right, top, or bottom to the middle. If the other peg is used to perform the jump, we reach a positioning with only one peg that is not located in the middle. Thus, the only four positions of this kind that are reachable are the ones where the last peg is located in the middle of one of the four sides of the cross.

Table 10.2: Solution for the game PEG-SOLITAIRE with n being the number of BDD nodes required to represent the $|\mathcal{S}|$ states achieving the specified reward.

Reward	n	$\lvert\mathcal{S}\rvert$
100	3,670,109	13,428,122
99	131	4
90	146,343,316	67,047,807
80	14,044,448	39,688,157
70	13,606,922	41,976,387
60	7,178,673	12,869,577
50	4,619,250	7,233,160
40	2,533,327	3,169,848
30	1,302,641	1,305,817
20	676,509	572,871
10	332,403	228,492
0	175,292	116,057
Total	6,837,289	187,636,299

10.4.2 Two-Player Games

In case of two-player games we compare only our own algorithms, namely the one based on strong pre-images (Algorithm 10.3; here denoted as *spi approach*) and the layered approach (Algorithm 10.6). The older approach for zero-sum games is not used because on the one hand many games allow for more than just win / loss / draw, and on the other hand in some preliminary experiments it performed a lot worse than the newer approaches. Similar to the single-player case we also experimented with the omission of the forward search in case of the spi approach.

The detailed runtime results for the solved games are depicted in Table 10.3. Concerning the effect of the forward search in the spi approach we can see that for most games the runtime increases drastically when omitting forward search, so that some games such as SHEEP AND WOLF cannot be solved in two hours, for which the spi approach with forward search takes less than a minute. In a number of other domains the BDDs virtually explode if they are not restricted to the set of reachable states, so that the available memory is not sufficient to solve the game in such a manner. All in all, this behavior is very similar to the single-player case.

In the comparison of the spi and the layered approach we can see a rather clear trend. The more complex games, i. e., those that take more than ten seconds to be solved, can be solved a lot faster using the layered approach, while for the smaller games the overhead of the disk accesses increases the runtime of that approach. Here the factors of the runtimes of the spi and the layered approach range from 0.264 (i. e., the spi approach is faster by a factor of more than 3.5) to 7.312. However, the differences in runtimes range from 3.2 seconds in favor of the spi approach in `nim4` to more than 50 minutes in favor of the layered approach in `tictactoelargesuicide`—and the latter value would even higher if we knew how long the spi approach takes in the two games it did not solve in the two hour limit. Summing up the times for all the problems solved by both approaches we arrive at 133 minutes for the spi approach and 32 minutes for the layered approach. Thus, similar to the single player case, here the layered approach is definitely the method of choice in practice.

Effect of the Incremental Progress Measure

We can see an effect for games not having an incremental progress measure. Examples for these are CHOMP and NIM. CHOMP was introduced by Gale (1974) as a two-dimensional version of Schuh's GAME OF DIVISORS (1952). It is a two-player game, in which the players take turns biting pieces off a chocolate bar. To bite some pieces off the bar, they name the coordinates (i, j) of an existing piece. All pieces with coordinates (x, y) at least as large as the specified ones, i. e., with $x \geq i$ and $y \geq j$ will be removed. The

Table 10.3: Runtimes for the solved two-player games. All times in [m:ss], averaged over ten runs. The maximal distance of a state to the initial state is denoted by d_{max} and the number of reachable states by $|\mathcal{S}|$. For games without incremental progress measure we give two values for for d_{max} and $|\mathcal{S}|$, the first one, without parentheses, being the number of layers and states if we remove duplicates, the second one, in parentheses, if we do not.

Game	Optimal Rewards	spi Approach	spi Approach (no Forward)	Layered Approach	Factor spi / Layered	d_{max}	$\lvert\mathcal{S}\rvert$
tictactoe-init1	100 – 0	0:00.327	0:00.332	0:00.663	0.493	5	73
nim1	100 – 0	0:00.367	0:00.377	0:00.985	0.373	5 (12)	344 (756)
tictactoe	50 – 50	0:00.457	0:00.742	0:00.990	0.462	9	5,478
gt_centipede	5 – 0	0:00.323	0:00.380	0:01.006	0.321	18	37
toetictac	50 – 50	0:00.593	0:02.944	0:01.030	0.576	9	5,478
sum15	50 – 50	0:00.458	0:00.629	0:01.037	0.442	9	5,478
eotcatcit	50 – 50	0:00.497	0:00.823	0:01.088	0.457	9	5,478
gt_attrition	80 – 80	0:00.391	0:00.512	0:01.479	0.264	33	66
tictactoe_orthogonal	50 – 50	0:01.092	0:01.662	0:01.606	0.680	9	5,620
nim2	0 – 100	0:00.509	0:00.510	0:01.709	0.298	5 (24)	2,162 (10,725)
tictactoeparallel	50 – 50	0:04.394	0:47.663	0:03.385	1.298	9	5,996,090
grid_game2	60 – 30	0:03.248	0:02.515	0:04.508	0.720	49	668,285
nim3	100 – 0	0:01.939	0:01.910	0:05.097	0.380	5 (63)	129,776 (1,866,488)
nim4	0 – 100	0:02.223	0:02.208	0:05.423	0.410	5 (64)	149,042 (2,179,905)
minichess-evilconjuncts	100 – 0	0:05.917	o.o.m.	0:05.896	1.004	9	4,573
minichess	100 – 0	0:05.719	o.o.m.	0:05.941	0.963	9	4,573
chomp	100 – 0	0:03.773	o.o.m.	0:06.254	0.603	8 (56)	12,868 (162,591)
double_tictactoe_dengji	50 – 50	0:11.304	0:46.021	0:06.536	1.729	18	81,803,709
sheep_and_wolf	0 – 100	0:47.580	o.o.t.	0:16.598	2.867	54	783,163
catcha_mouse	100 – 0	1:19.330	o.o.t.	0:20.959	3.785	20	551,257,781,148
numbertictactoe	100 – 0	3:25.803	52:57.811	1:05.180	3.157	9	9,003,966
clobber	42 – 0	55:05.876	o.o.t.	7:32.072	7.312	17	26,787,440
tictactoelarge	50 – 50	72:30.589	o.o.t.	22:14.458	3.260	25	158,977,021,146
tictactoelargesuicide	50 – 50	o.o.t.	o.o.m.	47:42.062	—	25	158,977,021,146
CephalopodMicro	100 – 0	o.o.t.	o.o.t.	95:56.190	—	54 (61)	513,747,140 (1,401,231,054)

piece on position $(1, 1)$, i. e., the very last piece of the bar, is poisoned so that the player to bite this loses the game.

This game's GDL specification was first introduced in the 2009 GGP competition, where it consisted of a chocolate bar of 8×7 pieces, the same version we used in our experiments. As the players can bite any number of pieces it is clear that duplicates may reside in different layers, so that no incremental progress measure exists—there is only a (non-incremental) progress measure where the progress corresponds to the number of pieces already bitten off the bar. For this game we see that the time required to solve it increases from nearly 3.8 seconds with the spi approach to nearly 6.3 seconds with the layered approach. This comes from the fact that the number of states increases from $12{,}868$ in 9 layers, where the duplicates are removed after each BFS step, to $162{,}591$ states in 57 layers, where all duplicates are retained, so that a lot more (backward as well as forward) steps are required and a lot more states must be solved.

A similar observation holds for NIM. Here, the most complex version we tested is `nim4`, which consists of four stacks with 12, 12, 20, and 20 matches, respectively. In the duplicate free spi approach only $149{,}042$ states were found in five BFS steps (six layers in total), while the layered approach must analyze $2{,}179{,}905$ states in 65 layers, so that the runtime increases from roughly 2.2 seconds to 5.4 seconds.

A small version of CEPHALOPOD[33] is a surprising exception. In this game the two players take turns placing dice on a rectangular board that is initially empty. If a die is placed (horizontally or vertically) adjacent to two dice showing a total of at most six pips they are removed from the board and the new die shows the sum of their pips. If it is placed adjacent to three or four dice of which two, three or four dice show a total of at most six pips the player may decide how many dice to capture (at least two) and the new die shows as many pips as the captured dice did. In all the other cases the newly inserted die shows one pip. The game ends when the board is filled and the player having more dice on the board wins the game. The default version consists of 5×5 cells, so that no draw is possible.

The small version that we used consists of 3×3 cells. This game does not contain an incremental progress measure. Counting the total number of pips shown on the dice can remain the same over one or more consecutive steps, if dice are captured. The number of dice on the board actually decreases by one, two or even three with each capture. Overall, there are $531{,}747{,}140$ unique states in 54 layers, but the layered approach must analyze $1{,}401{,}231{,}054$ states in 61 layers. Nevertheless,layered approach is faster in solving the game. It can find a solution after nearly 96 minutes, while the spi approach one cannot find one in the allowed 120 minutes. Thus, it seems that the overhead due to the increased number of states and layers to be solved is not as big as the savings which come with the omission of strong pre-images and the smaller sets of reachable states for the various layers compared to the overall set.

For the more complex games having an incremental progress measure such as CATCH A MOUSE, two versions of TIC-TAC-TOE played on a 5×5 board with the goal to find a line of length five, or CLOBBER played on a 5×4 board the runtime decreases with the layered approach by a factor of up to seven.

CLOBBER

The highest factor comes with the game CLOBBER (Albert et al., 2005). This game is typically played on a chess-like board of various dimensions, which is completely filled with the players' pieces (white and black), often in an alternating manner. White is to perform the first move. During its turn the active player can move only one of its own pieces onto a (horizontally or vertically) adjacent cell that is occupied by one of the opponent's pieces. That will be removed from the board and the active player's piece moved to that cell, leaving its original cell empty. The game ends when a player cannot perform any move and is won for the player that performed the last possible move.

In our experiments, the game is played on a 5×4 board. An incremental progress measure is given by the number of removed pieces. Though only $26{,}787{,}440$ states are reachable in 18 layers the older approach takes nearly one hour to solve the game, while the layered approach takes only 7.5 minutes, resulting in a factor of more than seven.

An explanation for the high runtimes compared to the small number of states might be that the moves are very difficult for symbolic search. As we have seen in the discussion on the complexity of CONNECT

[33] See `http://www.marksteeregames.com/Cephalopod_rules.html` for a short description of the game by the author.

FOUR's termination criterion in Section 4.3.2, especially in Lemma 4.7 on page 35, a BDD representing a situation where two horizontally or vertically adjacent cells are occupied is of exponential size. In CLOBBER any move is concerned with moving to a horizontally or vertically adjacent cell, which must be occupied by an opponent's piece, so that the BDD representing the transition relation is of exponential size. As the transition relation is used in every image calculation, which in itself is already NP-complete, it makes sense that the runtime is a lot higher than it is in other games of similar complexity in the number of states.

CONNECT FOUR and CUBICUP

We also experimented with some even more complex games, which we cannot solve in the two hour limit. Two such games are CONNECT FOUR and CUBICUP, both of which incorporate an incremental progress measure.

For CONNECT FOUR we evaluated the approaches on two smaller boards, namely 5×6 and 6×6. The 5×6 version can be solved by both approaches, the spi and the layered one. The former takes about 139 minutes, while the latter finds the optimal solution, a draw, in roughly 30 minutes. For the 6×6 version the available memory of $24\,\text{GB}$ is too small for the spi approach, while it suffices for the layered approach, which can find a strong solution—the game is won by the second player—in nearly 564 minutes. The full 6×7 version of the game is still too complex for us. Even on another machine with $64\,\text{GB}$ of RAM we were not able to solve more than nine of the 43 layers. This is not surprising as we have detailed earlier in Section 4.3, because the BDD for representing the termination criterion is rather large and it is required in every step. Similarly, the BDDs for the states won by one player are also of exponential size, so that the solved BDDs are a lot larger than the BDDs for the reachable layers alone.

In CUBICUP[34] cubes are stacked with a corner up on top of each other on a three-dimensional board. A new cube may only be placed in positions where the three touching cubes below it are already placed. If one player creates a situation where these three neighbors have the same color, it is called a *Cubi Cup*. In this case, the next player has to place the cube in this position and remains active for another turn. The player to place the last cube wins—unless the three touching cubes produce a Cubi Cup of the opponent's color; in that case the game ends in a draw.

Due to the rule of one player needing to perform several moves in a row it is clear that in one BFS layer both players may be active (in different states), so that we need to use the extension of the layered approach that works for this kind of games, which we proposed earlier.

We are able to solve an instance of CUBICUP with an edge length of five cubes. Using the spi approach we are not able to solve this instance, as it needs too much memory. After nearly ten hours of computation less than $60\,\%$ of all states are solved but the program starts swapping. With the layered approach things look better, as it is able to find the strong solution—the game is won by the first player—for the $7{,}369{,}419{,}770$ states in roughly 565 minutes[35].

[34] See `http://english.cubiteam.com` for a short description of the game by the authors.

[35] Unfortunately, we had to stop the solving several times and restart with the last not completely solved layer, as somehow the implementation for loading BDDs using JavaBDD and CUDD seems to contain a memory leak, which so far we could not locate. No such leak appears in the spi approach, as it does not load or store any BDDs.

Chapter 11

Playing General Games

> The Wheel of Time turns and Ages come and pass, leaving memories that become legend. Legend fades to myth, and even myth is long forgotten when the Age that gave it birth comes once again.
>
> Robert Jordan, *The Wheel of Time* Series

The idea of general game playing is to play any game that can be described by the input language GDL as good as possible. Since 2005 an annual competition is held at an international AI conference—either AAAI or IJCAI—where a number of general game players play against each other trying to score the highest number of points. In this competition (Genesereth et al., 2005) the scenario is as follows. There is one server, which has control of all the games. It sends messages to the participating players (also called agents). These messages initially include the game's name, the agent's role in the game, the GDL description of the game, the startup time and the move time. The agents can use the startup time for any calculations they might need before the actual game starts. After this, the players have the move time, which often ranges from a few seconds up to as much as one or two minutes, to choose the next move. Each player sends its move to the server. Once all agents have sent their moves—or the move time is up, in which case the moves of the agents that did not sent a legal move will be set to the first legal one—the server informs them of the chosen moves of all players. Afterward another move must be sent within the next move time and so on. Finally, when the game is finished, the server informs the agents of this, so that they can stop their players.

In a competition all players run on their own machines, i. e., they might be run on a Laptop with a weak CPU and very limited RAM, or a large cluster with many CPUs (each possibly having several cores) and an immense amount of RAM, or anything in between, so that a fair comparison between the different approaches is rather difficult or straight impossible. Thus, the best algorithm will not necessarily win a competition—unless it is a lot better than all the others that might run on better hardware. Nevertheless, in the years since the establishment of the competition in 2005 mainly two approaches have been successfully applied. In the early years the best players used minimax based approaches, while in recent years nearly all players have switched to the simulation based UCT approach (Kocsis and Szepesvári, 2006).

In the following, we will start by introducing the algorithm UCT (Section 11.1), which we use in our player. In some cases specialized players apply it successfully in their respective domain. Three of these we will briefly describe. Furthermore, we will point out a number of ways to parallelize UCT. Then, we will briefly introduce the most successful general game players in the last few years (Section 11.2). Afterward, we will present our implementation of a general game player (Section 11.3) and provide some experimental evaluation of different settings of it (Section 11.4).

11.1 The Algorithm UCT

Early approaches to general game playing, for example used in CLUNEPLAYER (Clune, 2007), FLUXPLAYER (Schiffel and Thielscher, 2007b), or UTEXASLARG (Kuhlmann et al., 2006), often made use of classical minimax search (von Neumann and Morgenstern, 1944) with several extensions, the most well-known of these being $\alpha\beta$ pruning (Knuth and Moore, 1975). One of the problems is that even with the use of those extensions an immense number of states must be analyzed in order to find a correct result. Furthermore, minimax in itself is not well-suited as an algorithm for playing games, as it is actually used to solve them. In the competition setting it is important to quickly determine which move to take, and this cannot be done by the minimax approach as it knows the outcome of all possible moves only at the very end of the solving process.

In order to use minimax for playing games the use of an *evaluation function*, in this context often also called a *heuristic*, is essential. It determines an estimate on the outcome of the game for any state without further searching the successor states. Enhanced by an evaluation function minimax thus no longer needs to search the state space completely but rather can stop at any given state. This state it can treat as a terminal one and take the estimate provided by the evaluation function as its outcome. A so-called *anytime algorithm*, i. e., an algorithm that can return a desired result at any given time, can be implemented by using minimax together with an evaluation function and *iterative deepening* search (see, e. g., Russell and Norvig, 2010). When using iterative deepening several minimax searches are performed successively. Starting with a maximal depth of one, the search treats any state found in the given depth as a terminal one and thus uses the evaluation function to estimate the outcome. When one complete search to the specified depth is finished the depth is increased and the algorithm starts over.

The main problem for the minimax approach in general game playing is finding an appropriate evaluation function. In specialized players for games such as CHESS or GO lots of expert knowledge and some hand-crafted heuristics are used to come up with efficient evaluation functions, but this is not possible in the domain of general game playing. Thus, nowadays most players reside to another anytime algorithm, which is not dependent on efficient evaluation functions. This approach is UCT (Kocsis and Szepesvári, 2006) (short for Upper Confidence Bounds applied to Trees).

The UCT algorithm is an extension of UCB1 (Auer et al., 2002) (short for Upper Confidence Bounds) that enables it to operate on trees. Furthermore, it makes use of Monte-Carlo searches.

In the remainder of this section we will introduce UCB1 (Section 11.1.1) and Monte-Carlo searches (Section 11.1.2), introduce UCT as the combination of the two (Section 11.1.3), present ways to use UCT in specialized players (Section 11.1.4) and provide pointers to a parallelization of UCT (Section 11.1.5).

11.1.1 UCB1

UCB1 (Auer et al., 2002) was proposed to handle multi-armed bandit problems. In such a problem it is assumed that we are confronted with a slot machine having a number of arms (opposed to the classical one-armed bandit machines known from casinos). For each arm the machine has a fixed probability distribution for the possible rewards and we would like to maximize our reward, but of course we do not know the distributions. Thus, we can try to find out more information on the actual distribution of one arm by pulling it and observing the result or hope that we know enough of the complete machine and pull the arm that we suppose provides us with the best reward. This confronts us with the so-called exploration versus exploitation problem. Either we can explore other arms to gain further knowledge on them, hoping to find one that is even better than the one we assume to be the best, or we can exploit the knowledge we have by pulling the—as far as we know—best arm in order to achieve the highest possible reward. Here we can distinguish between the short-term reward and the long-term reward. By exploring the assumed sub-optimal arms we reduce the short-term reward but might fare better in the long run, while by exploiting the generated knowledge we maximize the short-term reward but at the same time might stay with a sub-optimal arm for a long time, thus reducing the long-term reward. That is where the UCB1 formula

$$U(a) = Q(a) + C\sqrt{\frac{\log N}{N(a)}} \tag{11.1}$$

comes in. It determines the UCB1 value U for an arm a given the average reward Q of that arm found so far, the total number of times N we have chosen any arm and the number of times $N(a)$ we have pulled arm a. With each update the value $N(a)$ for the chosen arm a increases stronger than that of N, so that for this the denominator increases by a larger amount than the nominator and thus overall the second term, called the UCB bonus, decreases. For all other arms only the nominator increases, so that the UCB bonus increases slightly as well.

The additional constant C determines the degree of exploration versus exploitation by weighting the terms. For a large value of C the UCB bonus will become more important, so that we will prefer to gain new knowledge on the average rewards of all—even the supposedly sub-optimal—arms, while for a small value the average reward becomes more important, so that we will use what we know and will more often pull the arm that we have observed to be best so far. In the limit, for $C = \infty$ we will pull each arm equally often, completely independent of the actual averages, while for $C = 0$ we will always pull the arm with the highest average, no matter the UCB bonus.

11.1.2 Monte-Carlo Search

Basic Monte-Carlo search is a simple approach to quickly gain some knowledge on good moves that does not require much memory. Given the current state it simply performs purely random runs (simulations) to a terminal state and updates the average reward received for the first chosen move. The information of course is not too reliable, but it can be generated a lot faster than a complete minimax search and the search can be stopped at any time. Once stopped the move with the highest average value is chosen.

Two facts about this approach are disadvantageous. First, the search is performed completely at random instead of being directed toward promising areas. Second, no knowledge is stored from one run to another. Especially in the domain of general game playing this means that we might have a good estimation of the first move, because we can use not only a full move time but also the full startup time to evaluate the initial state, but once the game proceeds we start evaluating the move's successor states from nothing, so that we have to regenerate all the knowledge we already had.

A simple extension to overcome the second problem is to use some kind of memory, e. g., in form of a game tree. We could store the complete runs (or at least the first new node of each run) in the chosen data structure and update the average rewards of all moves of all stored states whenever we have performed a run that used this move. This way, when the game progresses we can use the information we have generated in all the previous states and thus make a more informed choice.

Nevertheless, the problem of the purely random runs still exists, and that is where UCT comes in.

11.1.3 UCT

As we have mentioned before, UCT (Kocsis and Szepesvári, 2006) is the combination of UCB1 (in the context of tree search) and Monte-Carlo searches. Similar to the extension to Monte-Carlo we have mentioned, UCT stores part of the game tree in memory (cf. Figure 11.1). Starting at the current state it traverses the tree and in each state it visits it evaluates all successor states according to the UCB1 formula 11.1, more precisely according to

$$U(s,m) = Q(s,m) + C\sqrt{\frac{\log N(s)}{N(s,m)}} \tag{11.2}$$

where $U(s,m)$ is the UCT value of move m in state s, $Q(s,m)$ the average reward achieved when choosing move m in state s, C the constant for controlling the exploration versus exploitation ratio, $N(s)$ the number of times state s was visited, and $N(s,m)$ the number of times that move m was chosen when in state s. In the traversal it always chooses the move with maximal UCT value, if all have already been expanded at least once. Otherwise it randomly chooses one of the unexpanded moves. The traversal within the tree is often called the *Selection* phase.

When the tree is traversed and a leaf node is reached, i. e., a node that corresponds to an unexpanded state, a Monte-Carlo search is performed. This phase is often called the *Simulation* phase. It starts at the leaf node and from then on chooses moves randomly until a terminal state is reached. Once that is the case either the complete simulation run, or only the first new node of this run (Coulom, 2006), is added to the

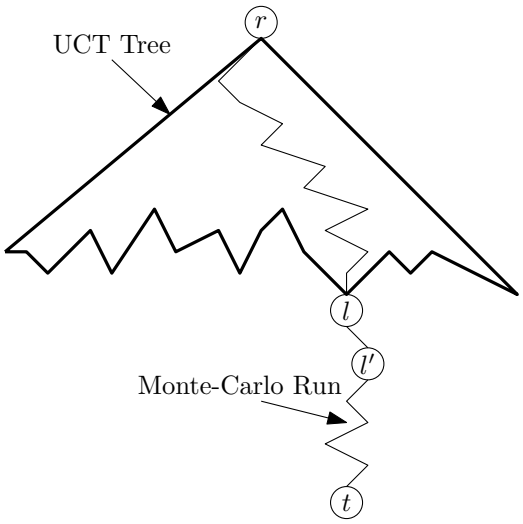


Figure 11.1: An overview of the UCT algorithm. Each run starts at the current state, typically the root r of the UCT tree, and traverses the tree according to the UCT formula 11.2 until a leaf node l is reached. Then it starts a Monte-Carlo run that ends in a terminal state t. When this run is done either the complete run, or only the first node l' of it is added to the tree.

UCT tree. This is called the *Expansion* step. The idea behind the latter approach is to use less memory, because in some games it might very well be that the tree's size exceeds the available memory, if the runs are reasonably fast.

Afterward the average rewards of all moves taken within the UCT tree are updated according to the outcome of this newest run, so that in the next run new UCT values must be calculated and typically another traversal is performed. This final phase is called the *Back-Propagation* phase.

Similar to UCB1, in UCT the constant C also controls the ratio between exploration and exploitation. If we prefer to explore potentially worse moves, hoping that these might still be better than the supposedly best one found so far, we use a higher value of C. In the limit, when $C = \infty$, we will always choose all moves equally often, independent of the values of the moves. In other words, we are searching in the breadth. If we prefer to exploit the knowledge we have, we use a smaller value of C in order to perform the move with the highest average reward more often. In the limit, when $C = 0$, we will always choose the move with the highest average, so that we are searching in the depth, and after some iterations will only deviate from that path if the average value of any state on the path drops below that of one of its siblings. Thus, in UCT we not only exploit existing knowledge but also verify it, as with each run we get a better estimate on the true values of all the states on the path.

11.1.4 UCT Outside General Game Playing

UCT is not only used in general game playing but also applied successfully in some more traditional game players, especially in GO. In the following we provide some details on UCT in the context of three games, namely GO, AMAZONS, and KALAH.

GO

GO is a board game that originated in China, but nowadays it is well known all over the world. We will start by providing a very brief overview over the basic rules, leaving out lots of the details that are of no importance in this section.

GO is played on boards of sizes 9×9 to 19×19. The two players (black and white) take turns placing stones on the board. These cannot be moved once placed, but can be *captured* and thus removed from the board. The game is won by the player that occupies the largest number of cells.

Due to its large branching factor (several hundred in the early moves on a 19×19 board vs. a few dozen in CHESS) and no known efficient evaluation function the classical minimax and $\alpha\beta$ searches did not bring computer GO players to a strength above average amateurs. Thus, similar to general game playing a different approach to determine good moves was needed. The approach that is most successful today is the use of UCT.

One of the first computer GO players using UCT was MOGO (Gelly and Wang, 2006). Initially, it performed UCT without many specializations. The main differences reported in the paper are to add only one new node to the UCT tree for each simulation run instead of the full run, similar to the approach used in CRAZYSTONE (Coulom, 2006), and a default value for expanding moves again even if other sibling moves have not yet been expanded. If this value is sufficiently small (but not too small) this results in promising nodes being explored several times before other ones are explored a first time, so that they can be exploited earlier. This especially helps due to the large branching factor, as otherwise several hundred nodes would have to be expanded first before the knowledge could be used further. Gelly and Wang (2006) also are the first to propose a parallelization of UCT. They implemented a parallel version using shared memory and mutexes to make sure that no two processes change values within the tree concurrently.

Later, several improvements to UCT were implemented in MOGO (Gelly et al., 2006). Some of these concern the improvement of the simulation runs by implementation of game specific techniques. These contain the choice of a random move saving a group with only one liberty (i. e., only one position to extend this group; a group with zero liberties is removed from the board), an *interesting* move in one of the eight positions surrounding the last move (a position is considered interesting if it establishes certain patterns), or capturing moves somewhere on the board. Pruning techniques were used to reduce the number of possible moves to only those within certain zones. Other improvements concern the search within the tree, including further pruning techniques, allowing only moves within some groups, as well as new initial values for moves newly added to the UCT tree. While before all unexplored moves had to be expanded once before the UCT formula was applied, Gelly et al. (2006) proposed to use some initial values for those moves. One idea they applied is to set the values to the same as in the equal moves of the grandparent (were applicable).

Gelly and Silver (2007) proposed the use of the *rapid action value estimation* (RAVE), which is now also used in the most successful general game players. RAVE corresponds to the *all-moves-as-first* heuristic (Brügmann, 1993). In the RAVE extension a new average, Q_{RAVE}, is introduced for all possible moves of states already in the UCT tree. This average is updated whenever a move m in state s, that was not chosen, is chosen in a later state (on the run from the root r to the found terminal state t). It stores the average of all these occurrences. It is good to quickly get some estimate on the outcome of moves. According to the authors, apart from GO this also works in other games where sequences of moves can be transposed. While this estimate is helpful at the beginning, when a node is still explored only a few times, it might hinder later on, as the value of a move typically depends on the precise state it is selected in, so that in the long run the impact of Q_{RAVE} should decrease as the number of expansions increases. This can be achieved by weighting the two averages linearly, i. e.,

$$\beta(s) \times Q_{RAVE}(s,m) + (1 - \beta(s)) \times Q(s,m) \tag{11.3}$$

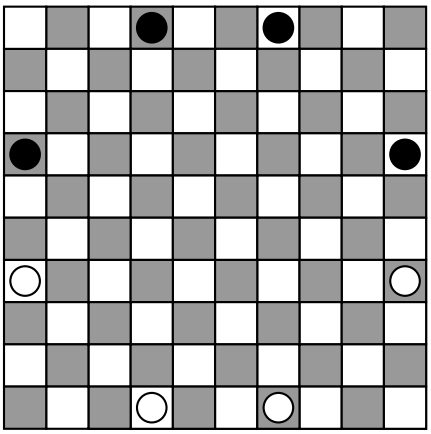
with

$$\beta(s) = \sqrt{\frac{k}{3n(s) + k}}$$

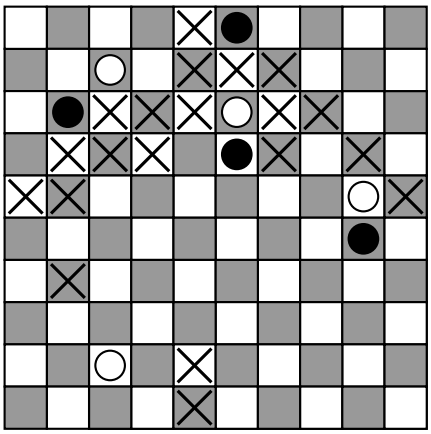
and $n(s)$ being the number of times state s has been visited and k the so called *equivalence parameter*. It controls how often a state must be visited until both estimates are weighted equally. The result from equation 11.3 is then used as the first term of the UCT formula 11.2.

In the same paper the authors proposed further methods to insert prior knowledge into the search. That is, they initialized the value of a newly added node to some prior value, and the number of expansions of that node to the number of runs UCT would require to gain that information. This prior knowledge can be gained for example by choosing the values from the grandparent (as in the previous paper), by some handcrafted heuristic, or by some learning methods. In this case the authors used temporal-difference learning (Sutton, 1988).

These improvements finally made MOGO the first computer GO player to achieve a strong master level (3-*dan*) on 9×9 boards (Gelly and Silver, 2008).



(a) Initial state of AMAZONS.



(b) Later state in AMAZONS.

Figure 11.2: The game AMAZONS. The circles represent the amazons of the two players, the crosses the positions of arrows.

AMAZONS

Another game where UCT has been successfully applied is AMAZONS. It is played on a 10×10 board with four game pieces, called *amazons*, for each player. The initial state is depicted in Figure 11.2a. The players take turns choosing their moves. Such a move consists of two steps. In the first one the current player chooses one of its amazons and moves it just like a queen in CHESS. Afterward that amazon shoots an arrow onto one location where it might move now. That location cannot be moved on or over anymore and stays this way until the game ends (see Figure 11.2b for a state after a number of steps). It ends when one player cannot move any of his amazons. The score is often calculated to be the number of moves the opponent still can take.

Kloetzer et al. (2007) used UCT in AMAZONS and also added some game-specific improvements. The most successful were to split the moves, i. e., making the moving of an amazon and the firing of an arrow two moves instead of one, as well as a combined evaluation, i. e., to evaluate the average score obtainable from a position as well as the average win-loss-ratio.

Further improvements concerned the Monte-Carlo runs. Here, some knowledge concerning liberty and isolation was added. If an own amazon has a liberty of one or two, i. e., one or two directions it can move, it should be moved immediately. If an opponent's amazon has a liberty of one or two it should be enclosed if possible. The isolation is somewhat more complex. An amazon is isolated if it cannot reach a cell an opponent's amazon might reach in any number of steps. In that case it should not be moved if possible.

They also proposed the use of an evaluation function together with UCT. Instead of performing a Monte-Carlo run to the end of the game it is only performed for a fixed number of steps. This means that no precise value is known, so that the combination of score and win-loss-ratio cannot be propagated to the UCT tree, but instead an evaluation function can be used.

Lorentz (2008) proposed further improvements to the UCT algorithm in AMAZONS. One idea is to stop the Monte-Carlo runs early, e. g., when all amazons are isolated. Unfortunately, this requires lots of additional calculations, which reduces the number of simulation runs. Thus, he decided to stop not necessarily at a terminal state, but after a Monte-Carlo run has reached a fixed length. This means that an efficient evaluation function must be used.

Another improvement comes with the choice of moves to explore. Instead of exploring every possible move (of which typically around $1{,}000$ are available) only 400 are examined. To determine the best 400 moves, all successors of a state must be generated and evaluated. As this is a time-consuming step he further proposed to not expand a leaf node of the UCT tree when it is reached the first time but only after it is visited a fixed number of times, in this case 40, similar to an approach by Coulom (2006).

The number of moves to be considered is even further reduced by the use of what the author calls *progressive widening* (Cazenave, 2001; Coulom, 2007; Chaslot et al., 2008b). This means that at first only

the five most promising moves are considered. After a number of visits the next five moves are considered and so on, until finally all 400 moves may be used.

These extensions to UCT along with some more game-specific enhancements and the use of a new evaluation function, which is less powerful but faster to calculate than one used in a minimax based player, results in a UCT player that was able to outperform a minimax based player. In this case, the UCT player won about $80\,\%$ of the matches against the other.

KALAH

For the game KALAH (cf. Section 10.1) Ramanujan and Selman (2011) recently studied the effectiveness of UCT. While minimax works reasonably well in this game using a rather simple heuristic for the evaluation function (i. e., the difference in the number of captured pieces (Gifford et al., 2008)) UCT works similarly well given a good UCT constant C, even without further improvements. This is a bit surprising, as so far for most games either minimax approaches or UCT worked well, but only rarely both were comparable.

They also tried combining both approaches. The first one, UCT_H, does not use the normal Monte-Carlo runs but instead evaluates the first successor of a leaf node using the same evaluation function that is used in the minimax approach. The second one, UCTMAX_H, does not store the average values but the minimax values based on the same evaluation function, i. e., it sets the value $Q(s)$ of a state s to either the maximum of the Q values of all successors (multiplied with $N(s)$, the number of visits of state s) if in s the own player is active, and to the minimum of all successors multiplied with $N(s)$ if the opponent is active in s.

In this game, UCTMAX_H is the best of the evaluated algorithms (i. e., minimax, UCT, UCT_H, and UCTMAX_H). According to Coulom (2006) the use of the minimax values works better than the average values if nodes are expanded sufficiently often or if a good evaluation function is used, which is the case in this game.

Additionally, they found out that UCT works better than minimax if the search trees do not contain too many terminal states. In the case of KALAH or GO this is typically the case, as the terminal states are rather deep in the game tree, while in CHESS first terminal states appear after only a few steps. This might thus be an additional explanation why UCT works worse than minimax in CHESS (Ramanujan et al., 2010a,b), apart from the fact that in CHESS good evaluation functions are known.

11.1.5 Parallelization of UCT

The Monte-Carlo runs used in UCT are quite independent of each other, so that it seems that a parallelization of UCT is rather trivial. Of course, there are some simple ways to parallelize UCT, but not all of these approaches work in a satisfying fashion.

One of the first short descriptions of a parallel version was in the first paper on MOGO (Gelly and Wang, 2006). There, the authors use a shared memory architecture along with mutexes to ensure that the values within the UCT tree are not changed concurrently.

Other early work on parallel UCT is due to Cazenave and Jouandeau (2007), who proposed three approaches for the parallelization of UCT, namely *single-run parallelization*, *multiple-runs parallelization*, and *at-the-leaves parallelization*.

In the single-run parallelization, nowadays also called *root parallelization*, a master process creates a number of slave processes. There is no communication whatsoever between these processes, especially no shared memory. Each process stores its own UCT tree and performs the normal UCT algorithm in it. After a timeout or after a predefined number of runs has been finished each process sends the information for the root node, i. e., for each possible move at the root the number of times it was chosen along with the average outcome when following it, to the master. That combines the information of all processes and then calculates the best move to perform according to the overall averages.

Surprisingly, the idea behind the single-run parallelization, i. e., the use of an ensemble of UCT trees, brings advantages even in the case of single-core machines. When using such an ensemble, the player achieves better results than with a single tree containing as many nodes as all trees of the ensemble combined. This effect seems to be relevant only in few games (e. g., GO (Chaslot et al., 2008a) or KLONDIKE (Bjarnason et al., 2009)), while in an experiment on a larger number of domains no significant evidence could be observed (Fern and Lewis, 2011).

The multiple-runs parallelization is similar to the single-run version, though in an iterated form. Again, a master creates a number of slave processes that perform UCT independently of each other using their own UCT tree. After some time or after a predefined number of runs has been performed they send their results for the root node to the master, which combines these. Once it has the results of all slaves it broadcasts the new root results. Each slave then updates the root of its UCT tree accordingly (the number of expansions for each move are divided by the number of slave processes) and goes on with its UCT search. This process of communicating the root results goes on until some total timeout has been reached. At that time the master process will choose the next move to take.

In the experiments performed by Cazenave and Jouandeau (2007) the multiple-runs parallelization was at least comparable and sometimes even slightly better than the single-run parallelization, but we did not find any newer research on this approach.

In the at-the-leaves parallelization only one UCT tree is used. The master process searches this tree according to the UCT formula and once it reaches a leaf, it spawns a number of slave processes, which perform only the Monte-Carlo runs. When a certain timeout is reached or the processes have performed a predefined number of Monte-Carlo runs they are stopped and send the average reward along with the number of runs to the master. That updates the leaf node accordingly and propagates the results along the chosen path in the UCT tree to the root, before starting another UCT run.

One shortcoming of this approach is that some Monte-Carlo runs might finish faster than others, so that the some processes will have to wait while others are still working. To overcome this problem, Cazenave and Jouandeau (2008) proposed an improvement, where the master does not wait for all processes to finish. Instead, it waits for the first process to finish, propagates its result, performs the next selection phase based on the updated UCT tree and starts the process to perform the next Monte-Carlo run starting at the reached leaf node. This way, after an initialization step, the Monte-Carlo runs can be performed in parallel starting at different leaves of the UCT tree.

All these approaches require some communication between the various processes and work well especially on large clusters, as the trees are not shared between the processes. As nowadays multi-core processors are becoming more common, so that the processes can make use of shared memory, other approaches can be thought of as well.

Chaslot et al. (2008a) proposed such a shared memory approach, which they call *tree parallelization*. In this approach, all processes share the same UCT tree and perform UCT as usual. They also present three approaches to omit possible conflicts between the processes. The first one uses a *global mutex*, so that only one process can be active within the UCT tree. The others either need to wait at the root node before they can start or they perform the Monte-Carlo runs. This approach matches the one by Gelly and Wang (2006). As it results in quite an overhead due to the need for waiting, another idea is to use *local mutexes*. For this each node of the UCT tree incorporates a mutex which will be locked (and later unlocked) when a process operates on this node. The last approach is not concerned with the location of mutexes but rather increases the probability that processes searching through the UCT tree in parallel do so in different parts of the tree. Instead of updating the averages only during the backpropagation they also update them on the way down towards the leaves. They update them by using a *virtual loss*, so that the counter is increased and the average is updated. This loss is later removed in the backpropagation step and replaced by the correct result.

The approach of Enzenberger and Müller (2009) is in a similar direction, though their approach is *lock-free*, i. e., they do not use any mutexes. They use pre-allocated memory arrays for each process. Such an array stores the nodes of the UCT tree. When a leaf node is expanded the successors are generated in the local memory array of the process. In the parent node the pointer to the first successor as well as the number of successors are set once all successors are created. It might very well be that two processes expand the same leaf node at the same time, so that this approach yields some memory overhead, as the successors generated by only one process will be used later on. It is assumed that such problems will happen only rarely.

During the update of the averages and counters they also omit mutexes. This way it might be that for some node a new average will be calculated but the counter is not incremented or the counter will be incremented but the new average is not stored due to the concurrency of the processes. In practice this does not pose much of a problem, so that they intentionally ignore these effects and can make use of maximal performance as at no time any process has to wait for another to finish.

We proposed an approach (Edelkamp et al., 2010a) that is quite similar to the previous one. We also use pre-allocated memory, but only one array for the complete tree. To prevent inconsistencies we use one mutex which we lock to increment the pointer to the last used cell of this memory block. Then we generate a successor of the found leaf node and insert it to the position where the pointer previously pointed. The pointers to these successors are stored in an array which is then passed to the leaf node (along with the number of successors). This way no two processes can extend the tree simultaneously, but all other operations can run in parallel and not much time is spent waiting for some mutex to be unlocked.

Similar to the idea of virtual loss, we already update the number of visits of the nodes during the search in the UCT tree, so that the other processes may choose other moves and thus not end at the same leaf node. The difference here is that the averages are not touched until after the Monte-Carlo run, so that the effect of these losses is less strong (it influences only the UCT bonus), but there is no need for recalculation.

Another approach we proposed is the *set-based parallelization*. For this we use a priority queue based on weak heaps (Edelkamp and Wegener, 2000). It stores the nodes of the UCT tree that are previously calculated in a complete BFS up to a certain layer. More precisely, it contains the indices of these nodes along with their corresponding number of expansions, the average reward, and the resulting UCT value. It is organized according to these UCT values. The first k of these nodes are chosen and removed from the queue and each process starts at one of them to perform a normal UCT run. After it has finished it reinserts the corresponding node into the queue (though only the number of expansions along with the average reward are set) and takes the next one of the k chosen nodes. Once all k nodes have been expanded the UCT values of all the nodes in the queue are recalculated (as they all change once one run for any of them is performed) and the queue is reorganized according to the new values.

11.2 Other General Game Players

As we have mentioned before, earlier approaches to general game playing (e. g., Kuhlmann et al., 2006; Schiffel and Thielscher, 2007b; Clune, 2007) mainly made use of evaluation functions in the context of minimax based search, while newer ones (e. g., Finnsson and Björnsson, 2008; Méhat and Cazenave, 2010) are typically based on simulation based methods such as UCT. In the following we will describe players for both of these approaches.

11.2.1 Evaluation Function Based Approaches

In the first few years, the most successful players were based on minimax search using iterative deepening to be able to send answers early along with functions to evaluate those states that could not be completely solved. Here we will present three of these players, but not in the chronological order but rather in the order of increasing complexity with respect to the generated evaluation function.

UTEXASLARG

One of the first papers on this topic is due to Kuhlmann et al. (2006) of the Learning Agents Research Group of the University of Texas, the authors of the GGP agent UTEXASLARG. Their search is based on $\alpha\beta$ pruning (Knuth and Moore, 1975), which is a faster and more memory efficient extension of the minimax algorithm. For those searches where they do not use a heuristic, the terminal nodes are visited in a depth-first manner. If a heuristic is present, they make use of iterative deepening (Korf, 1985). To further enhance the search, they use transposition tables (Reinefeld and Marsland, 1994) and the history heuristic (Schaeffer, 1983).

To be able to handle multiplayer games in the minimax setting they group all players into two teams, the team that cooperates with the player their agent controls and the opponents. Nodes where it is one of the cooperating players' turn to choose among a number of moves are treated as maximization nodes, the others as minimization nodes.

Another extension to classical minimax search is for simultaneous move games. In this situation they assume the opponents to choose their moves at random and choose the action that (together with their team mates' moves) maximizes the expected reward.

To come up with a heuristic they mainly identify successor relations and boards. For the former they only allow predicates of arity two (such as `(succ 1 2)`), for the latter predicates of arity three (such as `(cell a 1 wr)`). With the help of the successor function they also identify counters, for which they look for a GDL rule of the form

```
(<= (next (<counter> ?<var1>))
    (true (<counter> ?<var2>))
    (<successor> ?<var1> ?<var2>))
```

with `<counter>` being the identified counter predicate, `<successor>` the previously found successor relation and `?<var1>` and `?<var2>` two variables. For the boards they identify the arguments that specify the coordinates of a position on the board and the markers that occupy the positions. For the coordinates they also can tell if they are ordered, which is the case if they are part of a successor relation. Furthermore, they distinguish between markers and pieces. The latter can occupy only one position in one state, while the former can be on any number of positions at the same time.

They identify all these structures by simulating a number of purely random moves, similar to what we do to find the sets of mutually exclusive fluents during our instantiation process (see Section 9.1.4). The coordinates of a board are the input parameters, the markers the output parameters.

Additionally they identify all the players that should be in league with the one their agent represents. For this they compare the rewards at the end of such a simulated game. If they always get the same rewards, they are in the same team, otherwise they are not.

Using these structures they can calculate certain features such as the x- and y-coordinates of pieces, the (Manhattan) distance between each pair of pieces, the sum of these pair-wise distances, or the number of markers of each type.

With these features they generate a set of candidate heuristics, each of which is the maximization or the minimization of one feature. These candidate heuristics they implemented as board evaluation functions, which can be used to evaluate the given state.

Their agent consists of a main process, which spawns several slave processes. Each is assigned the current state and one of the candidate heuristics to use for the internal search. Each slave informs the main process of the best action it could find so far. Additionally, at least one slave performs exhaustive search, i. e., it does not use a heuristic.

The decision, which of the proposed actions to choose, is not trivial. Of course, if the exhaustive search was successful, that action should be chosen, as it is supposed to be optimal. Unfortunately, the authors do not provide any more insight into the choice of the best action in case the game is not solved.

FLUXPLAYER

The winner of the second general game playing competition in 2006, FLUXPLAYER (Schiffel and Thielscher, 2007a,b), uses a similar approach. It also performs a depth-first based minimax search with iterative deepening along with transposition tables and the history heuristic and uses a heuristic evaluation function for those non-terminal states that it will not expand in the current iteration, but there are some more or less subtle differences.

For multiplayer turn-taking games it handles each node of the search tree as a maximization node for the active player. This corresponds to the assumption that each player wants to maximize his own reward, no matter what the opponents will get.

In case of simultaneous move games it serializes the moves of the players and performs its own move first. The opponents are then assumed to know the chosen move. The advantage of this approach is that the classical pruning techniques can be applied, while the downside is that it may lead to suboptimal play (e. g., in ROSHAMBO, or ROCK-PAPER-SCISSORS, the information of the move of one player easily enables the opponent to win the game).

Furthermore, it does not perform pure iterative deepening search, but uses non-uniform depth-first search, i. e., it uses the depth limit of the current iteration only for the move that was evaluated with the highest value in a previous search, while the limit for the other ones is gradually smaller. This is supposed to help especially in games with a high branching factor, as the maximal search depth will be higher than in classical iterative deepening search.

The main difference comes with the heuristic evaluation function. FLUXPLAYER uses a similar approach to find the structures of the game as UTEXASLARG, but can also detect game boards of any dimension and with any number of arguments specifying the tokens placed on the positions, i. e., it allows for arbitrary numbers of input and output parameters for each predicate. Furthermore, the heuristic function mainly depends on the similarity to a terminal state and the rewards that might be achieved. It tries to evaluate how close it is to a terminal state that corresponds to a win for it. For this it uses methods from fuzzy logic in order to handle partially satisfied formulas.

CLUNEPLAYER

The third approach in a very similar direction is due to Clune (2007, 2008) whose agent CLUNEPLAYER won the first competition in 2005 and ended second in 2006 and 2007. Again, there are some differences to the other approaches.

In a first step he identifies certain expressions of the game at hand. One of the main features of this is that he determines which constants can be set as which arguments of the expressions, if they incorporate variables. These are substituted for the variables to generate more specific candidate expressions.

From these expressions he creates candidate features, which represent various interpretations of the expressions.

A first interpretation is the solution cardinality interpretation. This corresponds to the number of distinct solutions to a given expression in a given state, such as the number of black pieces on the board in a state of the game of CHECKERS.

The second interpretation is the symbol distance interpretation. For this he uses the binary axioms to construct a graph over the constants for the two parameters. The vertices of this graph are the constants, and an edge is placed between two of these if they are the two arguments of one such binary axiom. Examples would be the successor relation or the axiom to determine the next rank in the game of CHECKERS. The symbol distance is then the shortest path distance between two symbols (the constants) in the constructed graph. With this, the distance between two cells on a board can be calculated.

The final interpretation is the partial solution interpretation. This applies to expressions containing conjunctions and disjunctions. For a conjunction, it gives the number of satisfied conjuncts, so that this is somewhat similar to Schiffel and Thielscher's (2007b) degree of goal attainment.

For each candidate expression he generates corresponding features by applying the possible interpretations. He also calculates some additional features by observing symmetries in the game's description.

To find out which of the generated candidate features are more relevant he uses the measure of stability. A feature that does not wildly oscillate between succeeding states is supposed to be more stable than one that does so.

For the calculation of the stability of a feature he performs a random search to generate a number of reachable game states. For each feature he calculates the value for each of the generated states. Additionally, he computes the total variance of the feature over all generated states as well as the adjacent variance, which is the sum of the squares of the differences of the values of the feature in subsequent states divided by the number of pairs of subsequent states. The stability is the total variance divided by the adjacent variance, so that for features that wildly oscillate the stability will be low, while it will be higher if it changes only slightly between subsequent states.

For the heuristic evaluation function he takes into account the approximation of the reward, the degree of control the player has along with the stability of this feature and the probability that a state is terminal along with the stability of this feature. For the approximation of the reward he uses only those features that are based on expressions that influence the `goal` rules. The control function is intended to capture the number of available moves for the players—the player with more possible moves has more control over the game. Finally, the termination property is used to determine the importance of reward and control, e. g., in the beginning of a game it might be more important to gain more control over the game, while in the endgame getting a higher reward is of paramount importance.

Combining these properties results in a heuristic evaluation function that captures the game in a better way than the other approaches. While Kuhlmann et al. (2006) use a number of heuristics and must choose which returned the best move and Schiffel and Thielscher (2007b) look only at the possible outcomes hoping that achieving a large similarity to the winning goal state earlier will assure higher rewards, here a

more flexible heuristic is applied. The result is that for example in CHESS it identifies the differences in the number of different pieces of the two players as being important (with the number of queens being most important and the number of pawns the least). In OTHELLO it does not use the number of tokens in the player's color as a good estimate, which is highly unstable, but prefers those states where the corners are occupied by the player, which is a much more stable feature and yields a much better guidance to a terminal state achieving a higher reward.

11.2.2 Simulation Based Approaches

After the early successes of the evaluation function based players, since 2007 all the following competitions so far were won by a simulation based player, i. e., ones performing UCT. In 2007 and 2008, CADIAPLAYER (Finnsson and Björnsson, 2008) could win the competition, while in the two following years, 2009 and 2010, ARY (Méhat and Cazenave, 2010, 2011a) was successful, and in the last competition in 2011 the winner was TURBOTURTLE. Nevertheless, the evaluation function based players are not bad, as they are still close to the winners. In fact, in the years 2007 to 2009 the second best player was always a player using evaluation functions.

In the following, we will describe the most successful simulation based players.

CADIAPLAYER

With its victory at the third and fourth competitions in 2007 and 2008, CADIAPLAYER (Finnsson and Björnsson, 2008; Björnsson and Finnsson, 2009) was the first winner of the competition to use UCT for the task of finding a good move. Actually, it performs parallel UCT by using one master to handle the UCT tree, which delegates the Monte-Carlo run to a client process once it has reached a leaf node of the UCT tree.

The agent incorporates several enhancements over classical UCT.

For single-player games they initially tried to work with A* (Hart et al., 1968), IDA* (Korf, 1985), or approaches inspired by heuristic planners such as FF (Hoffmann and Nebel, 2001). Later on they changed back to using pure UCT search, but with some further enhancements for the single-player case. They store not only the average reward achieved when taking an action in a state, but also the maximal reward achieved. During the UCT search they still use the average for calculating the UCT value, but when sending a move to the game server they use the move achieving the highest maximal reward. Additionally, when they find a terminal state with a better reward than the best one found so far, they store the complete path in the UCT tree. This way, any good solution found can be recreated. Otherwise it might be that a good solution was found only once but the end of the solution path got lost after the Monte-Carlo run, so that the player might actually achieve fewer points than originally determined.

For multiplayer games they use one UCT tree for each player to simulate its intentions. As not all games are two-player zero-sum games the opponents cannot simply be awarded the negation of CADIAPLAYER's reward, but need to know the (average) outcome they will get by choosing some action.

They also use a discount factor, which is slightly smaller than one, when propagating the calculated rewards to the root. This way the results from shorter games are preferred, as for the longer ones the uncertainty, especially concerning the opponents' moves, increases. This is not necessary in the single-player case as there is no opponent that might take an unexpected move

Another enhancement, which they call *Move-Average Sampling Technique* (MAST) handles the choice of moves in the Monte-Carlo runs (as well as the unexplored moves in the UCT tree). For this they keep the average reward for each action that was encountered, independent of the state it was found in. They then choose a move m with probability $P(m)$ according to Gibbs sampling (Geman and Geman, 1984)

$$P(m) = \frac{e^{Q(m)/\tau}}{\sum_{m'=1}^{n} e^{Q(m')/\tau}}$$

with $Q(m)$ being the average reward achieved when choosing move m and τ a parameter to vary the distribution (lower values stretch it while higher values result in a more uniform distribution).

In a more recent study Finnsson and Björnsson (2010) evaluated a number of further techniques for choosing the moves in the Monte-Carlo runs as well as the unexpanded moves within the UCT tree, apart from MAST.

A small extension of MAST is *Tree-Only* MAST (TO-MAST), where they update the moves' averages only for those moves that were performed within the UCT tree, not those of the Monte-Carlo runs. The idea is that the moves within the tree are already guided to some degree based on the UCT decisions, while the ones in the Monte-Carlo runs are more random.

Another approach is *Predicate-Average Sampling Technique* (PAST), where they store the average rewards $Q(f, m)$ achieved when choosing move m in a state that contains fluent f. When choosing an action in the Monte-Carlo runs they calculate the same probability distribution $P(m)$ as in the MAST case only that $Q(m)$ is replaced by $Q(f', m)$, where f' is the fluent of the current state for which the highest average reward with move m was achieved.

Another technique they introduced is *Features-to-Action Sampling Technique* (FAST). For this they identify two kinds of features, piece types and board cells, by template matching. The piece type is only relevant if more than one type is present; otherwise cell locations are the relevant features.

They apply temporal-difference learning (Sutton, 1988) for learning the importance of the features, i. e., the values of the different pieces or the value of placing a piece into a specific cell. During a simulation the states $s_1, \ldots, s_n$ are visited. For each s_i of these states the delta rule

$$\vec{\delta} = \vec{\delta} + \alpha \left[R_i^{\lambda} - V(s_i) \right] \nabla_{\vec{\theta}} V(s_i)$$

is applied for learning, where R_i^{λ} is the λ-return, $V(s)$ the value function and $\nabla_{\vec{\theta}} V(s)$ its gradient. The reward at the end of a simulation is the difference of the rewards the players get according to the GDL rules. Between two simulations $\vec{\delta}$ is then used to update $\vec{\theta}$, the weight vector that is applied in the value function to linearly weigh and combine the found features $\vec{f}(s)$:

$$V(s) = \sum_{i=1}^{|\vec{f}|} \theta_i \times f_i(s).$$

For those games where different piece types have been found, each feature $f_i(s)$ corresponds to the number of pieces of a given type in state s. In those games where the cell locations are of relevance, each feature is binary and represents the fact that a player's piece is located in the corresponding cell or not.

To use the learned information during the Monte-Carlo runs, they additionally calculate the $Q(m)$ values for each move m, so that they can apply this information in the same way as, e. g., in MAST. For piece-type features it is calculated as

$$Q(m) = \begin{cases} -\left(2 \times \theta_{p(to)} + \theta_{p(from)}\right) & \text{if } m \text{ is a capture move} \\ -100 & \text{otherwise} \end{cases}$$

with $\theta_{p(to)}$ and $\theta_{p(from)}$ being the learned values of the pieces on the to and $from$ cells. This results in capture moves being preferred if they can be applied, and furthermore in capturing higher ranking pieces with lower ranking ones being preferred. If the cell locations are the relevant features, $Q(m)$ is calculated as

$$Q(m) = c \times \theta_{p(to)}$$

with $\theta_{p,to}$ being the weight for the feature of player p having a piece in cell to and c a positive constant.

As a last improvement to their UCT implementation they also adapted the Rapid Action Value Estimation (RAVE) proposed in the context of GO (Gelly and Silver, 2007).

The first approaches, i. e., MAST, TO-MAST, PAST, and FAST, are typically only applied during the Monte-Carlo runs while RAVE is only applied within the UCT tree, so that RAVE can be combined with any of the former ones.

Unfortunately, in their evaluation no clear trend can be determined. For example, in games such as SKIRMISH, where different pieces must be captured, FAST is clearly the best, while in others it is worse than any of the other approaches. Similarly, PAST is better than MAST in AMERICAN CHECKERS, but loses in the other domains tested. TO-MAST is better than MAST in most domains, but in OTHELLO it

wins only about $40\,\%$ of the games against a MAST based player, possibly because in that game a move such as placing a piece in a corner of the board is often good in any phase of the game, while such a move is not available in the beginning and thus not present in the UCT tree.

ARY

The player ARY (Méhat and Cazenave, 2010, 2011a), which won the GGP competition in 2009 and 2010, also uses a parallel version of UCT to find good moves. More precisely, it uses the root parallelization proposed by Cazenave and Jouandeau (2007). For single-player games the threads store the maximal reward achieved during any run over a certain node instead of the average reward. Also, each thread stores the complete play that achieved the best reward, so that a good solution can be retrieved at any time.

Concerning the move to be sent to the game master they evaluated four different schemes. The first one, called *Best*, always sends the move with the best (average or maximal) reward found by any of the threads. The approach *Sum* is the one we described earlier, i. e., the one that combines the roots of all threads and chooses the move with the best (average or maximal) reward after that combination. *Sum10* is very similar to this, only that instead of combining all moves of all threads, only the ten best moves of each thread are considered. Finally, *Raw* disregards the number of runs of the single threads in the combination step, calculating an unweighted average of the values returned by the threads.

In their evaluation the often-used Sum approach achieved better results than the Best approach in the case of two-player games. Sum10 is slightly better than Sum, but the difference is not significant enough to draw any real conclusions. Raw performs worse than Sum and often suffers from a lack of improvement due to the parallelization, which is not the case in the other approaches. Surprisingly, in simultaneous move games an increased number of threads does not bring much improvement by any of the approaches. Another interesting observation is that in many cases the result of using 16 threads is worse than when using only eight threads.

Additionally, they compared the results achieved by increasing the number of threads (for the best approach, i. e., Sum10) with the results achieved by increasing the runtime of the non-parallelized version. In contrast to what has been observed in some specialized game players (e. g., Chaslot et al., 2008a; Bjarnason et al., 2009), here a longer runtime of a single thread brings better results than more threads with the same total runtime, i. e., the sum of the runtimes of all threads.

In a later paper Méhat and Cazenave (2011b) compared the root parallelization approach with a tree parallelization. In this case, the tree parallelization does not use shared memory, but operates on a cluster of machines. Thus, only one thread, the master, stores the full UCT tree. It performs the search within the tree and once a leaf node is reached it starts another thread at that position to perform the Monte-Carlo run. Instead of waiting for the result it immediately performs another search within the UCT tree and starts another thread and so on. Once a thread is finished it informs the master of the result of the Monte-Carlo run and is available for another one. The master then updates its tree according to the achieved result and goes on starting new threads until the available time is over. In that case it chooses the move with the best average reward and sends that to the server.

When limiting both approaches to very few threads, i. e., one or two, the root parallelization approach is comparable to, or even better than, the tree parallelization approach. When the number of available threads is increased the tree parallelization gets clearly better in most of the tested games. The only exception is CONNECT FOUR, where the root parallelization achieved better results for any number of threads. In contrast to the root parallelization increasing the number of threads results in a clear advantage for the algorithm, even for the tested maximum of 16 threads, where the performance of the root parallelization tended to drop.

TURBOTURTLE

Unfortunately, so far nothing has been published on the winner of the competition in 2011, TURBOTURTLE by Sam Schreiber. According to private communications, TURBOTURTLE makes use of UCT, but it also tries to identify whether an opponent uses a UCT based algorithm and tries to exploit the problems of such an approach, though we do not know how it identifies such players or how it actually tries to exploit that knowledge.

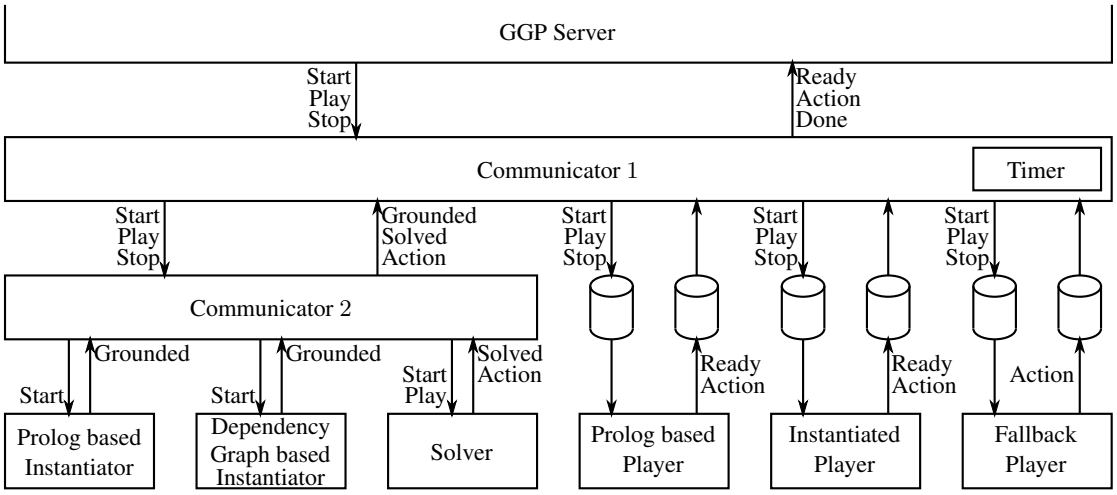


Figure 11.3: The overview of our general game playing agent GAMER.

11.3 The Design of the General Game Playing Agent GAMER

We implemented a general game playing agent, which we decided to call GAMER as well after the success of our action planner of the same name in the International Planning Competition IPC 2008. This agent can be seen as a hybrid player. On the one side is the solver, which takes over as soon as the optimal outcome for the current state is known, on the other side are a number of different players using UCT to find a good move. A schematic overview is provided in Figure 11.3.

We decided to use such a hybrid approach as we cannot be certain that the games can be solved in time (if at all). Still, we wanted to keep the solver as part of our agent, as it brings the possibility to solve a game and then play perfectly.

In the following we will describe the various parts of our agent in more detail. Before that we will start with a brief overview over the GGP server and the communication protocol that is required during one match.

11.3.1 The GGP Server

According to the communication protocol provided in the GDL specification (Love et al., 2006) there is one GGP server that stores all the games as well as the past matches, and handles the ongoing matches. At the beginning of a new match it sends the GDL description of the game to be played to all players using a *Start* message. This message includes the game description, the role each player plays in the current game, as well as two times, i. e., the startup and the move time.

Before the startup time is over the players are supposed to send a simple *Ready* message back to the server, just to denote that they are really online and running and to tell the server that they are now ready to start the actual game. Originally the idea might have been to give the players some time to start everything up and prepare for the beginning of the game, but nowadays most players use the full startup time for first calculations—in case of simulation based players for example for running a lot of simulations to generate a big UCT tree even before the first step.

When the startup time is over a loop of *Play* and *Action* messages starts. The server sends a *Play* message to each player, specifying what moves all players have chosen in the previous state (the first message contains only the string `nil`). The players use this information to update their internal state accordingly. The move time starts when the *Play* message is sent the players. They are supposed to send an *Action* message back to the server before this next move time ends. This contains the move the player has chosen to perform in the current state. If this move is illegal or the message is not received in time the server chooses one of the legal moves at random—though in practice it typically chooses the first legal move its inference mechanism returns. In the competition setting the players are only allowed up to three

errors (too late answers or illegal moves) in a single match; if they do more their score for this match will be set to zero, no matter the reward they actually get in the terminal state. Once the server has received or chosen moves for all participating players it sends another *Play* message.

When the moves of the players result in a terminal state the server sends a *Stop* message instead of a *Play* message. This again contains the moves chosen by all the players, so that the players might use this to calculate the final state and determine the outcome of the game. When they are done they are supposed to send an empty *Done* message to the server to denote that they are done with this match and ready to start the next one.

In the first few competitions a GGP server of the Stanford games group[36] was used, later one of TU Dresden[37] and in the last competition one by Sam Schreiber[38]. The latter two are open source and can be downloaded and installed in order to start a server on a personal machine.

11.3.2 The Communicators

We use two communicators, one to interface between the players and the server, another to interface between the first communicator and the solver and instantiators.

When a new *Start* message is received the first communicator starts the Prolog based player as well as a fallback player (Section 11.3.5), which is only important if something goes wrong in the normal players. It also sets a timer which will fire a short time before the startup time is over and during the normal play before a play time is over, so that an answer can be sent to the server in time.

Additionally it informs the internal second communicator that a new game is to be started.

This second communicator immediately starts the two instantiators (Section 11.3.3). When one of those is finished the communicator stops the other instantiator, starts the solver (Section 11.3.4) and informs the first communicator (by sending a *Grounded* message). If this message is received before the startup time is over that one stops the Prolog based player and instead starts the player using instantiated input (Section 11.3.5). When the solver is finished the second communicator sends a *Solved* message to the first communicator, which stops all running players, because from now on optimal moves according to the found solution can be played.

The *Play* and *Stop* messages are forwarded from the first communicator to the second one, which uses these to update the solver. The communication to the players is a bit more complicated, as somehow we did not get the network communication to work reliably between the communicator (implemented in Java) and the players (implemented in C++), so that we decided to switch over to the use of files on the hard disk. Thus, for each *Play* message a file containing the moves is created, which is evaluated by the players. They also inform the communicator of the move to take via files. The players write the best move so far whenever it changes, and the communicator reads it when the move time is over.

11.3.3 The Instantiators

As we have seen in the experimental evaluation of our two approaches to the instantiation of general games (Section 9.2 both ways of instantiating have their advantages and drawbacks, and we are not yet able to automatically determine when one will work better than the other. Thus, we decided to actually start both instantiators in parallel.

When one instantiator is finished it sends a *Grounded* message to the internal (second) communicator, which then does three things. First, it stops the other instantiator, so that more memory can be used by other processes, because the result of that is no longer important. Second, it starts the solver. Third, it relays the message to the outer (first) communicator, which in turn stops the Prolog based player and starts the one working on the instantiated input in case the startup time is not yet exceeded.

[36] `http://games.stanford.edu`
[37] `http://ggpserver.general-game-playing.de`
[38] `http://code.google.com/p/ggp-base`

11.3.4 The Solver

When the game at hand is fully instantiated and the solver has started the internal communicator informs it of all the moves that have been done so far, if any. The solver generates all the necessary BDDs, i. e., for the initial state, the transition relation, the termination criterion, and the reward rules. Then it applies the moves that were already performed to end up in the current state. When this is done it starts the forward search at that state. The idea behind this is that we do not care about a strong solution for those states that cannot be reached any more, but rather prefer to save some time in the solving process. Any step already performed might decrease the solving time drastically.

We might omit the forward search completely if we decided to use the strong pre-image based approach to solve the games. Nevertheless, in the evaluation of the solvers in Section 10.4.2 we saw that omitting the forward search actually harms the solution process as the runtime often increases a lot. Thus, we decided to use forward search anyway. Given this decision and the results of the layered approach to solving general games we further decided to use the layered approach, as this is not only faster but allows for faster checking if the current state has already been solved.

When the forward search is completed and the actual solving starts, after each backward step the solver checks if the current state has already been solved. Due to the layered approach all it has to do is compare the index of the last solved layer with the layer of the current state. To know the current layer the solver needs only count the number of moves already performed, i. e., the number of *Play* messages it has received.

Once it has found that the current state is solved it first of all informs the internal communicator about this by sending a *Solved* message. The communicator relays the message to the outer communicator which stops all players that might still be running, because with the solution for the current state and all possibly reachable successor states found we can play optimally as long as we follow the moves the solver sends.

To find the moves the solver searches the BDDs containing the solution of the current layer for the current state. From the filename it can determine the optimal outcome. In order to choose an optimal move it applies one move legal after the other until it has found one that results in a successor state that is part of a BDD of the solution achieving the same result in the successor layer. Here we make use of the fact that we stored the terminal states and the non-terminal states in different files. In case we do not get the maximal reward of 100 we will always stick to a move that results in a non-terminal state if that is possible without decreasing the calculated reward, hoping that the opponent will make a mistake enabling us to get a better reward. Given a chosen move it sends the corresponding *Action* message to the internal communicator, which relays it to the outer communicator, which in turn relays it to the server. The next *Play* message is relayed to the solver, which updates its current state, finds it in the solutions of the current layer, and proceeds as before. This is repeated until the game ends.

11.3.5 The Players

Apart from the solver we have three independent players to find a good move, one based on Prolog, another based on the instantiated input, and another one that is merely a fallback in case something goes completely wrong, so that at least some legal move will still be sent to the server.

The Prolog Based Player

The Prolog based player is a UCT player that uses Prolog to evaluate the formulas, i. e., find the legal moves in the current state, calculate the successor state and check if the state is terminal and which rewards the players get in such a case. For the logic evaluation we decided to use SWI Prolog[39], which provides an interface to C++, in which we implemented our player.

For opponent modeling in multiplayer games, we use as many UCT trees as there are players, similar to the approach by Finnsson and Björnsson (2008). In each tree we perform UCT from the corresponding player's point of view. The advantage of this approach over one modeling all players within the same tree is two-fold. First, in case of simultaneous move games we need fewer nodes. For a state where we have n active players with m legal moves per player, storing all possibilities in one tree would need m^n successor nodes, while we need only m nodes in each of the n trees. Second, using only one tree we normally would

[39] `http://www.swi-prolog.org`

need to calculate the average reward achieved when choosing a specific move by each player, so that we would have to combine the averages of several of the successors into one value, which would also be much more time consuming, as this is done automatically in each of the trees in our approach. Nevertheless, this approach also comes with a disadvantage, as we do not know the exact successor state given one player's move, so that we must use Prolog to calculate the successor state in every step within the UCT trees and cannot simply store a pointer to the successor as it might be done in the setting of only one tree for all players.

We found that Prolog's inference mechanism is rather slow in most games (we provided some results for pure Monte-Carlo search in Table 9.1 on page 118 for motivating the use of an instantiator), so that we do not only store the first new state of a Monte-Carlo run in the tree but rather the complete run if the move time is below a certain threshold—45 seconds seems to be a reasonable compromise. If the move time is rather short, the player performs only relatively few runs, so that there is no harm in storing the full runs in terms of memory, i. e., we do not run out of memory, but we can store more information which should help to perform better than with only one added state. If the move time is longer than the threshold we are able to perform more runs, so that the memory might run out if we stored all runs completely. Due to the increased number of runs we still extend the tree reasonably when only adding one new state per run so that in that case we still should be able to find good moves.

We also implemented parallelization. Unfortunately, we did not find a way to get SWI Prolog to work in a shared memory approach, because the knowledge bases of the different threads got mixed up. Thus, we decided to use independent processes and the message passing interface MPI, in this case Open MPI[40] (Gabriel et al., 2004), to send the messages between them. Due to this we are restricted to those parallelizations that do not share any memory, especially the root parallelization or the at-the-leaves parallelization, but this way we are in principle able to run our player on a large cluster of computers, though so far we played only on one (multi-core) machine.

We implemented an approach close to the root parallelization proposed by Cazenave and Jouandeau (2007). The master stores only the root of the tree along with the immediate successor nodes, as in these the counters and the averages of the corresponding moves are stored. The slave processes each have their own UCT trees and perform normal UCT. After each run they inform the master about the chosen moves and the achieved rewards for the players. The master takes this information and merges it into its nodes. After each update it checks if the best action has changed. If that is the case, it writes it into a file so that the communicator can read it once the time is up.

When the communicator informs the master about the chosen moves of all the players it broadcasts this information to all the slaves. Furthermore, it updates its tree by setting the chosen successor nodes as the new roots and generating their successors. Next it receives the number of visits and the average rewards for each successor from each slave. Based on this information it then calculates the total averages and broadcasts these to the slaves, so that those can start the search for the next move to play with a more informed root node.

The only difference between single- and multiplayer games comes with the fact that for single-player games we know that when we have found a path leading to the maximal reward of 100 points we have found an optimal solution, so that we stop the algorithm and submit these moves, one after the other, to the communicator.

We also implemented a simple version of a transposition table. This takes a state in form of a string as the key and returns the corresponding node within the UCT tree, if it exists. The advantage is of course that we need less memory than without it, as we do not need to store duplicates any longer. Also, the information generated on different paths traversing over the same state is combined in that state, so that it should be more precise.

The Instantiated Player

The UCT player that uses the instantiated input consists of three different approaches. We have UCT versions for single-player games, for non-simultaneous multiplayer games and for simultaneous multiplayer games.

[40] `http://www.open-mpi.org`

All versions are parallelized using C++ *pthreads*, so that we can make use of the shared memory. As long as we use only a single machine this is a reasonable approach. We use the algorithm previously described by Edelkamp et al. (2010a), which is similar to Chaslot et al.'s (2008a) tree parallelization with virtual loss.

We use a static allocation of a UCT tree, i. e., we allocate the memory for the nodes of the tree at the start of the algorithm and will never allocate new nodes afterward. The tree here is represented by an array of nodes, so that we can access a certain node by its index within the array. Each node stores the indices of the successor nodes—here some additional memory must be allocated at runtime—as well as their number, the average reward for each player, the number of times this state has been expanded, the hash value of this state, and—in case of a non-simultaneous multi-player game—the index of the player active in this state.

Additionally we store a *hash table* and a *state table*. The hash table stores the index of the node representing the state having the given hash value, while the state table stores the corresponding state. We use the hash table as a kind of transposition table in order to remove duplicates. Whenever a supposedly new state is to be inserted into the tree, we first of all calculate its hash value. For this we apply double hashing, i. e., we use two hash functions h_1 and h_2 and check both the hash table starting at positions $h_1 + i \times h_2$ mod $size$ with $size$ being the size of the tables, a large prime number (in our case 201,326,611), and $i \geq 1$ an integer to successively check different buckets. If the hash table has stored an index at the tested location, and it is the same state—which we can find out by checking the state table—we are done, as the state already is inserted in the tree. If the hash table has stored an index but the corresponding state differs from the new one we continue with the next bucket (by incrementing i). Otherwise, if the hash table has not stored an index at that location this state is new and the corresponding node can be added to the UCT tree, i. e., at the position of the next unused index. This index is also added to this position of the hash table and the actual state to the same position of the state table.

To handle the parallel accesses to the various tables we need two mutexes, one on the index of the next empty position in the UCT tree, and another one on the hash table. When we have reached a leaf of the UCT tree and expand it, we first check if the newly generated successor is really a new state, as we have described before, and lock the hash table so that no other process may access it at the same time. If it is no new state we unlock the hash table and then must only set the index of the successor correspondingly and continue the search within the tree to the next leaf. Otherwise we update the hash and state table, unlock the hash table, generate a new node for the new state, lock the index of the next empty position in the UCT tree, add the node to this position, increase the index to the next empty position and unlock it again. This way only processes that want to add new nodes at the same time may collide and they can only operate sequentially, while all other processes can continue their work undisturbed in parallel.

In the single-player version we store not only the average reward achieved but also the maximal reward ever found on this path. For the UCT search we randomly decide if we use the average score or the maximal score in the UCT formula. When writing the best move into the file for the communicator we always choose the move with the highest maximal reward.

During the instantiation process we already find out if a multiplayer game contains simultaneous moves or not, so that we can handle the two cases differently in this player. In contrast to our Prolog player, in non-simultaneous games we manage only one UCT tree. As in each state only one player can choose a move, we just have to check who is the active player in the state that is represented by the current node. For this we check the first successor node, which contains a pointer to the move that leads to it. This move stores the information who is the active player. For this player we then calculate the UCT values and choose the move with maximal value. Once the movetime is up, we choose the move for which we achieved the highest average reward.

The games containing simultaneous moves are handled similarly. Instead of combining all the successors with the same move for one of the players to generate the total average by choosing that move we simply set the active player randomly and then use the algorithm for turn-taking games, i. e., we choose the move with highest UCT value for that player. Because we choose the active player at random every time we reach a node we hope that overall the move that is best for all players will be chosen more often than the others, though of course we sacrifice a lot of possibly useful information in order to further speed up the search.

The Fallback Player

The last part of the complete agent is the fallback player. This is not actually a true player, but is merely used in case of an emergency.

Internally it stores the current state and updates it with each *Play* message that is received. After such an update it writes all the legal moves, which it finds by querying Prolog, into a file, which the first communicator will read. Before sending the next move to be taken to the server it checks if that move is one of those found to be legal by the fallback player. If it is, everything is fine. Otherwise there is either no move to be sent or it is not legal, so that some bug might have occurred. To still be able to achieve at least some points the communicator then chooses one of the legal moves at random and sends that to the server. If an error occurs early in the game that of course is a problem, and we will likely only play random moves from then onwards, so that we will have rather bad chances for achieving a good terminal state and gaining a high reward. If an error occurs later on it might very well be that random moves will suffice so that we will still get some points. Ignoring the information that a move is illegal and sending it anyway, or sending no move whatsoever, would result in an error of the agent; if an agent produces more than three errors, it gets no points, no matter the terminal state, so that sending random moves might actually help in such a critical case.

11.4 Experimental Evaluation

We performed a number of experiments with our player on our machine, i. e., an Intel Core $i7$ 920 CPU with 2.67 GHz and 24 GB RAM. The CPU is a quad-core CPU, but is treated by the operating system as an oct-core due to Hyperthreading.

In a first set of experiments (Section 11.4.1) we used only the Prolog based player and evaluated the effect of an increased number of threads on the total number of UCT runs and on the initial evaluation. Then we performed the same set of experiments using the instantiated player (Section 11.4.2). Finally, for a number of single- and non-simultaneous two-player games we evaluated the overall hybrid player (Section 11.4.3).

11.4.1 Evaluation of the Prolog Based Player

Here we are only interested in the influence that an increased number of threads or an increased runtime has on the average reward that is stored at the root node. Note that this average reward does not necessarily reflect the reward that will be reached when stopping the search and playing only based on the information already gathered. Especially in case of single-player games this value might be a lot smaller than one would expect, if many different terminal states result in a low reward and only few in a higher reward. However, the hope is that this average will increase the longer the player runs or the more threads are used to perform the UCT search. Even if the average value does not change too much over time we still can be certain that more runs generate a larger UCT tree, so that they will bring an advantage in the long term.

We evaluated the Prolog based player on six different games, three single-player games, namely FROGS AND TOADS, PEG-SOLITAIRE, and MAX KNIGHTS, as well as three non-simultaneous two-player games, namely CLOBBER (on a 4×5 board), CONNECT FOUR (on a 7×6 board), and SHEEP AND WOLF.

Before we get to the results, let us briefly describe the used games. PEG-SOLITAIRE, CLOBBER, and CONNECT FOUR were already described in the course of this thesis. FROGS AND TOADS, which is based on Loyd's FORE AND AFT puzzle (1914, p. 108), is played on a board consisting of two overlapping squared boards. Each board has an edge length of four cells and they overlap on one cell (cf. Figure 11.4). The frogs (here denoted by black circles) start on the left board, the toads (the white circles) on the right board. The goal is to swap the positions of the frogs and the toads. To achieve this the player may either move a piece to an adjacent cell if that is empty or jump over another piece onto an empty cell. Note that no pieces are ever removed, so that only one cell is empty throughout the game. According to Yücel (2009) it takes at least 115 steps to achieve the goal. In the GDL specification the game ends after 116 steps or if the desired goal is reached. The player gets a reward correlated to the number of pieces on the correct side of the board, so that a total of 31 different rewards are possible.

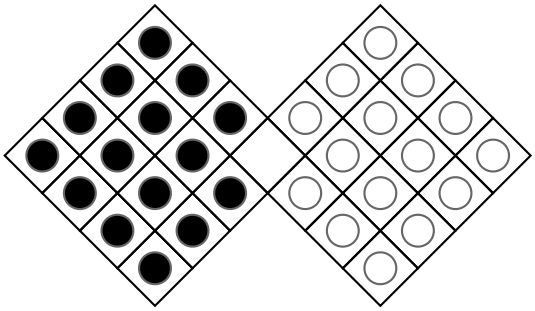


Figure 11.4: The game of FROGS AND TOADS.

The game MAX KNIGHTS was first used at the 2009 GGP competition. It is played on an 8×8 chess like board and the goal is to place as many knights on the board as possible. Similar to the QUEENS puzzle the idea is to place the knights in such a manner that they cannot capture each other. Once a knight is placed in a cell that is attacked by an already placed knight the game ends. Of course, for a human this game is rather trivial. At most 32 knights that cannot capture each other can be placed on the board, as they can only capture pieces on cells of the other color. Thus, it is save to place them all on cells of the same color.. In the GDL specification of this game the player gets a reward correlated to the number of placed knights, i. e., a32 different reward values are defined.

The game SHEEP AND WOLF is also played on an 8×8 chess like board, though in this game only the black cells are used. Initially, four white pieces (the sheep) are placed in the bottom row and one black piece (the wolf) in the topmost row. The sheep can only move forward one cell at a time, the wolf can move forward or backward, but also only one cell at a time. The wolf performs the first move. The goal for the sheep is to prevent the wolf from being able to move. Thus, if all adjacent cells are occupied by sheep the wolf has lost.[41] The goal of the wolf is just the inverse, i. e., it has to try to escape the sheep. The game ends when either the sheep have completely surrounded the wolf or the wolf has escaped. As the sheep can only move forward it is clear that the game ends after at most 56 moves, when all sheep have reached the topmost row. Again, this game is rather trivial for a human, as the sheep can always move in such a way that the wolf cannot get behind them, so that at the end they will be able to push it back to the topmost row and to occupy the adjacent cells. However, a simulation based player may quickly run into trouble because there are only relatively few terminal states that are won by the sheep while many playouts lead to the wolf escaping. The GDL specification distinguishes only between won and lost states, i. e., only two reward combinations are possible.

Concerning the possible rewards, in PEG-SOLITAIRE the player is awarded a number of points reversely correlated to the number of remaining pieces on the board (100 for one piece in the middle, 99 for one piece somewhere else, $100 - i$ for $1 + i$ pieces left on the board, with a minimum of 0 points for eleven or more pieces). In CLOBBER the players get a reward dependent on the number of moves it took them to finish the game. If the starting white player performed the last move it gets roughly $14i$ points if i white pieces are still on the board; if the black player performed the last move it gets roughly $16i$ points if i black pieces are still on the board. The losing player always gets zero points. CONNECT FOUR is a typical two-player zero-sum game with only won, lost, or drawn terminal states, so that only three reward combinations are possible.

All games are at least weakly winnable for all players, i. e., 100 points are possible if they all cooperate in order to maximize that player's points. Thus, for all single-player games it is possible to reach a terminal state with a reward of 100 points, while for the two-player games the outcomes are 42–0 in CLOBBER, 100–0 in CONNECT FOUR, and 0–100 in SHEEP AND WOLF if both players play optimally. All two-player games were evaluated from the point of view of the starting player.

[41] In other words, the sheep hunt the wolf. This sounds strange, but that is what this game is about.

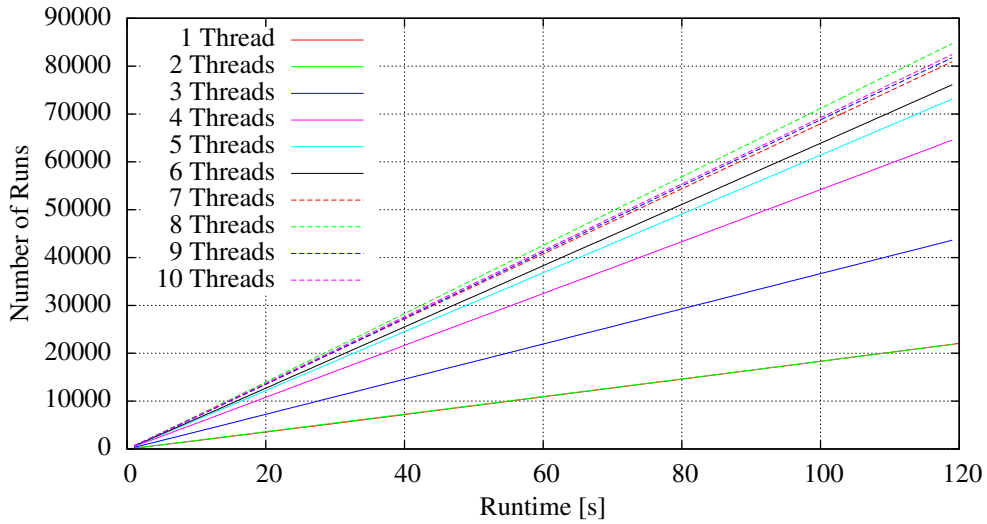


Figure 11.5: Number of UCT runs in CLOBBER by the Prolog based player, averaged over 10 runs.

Table 11.1: Number of UCT runs of the Prolog based player after two minutes, averaged over 10 runs.

Game	1 Thread	2 Threads	3 Threads	4 Threads	5 Threads	6 Threads
FROGS AND TOADS	745	761	1,511	2,256	2,482	2,531
MAX KNIGHTS	25,970	26,164	51,773	77,179	81,861	88,891
PEG-SOLITAIRE	7,917	8,138	15,925	23,770	25,260	28,401
CLOBBER	21,881	21,853	43,635	64,560	73,151	76,123
CONNECT FOUR	19,638	19,844	38,808	57,811	65,263	68,393
SHEEP AND WOLF	4,514	4,720	9,322	13,929	14,921	16,134

Number of Runs

Figure 11.5 depicts the number of UCT runs for varying number of threads during two minutes in the game CLOBBER. The first thing to notice is that the number of runs does not really differ between the use of one and two threads. This is due to the parallelization we chose. For the root parallelization we have a master process and a number of slave processes, which are the only ones that actually perform UCT runs, while the master process collects the results of all the slaves. In the case of a single thread we use a version without any parallelization, i. e., the only process performs all the searches and no master is necessary to collect the results. Thus, the cases for one and two threads are very similar.

Additionally it can be noted that using three threads, i. e., two slaves, the player can perform about twice as many runs as with only one or two threads, i. e., one slave. Similarly, for four threads, i. e., three slaves, the number of runs is increased to a factor of roughly three compared to the single slave version. For five and six threads, i. e., more than we have physical cores, there is still an increase, but it is less prominent. From then on the increase in number of runs gets smaller and smaller, so that for the later experiments we decided to stick to a setting with up to six threads.

In our implementation we have to call Prolog even from within the UCT tree, so that the runtime does not differ much between smaller or bigger UCT trees. Thus, it is plausible that for any number of threads the number of UCT runs increases linearly with the total runtime, which can be clearly seen from the figure.

We performed the same experiment for the other games as well, and all results look very similar. Thus, we do not provide the graphs but only the total number of runs performed after two minutes (cf. Table 11.1).

Single-Player Games

Concerning the average reward determined for the initial state the results are also similar for all six tested games, but actually quite unexpected. In all cases the number of threads has no big influence on the overall

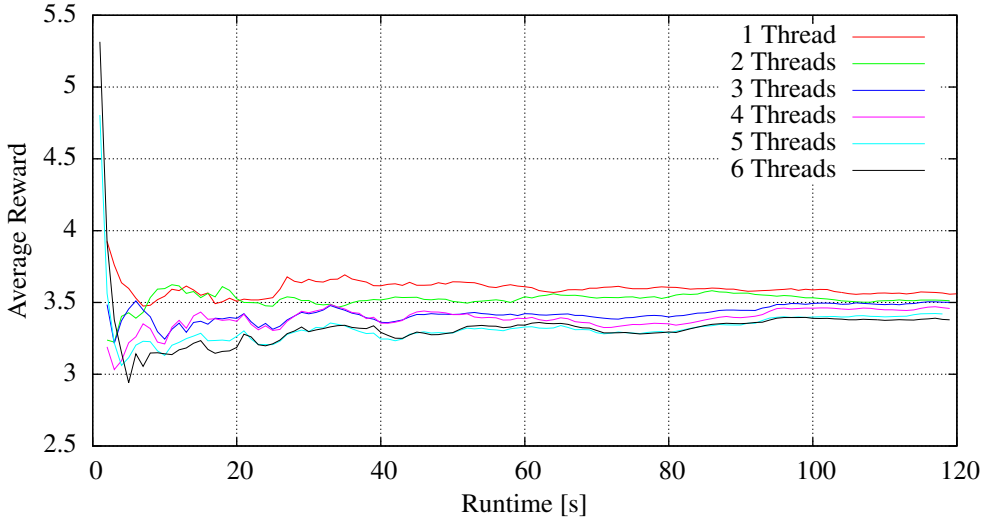


Figure 11.6: Average rewards for FROGS AND TOADS by the Prolog based player, averaged over 10 runs.

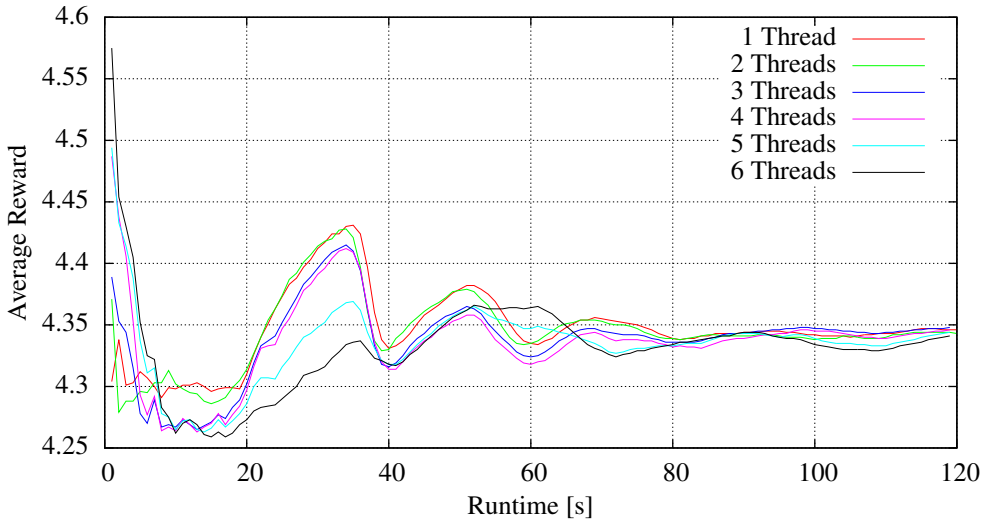


Figure 11.7: Average rewards for MAX KNIGHTS by the Prolog based player, averaged over 10 runs.

evaluation. Neither does it influence the time needed to reach a reasonably stable point, i. e., one where the average reward does not change much any more.

The length of the game steps makes FROGS AND TOADS rather complex. Due to this only few UCT runs can be performed (745 in two minutes for one thread), but still the results are very stable, even after about ten seconds (cf. Figure 11.6). Independent of the number of threads the average reward achieved is close to 3.5. An explanation for this bad result might be the complexity of the game, because there are many possible runs, and only very few result in a good outcome. In the random Monte-Carlo runs it might very well be that the player performs only moves or jumps within the square of pieces of the same color instead of trying to really swap sides. However, these are only the average rewards. Thus, playing based on the information already gathered after 120 seconds should enable the player to reach a much higher reward. What is quite surprising is that here an increased number of threads (and thus runs) slightly decreases the overall average reward. An explanation might be that at first more runs mean that larger parts of the game are analyzed, and thus more terminal states with lower rewards are found. It might be that with a lot more runs the UCT tree grows enough to guide the search to better terminal states more reliably.

At first glance the results for MAX KNIGHTS seem to show a rather strange behavior (cf. Figure 11.7). Here the average oscillates with a decreasing amplitude before finally settling at an average of roughly

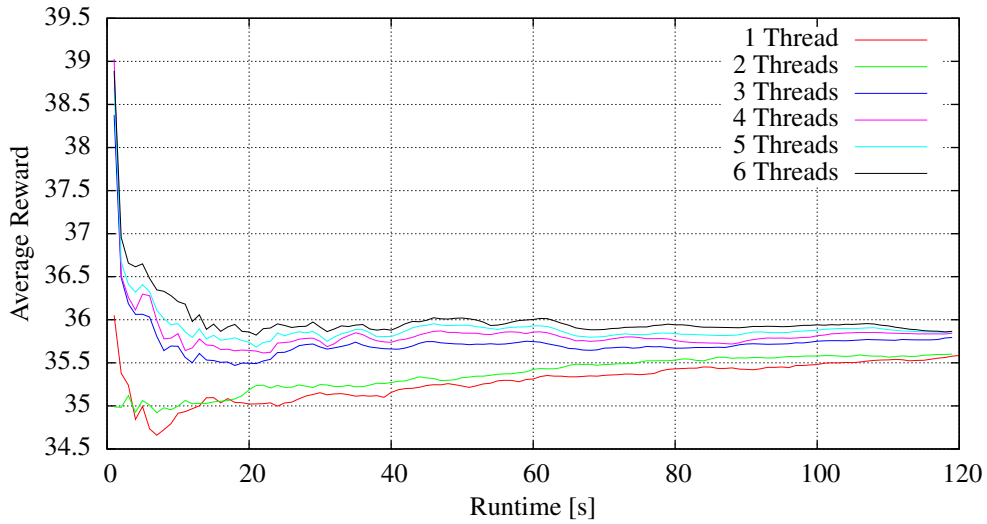


Figure 11.8: Average rewards for PEG-SOLITAIRE by the Prolog based player, averaged over 10 runs.

4.35 after about 80 seconds. With more threads the amplitude gets smaller. Nevertheless, for all cases it is actually quite small; the difference between the first two extrema is less than 0.2 points, so that there is not much difference, especially given the fact that here 100 points are optimal and the achieved average corresponds to the situation of having placed only five or six knights. Note that according to the GDL specification of this game it is actually possible to place a new knight anywhere on the board and only after a move it is determined whether the game ends (if the newly placed knight can capture one of the older ones), so that the random choice in the Monte-Carlo runs might again be an explanation for this bad average value. The more knights are already placed on the board the higher the probability to choose a move that results in the end of the game.

For PEG-SOLITAIRE (cf. Figure 11.8) the overall behavior is again similar to the first games. After roughly 20 seconds most versions are quite stable between 35.5 and 36 points. Only the versions with one and two threads take longer, and the determined average reward for both cases is still increasing after the timeout of two minutes. Also, their results are a bit below all the others throughout the complete runtime. In this case, if more threads are used then the results are a bit better, but again the difference is rather small. From the achieved average value it seems that in PEG-SOLITAIRE lots of playouts actually lead to terminal states achieving many points.

Two-Player Games

For the two-player games we assume that we are the starting player and thus the average reward corresponds to the average outcome of the first player in the terminal states that were reached during the UCT runs.

For CLOBBER (Figure 11.9) after 40 seconds the stable point is reached, and independent of the number of threads the achieved average reward is around 34.5 points. From the beginning the results did not differ much, as even from the first second of search onwards they were between 32 and 35.5. Given that the first player can achieve a reward of 42 in case of optimal play of both players the results of the UCT runs are actually quite close to the optimal estimate.

In CONNECT FOUR (Figure 11.10) the results differ a bit more than in CLOBBER. After about 80 seconds the version using only one thread and after about 40 seconds all other versions achieved a reasonably stable point where the average reward lies between 57 and 59 points. In this game the number of threads influenced the average outcome negatively again, as with more threads the average value is slightly decreased. However, this difference again might not be significant enough to allow strong conclusions.

As we have mentioned earlier, for a human the game SHEEP AND WOLF is rather easy to solve, while for a computer player, at least one based on UCT searches, this seems to be much more difficult (cf. Figure 11.11). Independent of the number of threads, the average reward achieved in the initial state lies between

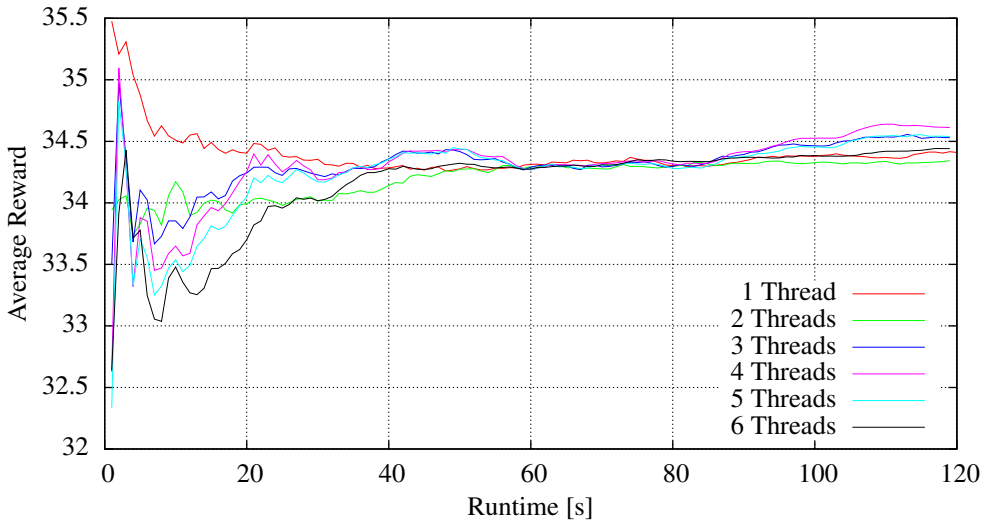


Figure 11.9: Average rewards for CLOBBER by the Prolog based player, averaged over 10 runs.

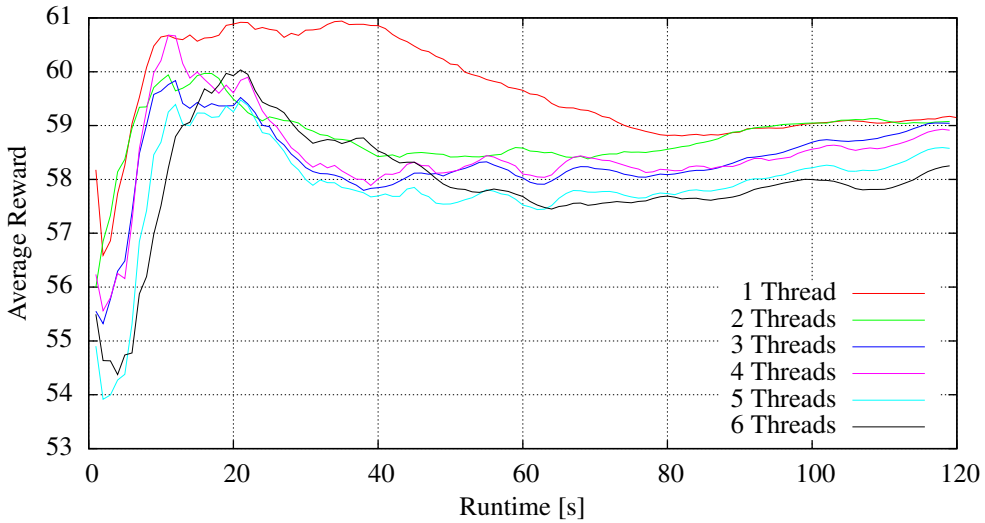


Figure 11.10: Average rewards for CONNECT FOUR by the Prolog based player, averaged over 10 runs.

90 and 95 points for the first player (the wolf), and is stabilized between 91 and 92 points after about 30 seconds. The explanation here clearly is that most lines of play lead to the wolf actually being able to get past the sheep, and only quite few prevent that from ever happening, and these playouts thus dominate the average reward found by the UCT player.

11.4.2 Evaluation of the Instantiated Player

For the instantiated player we used the same set of experiments, i. e., first experiments to find the influence of the number of used cores (ranging again from one to ten) on the number of UCT runs within two minutes for the six games used before, then the influence of the number of cores on the average (or maximal in case of single-player games) rewards within two minutes for the six games.

Number of Runs

Concerning the number of UCT runs we again show only one graph to show the influence of the number of threads over time (cf. Figure 11.12), because for all games the situation is very similar.

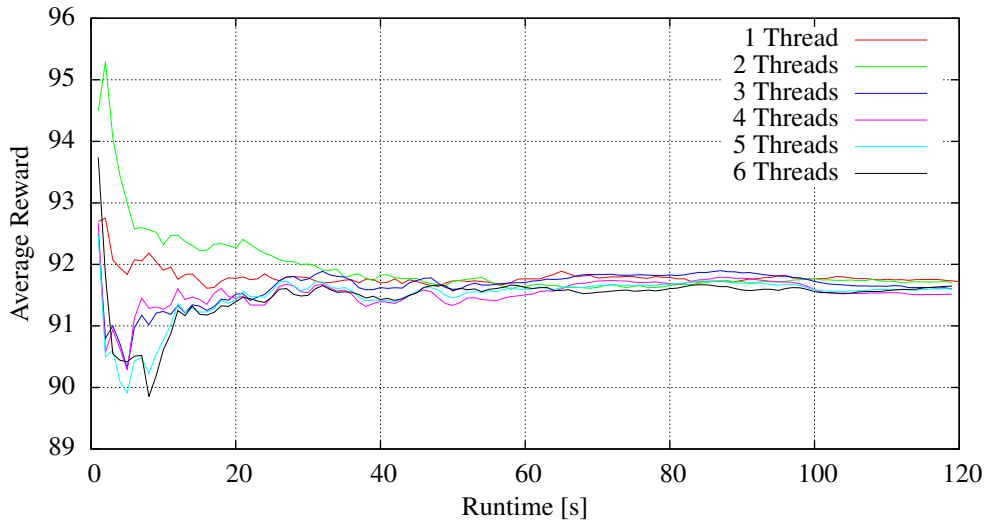


Figure 11.11: Average rewards for SHEEP AND WOLF by the Prolog based player, averaged over 10 runs.

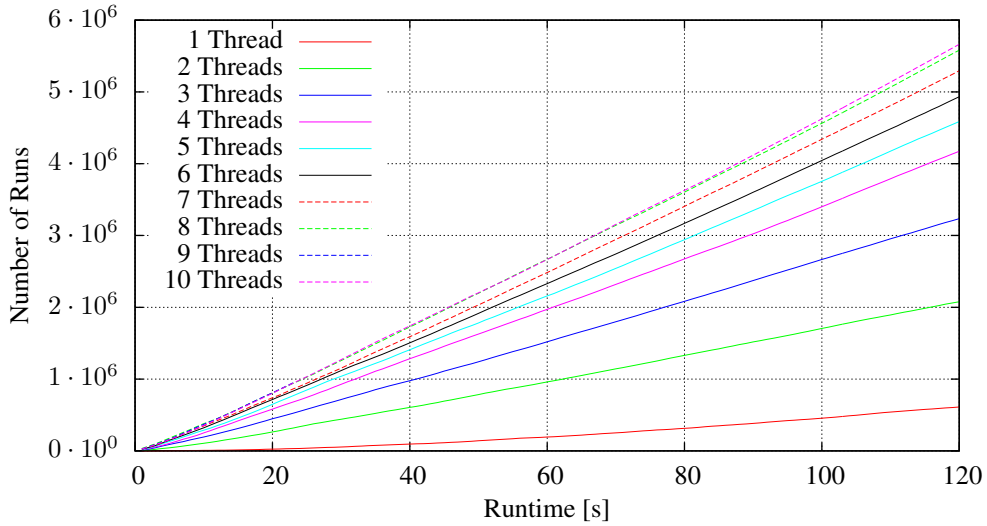


Figure 11.12: Number of UCT runs in CLOBBER by the instantiated player, averaged over 10 runs.

For one thread we get a linear increase in the number of runs with the time played. Though here we store the states within the UCT tree it seems that the overhead required by the Monte-Carlo runs, where the set of legal moves and the successor states must be calculated, is not too big, especially compared to the situation when using Prolog. Otherwise, we would expect to see more than a linear increase in the number of runs over time.

In this setting, when using two threads we gain more than a factor of two in the number of runs. This is likely because in all cases we use only one thread to determine the best move and write it to the hard disk whenever it has finished a UCT run. This way, only one thread is slowed down while the others can operate at full speed running only the UCT runs.

For three threads we arrive at roughly 1.5 times the number of runs as in the two thread setting, for four threads we can double the number of runs. From then on the increase is a lot smaller, similar to the setting in the Prolog based player.

In the other games the situation looks very similar, and the main difference comes with the number of runs for the different games. The results after two minutes are depicted in Table 11.2.

Compared to the Prolog based player we can see a clear improvement in the number of runs, ranging from a factor of 34 in FROGS AND TOADS to 180 in MAX KNIGHTS, though the numbers are not really

Table 11.2: Number of UCT runs of the instantiated player after two minutes, averaged over 10 runs.

Game	1 Thread	2 Threads	3 Threads	4 Threads	5 Threads	6 Threads
FROGS AND TOADS	20,421	38,550	55,262	72,105	79,592	86,800
MAX KNIGHTS	3,132,020	5,879,737	8,649,243	10,885,410	11,784,900	12,547,157
PEG-SOLITAIRE	1,158,908	2,162,479	3,254,122	4,306,140	4,753,630	5,123,357
CLOBBER	611,022	2,078,154	3,234,853	4,174,324	4,587,748	4,934,097
CONNECT FOUR	1,495,015	2,912,314	4,280,195	5,721,257	6,108,360	6,633,747
SHEEP AND WOLF	250,920	536,415	809,008	1,071,300	1,128,661	1,196,076

comparable as the algorithms do not only differ in the inference mechanism, but also in the used datastructures.

Single-Player Games

Concerning the average and maximal rewards, we again compared only the use of one to six threads, as we have seen that more threads do not bring too much more runs, and thus the average rewards will not differ much.

For the average rewards of FROGS AND TOADS (cf. Figure 11.13a) the version using two threads gets stable after about 70 seconds, all the others after about 30 seconds, though there is not much difference anyway. Initially, the average rewards were between 3.35 and 3.65, in the end they are close around 3.5 independent of the number of used threads. Overall, the situation is very similar to one in the Prolog based player.

Concerning the maximal rewards (cf. Figure 11.13b), which we store in the instantiated player as well, we can see that this value increases even at the timeout, though most increases happen during the first 40 seconds. In the end, the maximal reward ever seen ranges from 24 to 27. Thus, in case of single-player games it seems that this is a much better indicator of the reward that will be achieved in the end than the average, as the latter often clearly is negatively influenced by the runs that do not end in terminal states achieving better rewards.

In the average rewards for MAX KNIGHTS (cf. Figure 11.14a) we again see some oscillating, similar to the Prolog based case, though here the amplitudes are even smaller. The more threads we use the faster the amplitude drops. Thus, for one thread it takes about 80 seconds, for two roughly 60 seconds, for three about 40 seconds and for the others between 20 and 30 seconds. Overall, the average rewards lie slightly above 4.3.

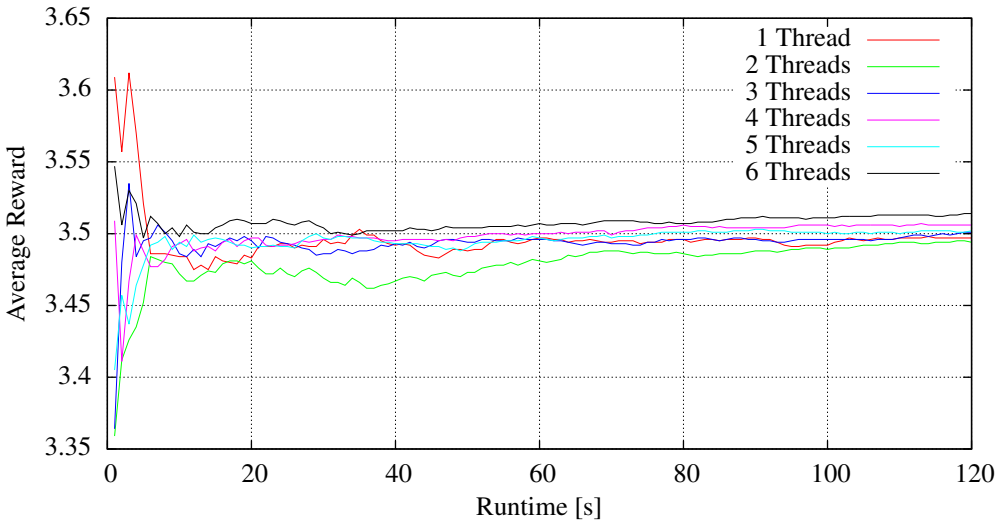
For the maximal rewards we see a similar picture (cf. Figure 11.14b) as we did for FROGS AND TOADS. Some increases occur even close before 120 seconds, though most happen in the first 40 or 60 seconds. The versions using one and two threads are worse than the other versions throughout the runtime. At the end, one or two threads found a maximal reward of 28 points, the others one of 30 points.

Starting with an average reward between 35 and 35.1 in PEG-SOLITAIRE (cf. Figure 11.15) for any number of threads the average reward slightly increases the full runtime, though all the time the versions using more threads get a slightly higher average reward. After 120 seconds the averages range from 35.25 to 35.45.

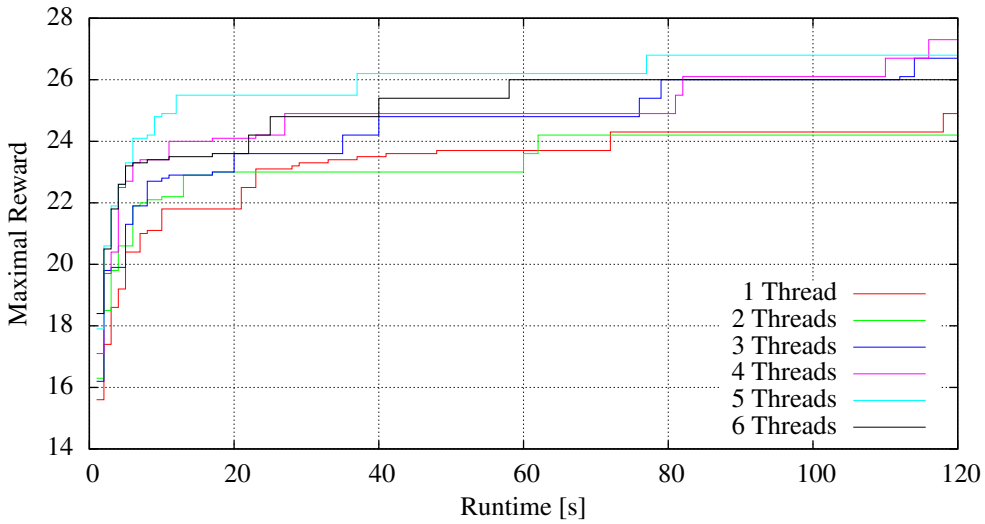
When looking only at the maximal rewards (cf. Figure 11.15b) we can see that during the first five seconds all versions found solutions achieving 90 points. With more time this value increases only slightly up to at most 92. Thus, it must be easy even for simulation based approaches to find good solutions where only two pegs are left on the board, though a situation with only one peg left is a lot more difficult to find. Also, from the average values of more than 35 it seems that the number of situations with smaller rewards is not as high as it seems to be in MAX KNIGHTS and FROGS AND TOADS.

Two-Player Games

For CLOBBER (cf. Figure 11.16) we can see that the results are quite stable after about 30 seconds in the single thread setting, though the average reward still increases slightly until the timeout. For more threads



(a) Average rewards.



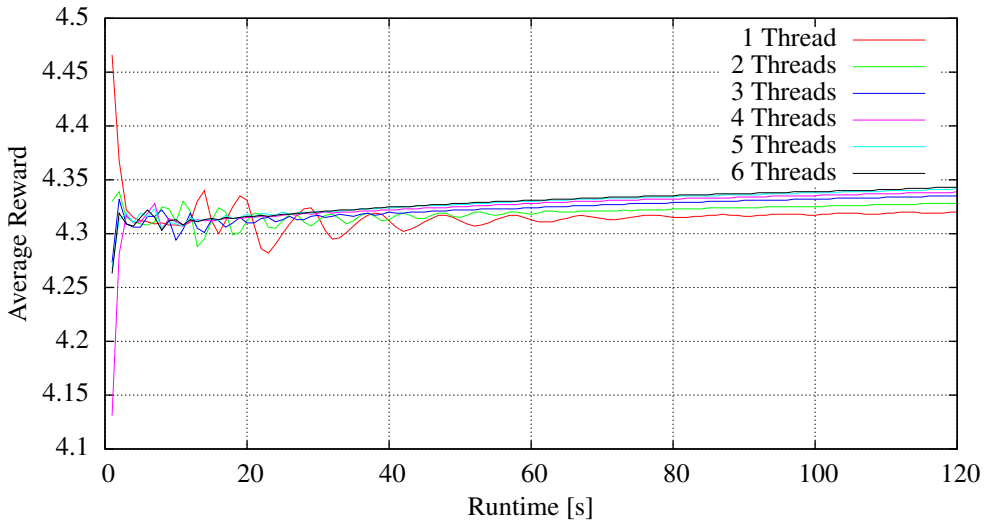
(b) Maximal rewards.

Figure 11.13: Average and maximal rewards for FROGS AND TOADS by the instantiated player, averaged over 10 runs.

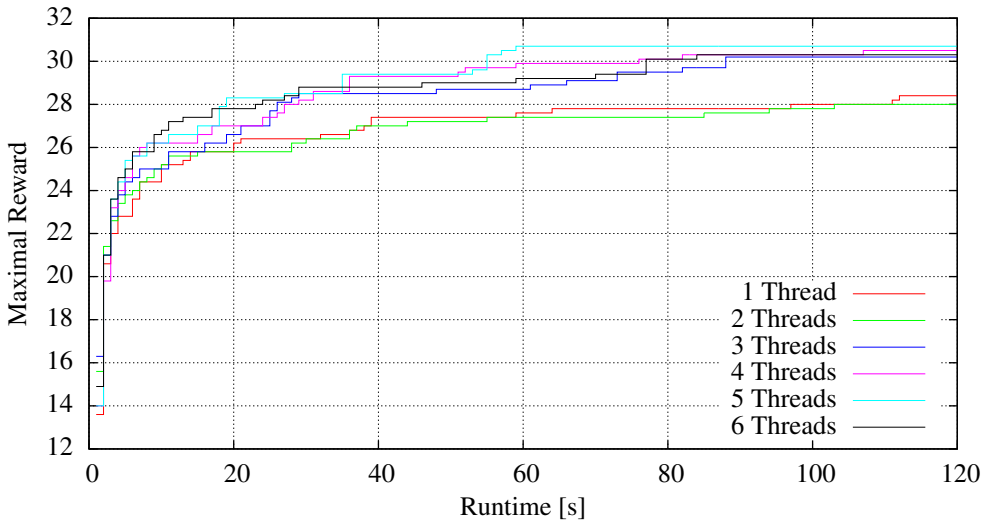
the curves are very smooth already from only roughly five seconds onwards, but in those cases we can see a similar increase in the average values. Here we can see a difference between the number of threads. The more threads we use the steeper the increase, though actually there is not much difference anyway. After 120 seconds the player arrives at an average value between 34.4 and 34.9. Compared to the Prolog based player we see that the averages became stable a lot faster, but the average rewards in the end are similar in both settings.

For CONNECT FOUR (cf. Figure 11.17) we can again see a negative influence by increasing the number of threads. When using more threads we arrive at a lower average value at the root node. In the one thread version the average value still decreases at the timeout, while for the others after 60 to 90 seconds the average rewards are quite stable. Here, the averages range from 57.5 for six threads to 60 for one thread, so that again the instantiated player's results are similar to those of the Prolog based player.

In SHEEP AND WOLF the average values do not change much from the very beginning when using more than one thread, though even when using only one the change is within 0.2 points. For more threads the



(a) Average rewards.



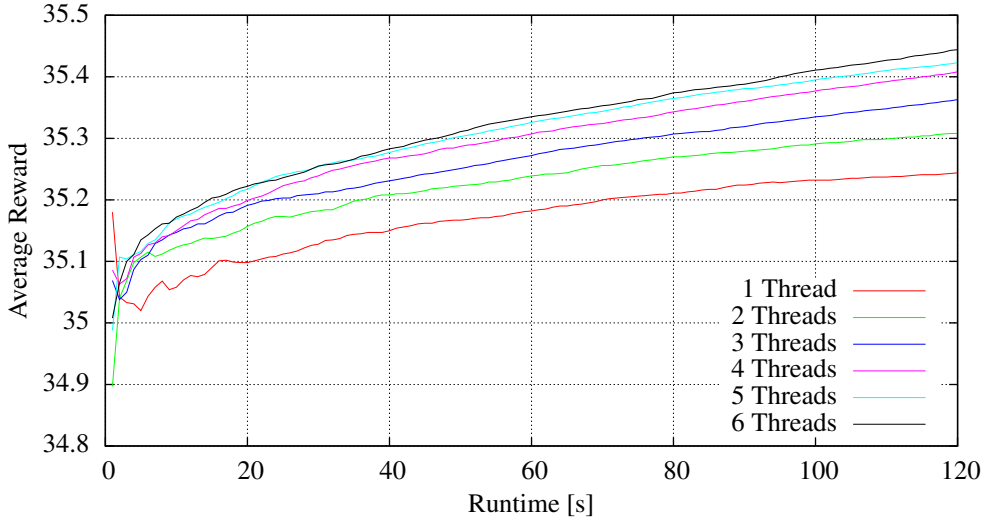
(b) Maximal rewards.

Figure 11.14: Average and maximal rewards for MAX KNIGHTS by the instantiated player, averaged over 10 runs.

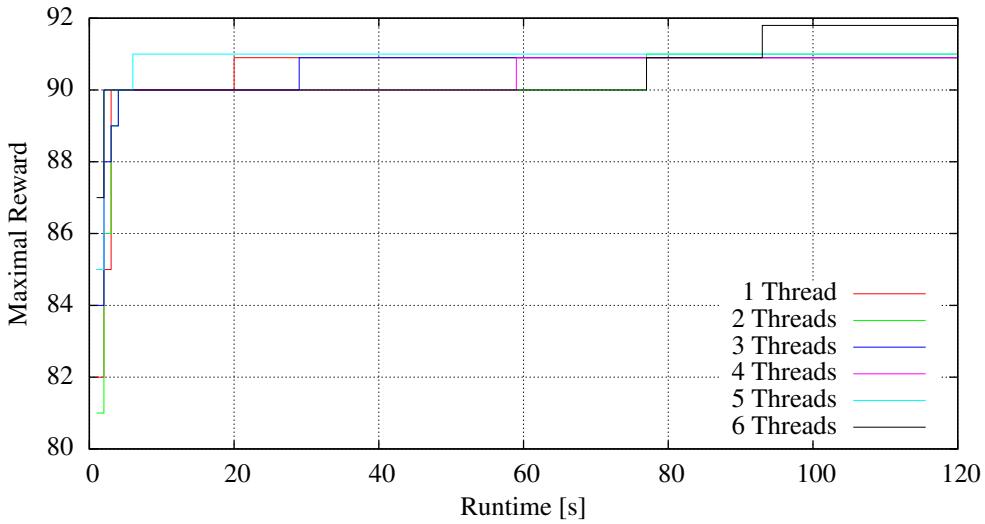
result of nearly 91.8 points is quite stable after about 20 seconds. Note that here, in the instantiated case, the player is also very far away from the first player's outcome of 0 points under the assumption of optimal play of both players and given that the average values do not change any more it is highly unlikely that it will get better later on.

11.4.3 Evaluation of the Full Hybrid Player

For the evaluation of the full player we played a number of different single- and two-player games. In case of single-player games we ran each game 100 times, in order to get significant results. For the two-player games we played with the hybrid player against the Prolog based player using only one thread. Each two-player game we ran 200 times, 100 times with the hybrid player being first player and 100 times with the hybrid player being second player. In all cases we used a startup time of 30 seconds and a move time of 10 seconds.



(a) Average rewards.



(b) Maximal rewards.

Figure 11.15: Average and maximal rewards for PEG-SOLITAIRE by the instantiated player, averaged over 10 runs.

As we have seen in the previous results, when using more than four threads the number of runs increases only slightly, from eight onwards there is no change whatsoever any more. Thus, we decided to use only one to four threads for the hybrid player's UCT algorithms, given that either the solver or the two instantiators would run in parallel anyway, so that we actually have five or even six threads running in parallel when using a UCT player with four threads.

Single-Player Games

For the single-player case we here used four different games. Beside FROGS AND TOADS, MAX KNIGHTS, and PEG-SOLITAIRE we also evaluated TPEG. The last one is nearly the same as PEG-SOLITAIRE, only that the initially empty cell is not in the middle of the board but one diagonally adjacent to it. Here, there is no distinction for the places where the last peg resides, so that we get $100 - 10i$ points for $i + 1$ pegs still on the board, with the minimum being 0 points for eleven or more pegs left. Additionally, the player is

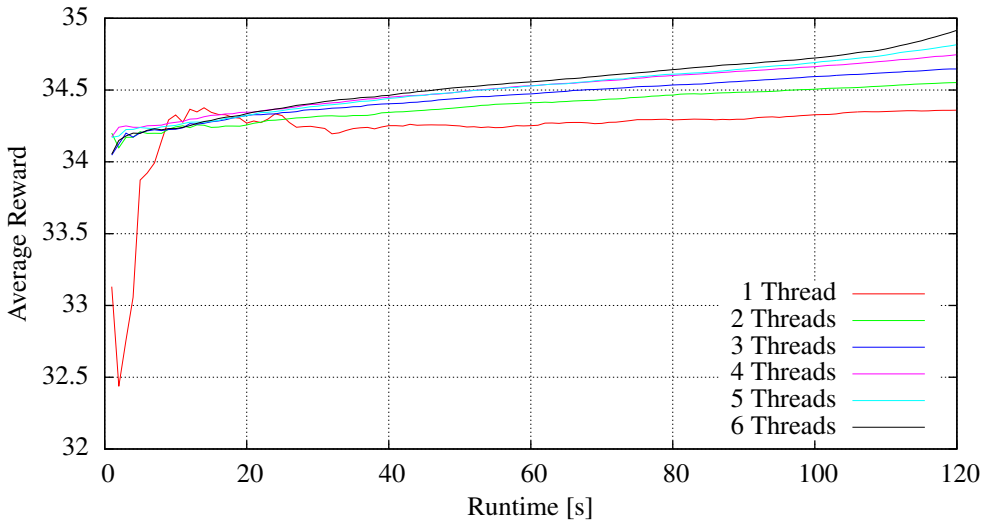


Figure 11.16: Average rewards for CLOBBER by the instantiated player, averaged over 10 runs.

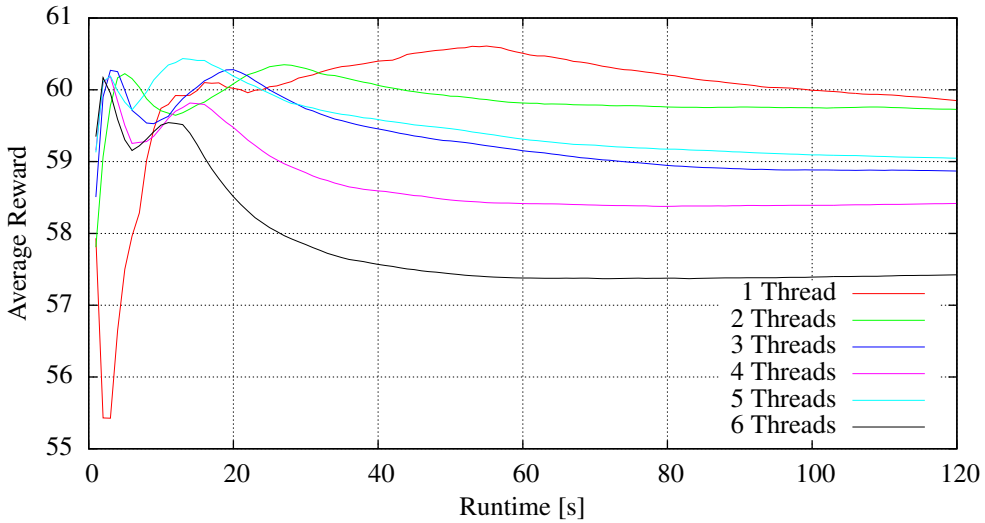


Figure 11.17: Average rewards for CONNECT FOUR by the instantiated player, averaged over 10 runs.

Table 11.3: Results of the player for single-player games comparing different number of threads, averaged over 100 runs each, along with the 95 % confidence intervals.

Game	1 Thread	2 Threads	3 Threads	4 Threads
FROGS AND TOADS	41.63 ± 1.32	42.08 ± 1.45	46.16 ± 1.44	46.91 ± 1.27
MAX KNIGHTS	73.85 ± 2.97	70.70 ± 2.89	75.80 ± 2.70	74.35 ± 2.63
PEG-SOLITAIRE	90.80 ± 0.60	91.50 ± 0.70	91.40 ± 0.68	91.80 ± 0.76
TPEG	97.40 ± 0.95	98.80 ± 0.64	98.30 ± 0.84	99.00 ± 0.59

allowed to commit suicide at any time, which immediately ends the game resulting in 15 points. The results for these four games are depicted in Table 11.3.

We can see that for FROGS AND TOADS we get a significant increase from one to four threads. In this game, the instantiator finished after about two steps, but the solver never finished, so that the Prolog based player was used the whole time.

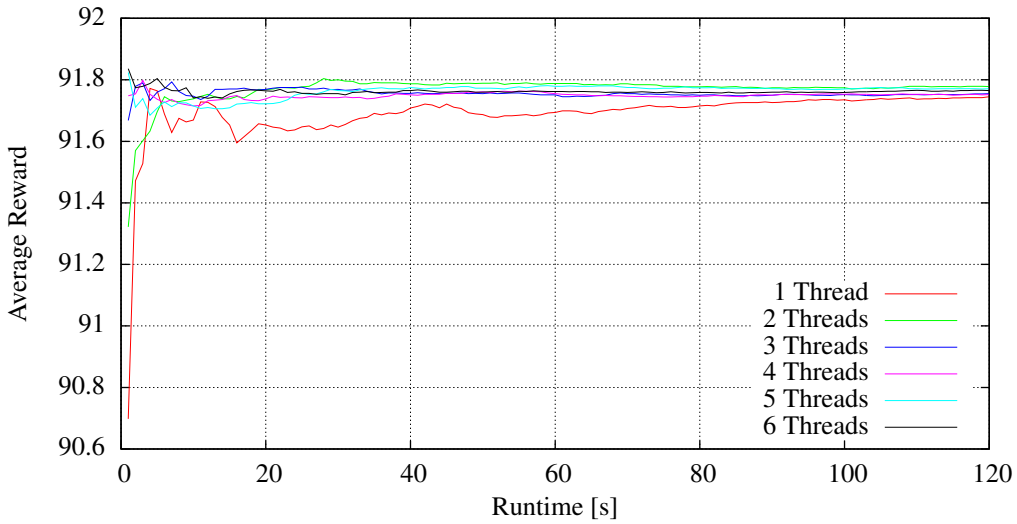


Figure 11.18: Average rewards for SHEEP AND WOLF by the instantiated player, averaged over 10 runs.

In MAX KNIGHTS the instantiator finished after 13 steps, but then the game was solved before the next step was done, so that the solver took over. From the results it seems that the Prolog based player played similar in all cases until the game was solved.

The instantiation of PEG-SOLITAIRE finished even before the first step was done, so that the instantiated player was started. The solver never finished, leaving the whole work to the player. In this game, the results do not differ significantly between varying numbers of threads.

Finally, in TPEG the situation is similar to the one in PEG-SOLITAIRE, i. e., the instantiator finished before the first step but the game was never solved, so that the whole time the instantiated player was activated. Here we see a small increase in the average outcomes from one to four threads. Of course, given that even for one thread we achieve 97.4 points on average prevents the versions using more threads to improve too much over this. Nevertheless, the four threaded version achieves an average of 99.0 points.

Two-Player Games

For the two-player setting we used six different games, namely CHOMP, CLOBBER, CONNECT FOUR, CUBICUP, QUAD, and SHEEP AND WOLF. Apart from QUAD we have already introduced all of these games. The GDL descriptions of CHOMP and CUBICUP allow only for won and lost states, and also for a draw in case of CUBICUP, so that only two respectively three different reward combinations are possible. Overall, apart from CLOBBER all games are zero-sum games.

The game QUAD was orignally invented by G. Keith Still in 1979. It is played on an 11×11 board, where the four corners are missing. Each player has 20 pieces of its color (the quads) as well as seven barriers. During its turn a player may place as many barriers as it likes (and has left) and at the end one quad, then it is the other player's turn. The goal is to establish a square spanned by four own quads (being the square's corners), which may be upright or diagonal and of any size.

In the GDL description we use the board is smaller (7×7 cells without the corners) and each player gets twelve quads and five barriers. The game ends when a player has achieved one of three types of squares (depicted in Figure 11.19), namely a straight one with an edge length of two, a straight one with an edge length of three or a diagonal one with an edge length of three. Alternatively, if no player was able to establish an allowed square and no player has any quads left the game ends as well.

The game is defined to be zero-sum with six different reward combinations. In case one of the players has established a square that player gets 100 points and the opponent 0. Otherwise if the starting player was able to get a square nearly finished, i. e., three corners of an allowed square are placed, it gets 95 points and the second player 5. In all the other cases the number of unplaced barriers of the players are important. If

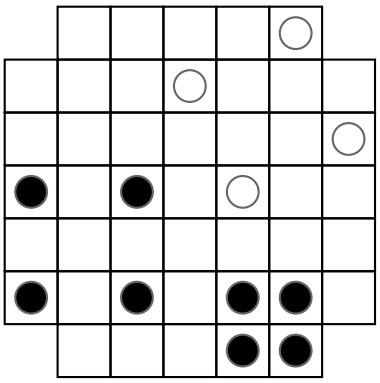


Figure 11.19: The game board of QUAD and the three allowed squares ending the game.

Table 11.4: Results of the player for two-player games comparing different number of threads, averaged over 100 runs each, along with the 95 % confidence intervals. Results denoted by "—" mean that we did not perform that experiment because the game was solved within the startup time, so that no difference in results is to be expected.

Game	1 Thread	2 Threads	3 Threads	4 Threads
CHOMP (1st player)	—	—	—	100.00 ± 0.00
CHOMP (2nd player)	—	—	—	99.00 ± 1.96
CLOBBER (1st player)	39.24 ± 6.90	34.77 ± 7.04	47.85 ± 6.62	45.59 ± 7.28
CLOBBER (1st player's opp.)	24.45 ± 5.90	30.68 ± 6.32	18.91 ± 5.63	22.42 ± 5.92
CLOBBER (2nd player)	62.32 ± 4.76	58.5 ± 5.22	63.85 ± 4.41	60.00 ± 4.63
CLOBBER (2nd player's opp.)	6.30 ± 3.57	8.46 ± 4.01	4.32 ± 2.91	5.15 ± 3.09
CONNECT FOUR (1st player)	43.00 ± 9.75	57.50 ± 9.59	62.00 ± 9.56	61.00 ± 9.61
CONNECT FOUR (2nd player)	22.50 ± 8.17	23.00 ± 8.29	29.00 ± 8.94	40.00 ± 9.65
CUBICUP (1st player)	31.00 ± 8.33	37.00 ± 8.77	50.50 ± 9.29	69.50 ± 7.87
CUBICUP (2nd player)	63.00 ± 8.77	67.00 ± 8.50	76.50 ± 7.81	86.00 ± 6.24
QUAD (1st player)	68.90 ± 9.10	52.00 ± 9.84	63.00 ± 9.51	76.00 ± 8.41
QUAD (2nd player)	47.00 ± 9.83	40.00 ± 9.65	54.00 ± 9.82	56.00 ± 9.78
SHEEP AND WOLF (1st player)	—	—	—	53.00 ± 9.83
SHEEP AND WOLF (2nd player)	—	—	—	100.00 ± 0.00

one player has more barriers left it gets 90 points and the opponent 10; if both have the same number of unplaced barriers both get 50 points.

Most of the games are zero-sum games, so from the average outcome of the 100 games from one player's point of view the average outcome of the other is immediate. Only in case of CLOBBER, where we have more general rewards, we cannot know both players' rewards given only those of one, so that we provide all four results (for the hybrid player being first player and the opponent, as well as for the hybrid player being second player and the opponent). All the average outcomes are depicted in Table 11.4.

In CHOMP the instantiator and the solver finish before the first step is performed. As the number of threads used is only relevant for the players we did not need to run this game for all settings of threads. The game is won by the first player, and thus our agent never loses if it is first player. Even if it is the second player it wins 99 % of the games, which shows that our Prolog based UCT player cannot handle it in a good manner and the idea to keep the solver waiting for a mistake by the opponent is beneficial, as otherwise it might have chosen to take the last piece immediately and thus lost the game.

For the game of CLOBBER the instantiator finishes before the first step, so that instead of the Prolog based player the instantiated player is used for most of the runtime. In the settings of up to three threads the solver finishes a few steps before the game is over (often one to three steps). As the game ends when a

player cannot perform any more moves, and this might happen after varying number of steps, it seems that in the four thread setting the player is able to reach a terminal after fewer steps.

The optimal outcome is $42 - 0$, so that in case of one and two threads the player often plays suboptimal if it is the first player, achieving only 39 and 35 points and allowing the seconds player to get 24 and 31 points on average, respectively. Nevertheless, note that the optimal outcome of 42 points for the first player is still within the $95\,\%$ confidence interval in case of one thread. For three threads the situation gets significantly better, so that the player gets 48 points and the opponent only 19. For four threads the situation is similar, with 46 versus 22 points.

When taking the role of the second player, the instantiated player is a lot better. Here it is able to get around 60 points in any setting, allowing the starting opponent to get only around six points. Surprisingly, here we see no significant influence by the number of used threads.

In CONNECT FOUR the instantiator finishes before the first step but the game is never solved, so that only the instantiated player is responsible for the moves. The optimal outcome is $100 - 0$. In case the hybrid player takes the starting player's role, when using only one thread the player's average is 43 points, so that overall the opponent wins more often. For two threads the average increases to 58, and for three and four threads the player is able to use its advantage and it plays significantly better than the opponent and arrives at averages of 62 and 61 points.

Surprisingly, when playing the second player's role, the situation looks bad for the hybrid player. The achieved average outcome improves with a higher number threads, but even for four threads it reaches only 40 points. This is clearly better than the optimal outcome, but still it seems that here the instantiated player needs four threads in order to play as well as the Prolog based player using only one thread, which is very unexpected given the higher number of UCT runs—the instantiated player with four threads can perform about 290 times the runs the Prolog based player with one thread can perform.

For CUBICUP we do not know the optimal outcome, though at least from a UCT player's point of view it seems that the second player has an advantage, which might make sense as the first player must establish one more cubi cup than the second one in order to win. Similar to the situation in CONNECT FOUR, the instantiator finishes before the first step but the solver never finishes. When using only one thread the hybrid player achieves an average outcome of 31 points as first player and one of 63 points as the second player. When using more threads it plays significantly better, with the best rewards being 70 points as the first player and 86 points as the second player when using four threads.

For QUAD, where the instantiator never finishes, the situation looks strange at first. Given only one thread the hybrid player achieves a higher average than with two threads. Actually, we would expect both settings to be similar, as in case of two threads actually only one thread performs the UCT searches, and the average number of UCT runs is the same in both cases. When looking at the confidence intervals we see that they overlap, so that we cannot be sure that the two threaded version really is worse than the single threaded one. When increasing the number of threads further, the results get better again, with 76 points as the first player and 56 points as the second player for the four threaded version being the maximum.

In SHEEP AND WOLF the situation is similar to the one in CHOMP. Again the game is solved before the first step so that it is played optimally. As we have pointed out before, the game is won by the second player, though UCT typically believes that it is a first player win. From the results we can see that the solver never makes a mistake when being the second player, so that it consistently wins. When being first player it cannot hope to win against an optimal player, but as UCT believes the game to be won by the first player it clearly does not play optimal, so that $53\,\%$ of the games are actually won based on the outcome of the solving process.

11.4.4 Conclusion

So far, our UCT players are pretty basic. Most of the improvements such as MAST (Finnsson and Björnsson, 2008) or RAVE (Gelly and Silver, 2007) are not yet used in them. However, we can say that the hybrid concept works very well. In case a game can be solved before the first move is played, GAMER is clearly able to use its advantage and plays optimally. Due to the decision to wait for an opponent's mistake it often is able to win in games that it would lose if both players played optimally. Even if the solving takes longer it is able to play better than using only UCT.

Concerning the number of runs the two UCT players scale nearly linearly when increasing the number threads or the runtime. Operating on a stronger machine with more processors or cores is thus very likely to further increase the number of runs. Furthermore, we have seen that the use of the instantiated player greatly increases the number of performed UCT runs compared to the Prolog based player. Even though the average rewards stored at the root node do not change much over time, the size of the UCT tree increases with each run, so that an increased number of runs often brings an advantage in practice.

Chapter 12

Conclusions and Future Work

The Road goes ever on and on
Down from the door where it began,
Now far ahead the Road has gone,
And I must follow if I can,
Pursuing it with eager feet,
Until it joins some larger way
Where many paths and errands meet.
And whither then? I cannot say.

John R.R. Tolkien, *The Fellowship of the Ring*
(*The Lord of the Rings*, Book 1)

To wrap up let us briefly recapitulate what this thesis was about. The overall topic was symbolic search, which we have applied in to AI topics, action planning and general game playing. Apart from some theoretical results on the BDD sizes we have proposed a number of new algorithms or improvements of existing ones.

For classical planning we have implemented an automated planner, which participated in two international planning competitions and was even able to win the first one. Though in the second competition it did not fare as well we were able to further enhance it, so that now it can nearly cope with explicit heuristic search based planners, at least in the set of general action cost domains of IPC 2011. For net-benefit planning we also implemented a new planner, which participated only in 2008 and was able to win that track.

For general game playing we have proposed an instantiator in order to get rid of the variables of the GDL input, so that its output can be used in symbolic search as well as in an explicit state player. Where most players so far resided to Prolog in order to infer knowledge about the game at hand we can use the instantiated input to generate the same knowledge in a much shorter time. Additionally we have proposed a number of techniques for strongly solving single- and non-simultaneous two-player games by use of symbolic algorithms. These algorithms are also used in the context of our player, which follows a hybrid approach. On the one hand it has the classical UCT based players, one using Prolog the other the instantiated input, on the other hand, it has the solver, which often can solve the games after few steps and then enables the player to play optimally.

In Section 12.1 we will recapitulate the contributions of this thesis in more detail.

Even though we have covered a wide range of possibilities, there are still many aspects where future research might bring interesting new results. In Section 12.2 we point out some of the possible future research avenues we can envision.

12.1 Contributions

With this thesis we have made a number of contributions. In this section we will briefly summarize all of them. While some are interrelated we will try to cover every one in a single small paragraph. These we have ordered not by their importance but by the order in which they appeared in the various chapters of this thesis.

Analysis of BDDs in Planning Domains

We have evaluated BDD sizes for representing the reachable states in the context of four classical planning domains, the $n^2 - 1$-PUZZLE, BLOCKSWORLD, GRIPPER, and SOKOBAN. We could distinguish two different cases, one in which the BDDs have at least an exponential number of nodes and one where they can be polynomial sized. The first case is apparent in the $n^2 - 1$-PUZZLE and BLOCKSWORLD, both of which are based on permutations for which BDDs were already known to work bad. GRIPPER is a problem of the second case, where the BDDs can bring an exponential saving if a good variable ordering is chosen. SOKOBAN also belongs to this case, but only if we accept a superset of the actually reachable states. For this second case of polynomial sized BDDs we have provided some new theoretical insights and have evaluated them experimentally as well.

Analysis of BDDs in CONNECT FOUR

In the context of the game CONNECT FOUR and the more general k-IN -A -ROW games we have established two results. First, if we ignore terminal states the set—or in case of CONNECT FOUR a superset—of the reachable states can be represented by polynomial sized BDDs. Secondly, the terminal states require a BDD of exponential size. Especially the proof of this might bring new insights for lots of other games, as the setting for which the BDD grows exponentially is very general. It requires only two horizontally or vertically adjacent cells to be occupied and appears in a similar form in lots of games.

Implementation of a Classical Planner

We have implemented a full-fledged planner for classical planning based on symbolic search. It originally performed BDDA* (Edelkamp and Reffel, 1998) in case of domains with general action costs and symbolic BFS in case of unit-cost domains, though we have seen that using BDDA* in the unit-cost cases slightly improved the results.

We participated in two international planning competitions. In 2008 we were able to win, while in 2011 we were quite far from the top. Nevertheless, the results of the 2011 competition have led to several further improvements in the planner, so that it is now quite competitive in the domains with general action costs, where it is nearly as good as the planners based on a whole bunch of heuristics, and can easily outperform those based on only one heuristic—mainly due to the savings in terms of memory.

Partial Pattern Databases for Classical Planning

We have proposed and implemented the symbolic version of partial pattern databases (Anderson et al., 2007). The calculation of those can be interrupted at any time, so that in our planner we use half the available time for the partial PDB generation and the remainder for the forward search using BDDA*.

Automatic Abstraction Calculation for Classical Planning

In our first implementation of the partial PDBs we did not apply any abstraction, so that the pattern actually corresponds to the full set of variables. However, we can create the (partial) PDBs for the case of abstraction as well.

Based on previous work by Haslum et al. (2007) we generated an automatic mechanism to abstract the planning task at hand. Especially, due to the use of BDDs we are often able to generate lots of different pattern databases in reasonably short time and evaluate them in order to decide which variables to take into the pattern and which to abstract away.

Connection Between Causal Graph and Variable Ordering

By observing the connection between the causal graph and the change of the dependency of different variables we were able to greatly improve the variable ordering of our planner, which helps it to find more solutions in a shorter time.

Symbolic Algorithm for Net-Benefit Planning

For net-benefit planning we have implemented a new symbolic algorithm to find optimal plans. Starting with a symbolic branch-and-bound algorithm we extended it step by step until finally we arrived at the desired goal. Net-benefit planning can be seen as closer to general game playing, especially if we do not use action costs. In those cases the only reward comes with the achieved soft goals, so that a similar approach might also be used in the context of general single-player games.

Based on this optimal symbolic net-benefit algorithm we also implemented an optimal net-benefit planner, with which we participated in IPC 2008, and were able to win that track as well. Unfortunately, in 2011 there were not enough participants, so that the track got canceled.

Instantiator for General Games

We have implemented two different approaches in order to instantiate general games. The problem with the game description language GDL is that it contains variables in Boolean formulas, so that most players use Prolog's inference mechanism or similar approaches in order to generate the important information. For BDDs we need fully grounded input. Thus, the use of an instantiator is very important for us.

The two versions we have proposed make use of Prolog and a dependency graph structure, respectively, in order to determine supersets of reachable fluents, moves, and axioms. Both bring advantages or disadvantages in certain domains, but so far we have not been able to automatically determine when to use which approach. Nevertheless, given both instantiators we can already instantiate a large number of games.

Symbolic Algorithm to Solve General Single-Player Games

We have proposed an approach to strongly solve general single-player games, i. e., to find the optimal outcome for every reachable state. This approach mainly makes use of a number of backward searches using symbolic BFS, with which we are able to solve many different games.

Symbolic Algorithms to Solve General Non-Simultaneous Two-Player Games

For strongly solving general non-simultaneous two-player games we were only aware of a symbolic approach for two-player zero-sum games. We proposed two new approaches for more general rewards. For both we need a total order of the possible reward combinations for both players. We have identified two reasonable orderings (either maximizing the own reward or maximizing the difference to the opponent's reward), for both of which we can see situations arising in a competition setting. Based on these orderings we solve the games in a retrograde manner.

The first approach makes use of strong pre-images, while the second one is dependent on a layered BFS in forward direction. For the strong pre-image based approach we have seen that we can skip the BFS in forward direction, but doing so often results in a longer runtime, so that this is not really a disadvantage of the layered approach. What is important for the latter is an incremental progress measure. If one exists we can be sure that it does not generate any duplicates, otherwise we might end up generating the same state over and over again, which requires us to also solve it over and over again. In the games that incorporate such an incremental progress measure the layered approach outperforms the first one, especially in case of more complex games. In those complex games the layered approach saves up to 50 minutes compared to the one using strong pre-images, while for the smaller games it loses only a few seconds, which does not matter in practice.

Implementation of a Hybrid Player

Though this thesis is mainly concerned with symbolic search, speaking of general game playing without an implementation of an actual player seems strange. Thus, we decided to create a hybrid player, i. e., one that uses the symbolic search solver on the one hand and a simple UCT based player on the other hand. Later we decided to extend this approach. For the solver we already were dependent on grounded input, so that we had to run the instantiators anyway. We decided to use the grounded input for another UCT player, one that does not require any Prolog inferences but can find the important information a lot faster by using the instantiated input. This helped to improve the speed with which the various UCT runs were performed.

Our main focus in this work was on symbolic search, which clearly does not work well in a simulation based player. Thus, so far the players are still pretty basic. However, we have shown that the hybrid concept already works very well. Given this concept we can already see several ways to improve it, for example by using the result of the solver as an endgame database for the players.

12.2 Future Work

We can envision several possibilities to continue research from the basis we have created with this thesis. There are still some open questions and possible extensions to our approaches, some of which we will present in this section.

First of all, some more insights into the workings of BDDs seem mandatory. There are cases where BDDs work well, but there are also cases where BDDs work bad, sometimes even worse than explicit-state approaches. In this thesis we found some criteria when BDDs keep to polynomial size and when they tend to grow exponentially. Especially the result for the termination criterion of CONNECT FOUR might give us important pointers to automatically decide whether to use BDDs or not. Nevertheless, there are many domains for which we see certain behaviors—good or bad—of the BDDs, but we do not know why these behaviors appear. Thus, more research in this direction will be very helpful for any decision of BDDs versus explicit-state search.

Given the BDDs for the sets of reachable states we might use them as perfect hash functions in order to improve explicit search. Such hash functions have been proposed by Dietzfelbinger and Edelkamp (2009), but so far they have not been implemented. We expect that with them we could bring explicit and symbolic search closer together and this way maybe solve more games.

In action planning the kind of abstraction we use often decreases performance compared to the full pattern. In another approach (Edelkamp et al., 2012b) we already tried to combine the merge-and-shrink heuristic (Helmert et al., 2007) with our planner. However, the results so far were not too convincing. Given that we often find more solutions if we do not abstract we decided to implement a symbolic bidirectional version of Dijkstra's algorithm (Edelkamp et al., 2012a), where the direction of the next step is determined based on the runtimes of the last steps, similar to the bidirectional BFS case. Surprisingly, that is the approach where we found the highest number of solutions in the IPC 2011 benchmarks. Thus, it seems that either blind search works better in case of symbolic search than the more informed heuristics, or we just did not find a good heuristic so far. Some further research in this direction seems necessary in order to get a clearer picture.

In general game playing we can use the BDDs generated by the solver as endgame databases, so that we can make use of them even before the actual solving is finished. Instead of performing the Monte-Carlo runs to a terminal state they can stop when a state is reached that is already solved. For the layered approach it is easy to decide whether the current state of the Monte-Carlo run is solved, as we need only to know the BFS layer it is in and the solved BFS layer closest to the initial state. Nevertheless, here a problem might be the forward search of the solver, as in lots of games that already takes longer than the complete game time. For the approach based on strong pre-images we can skip the forward search. As we have seen in the evaluation this is not reasonable if we want to completely solve the game. Still, it might bring us some steps from any terminal state backwards toward the initial state. In theory, each step the Monte-Carlo runs do not need to take will save some time, and this saved time of course sums up over many runs. But for this to work we need some way to identify if the state of a Monte-Carlo run is already solved or not. In other words, we must evaluate the BDDs containing the solved states, and this might very well take a lot of time.

Thus, using this kind of endgame database will only make sense if the overhead due to the lookup within the BDDs is compensated by the reduced number of steps of the Monte-Carlo runs.

In a previous paper (Edelkamp and Kissmann, 2008e) we have implemented the results of the strong pre-image based solver as an endgame database for a UCT player. In that case we have implemented a player using BDDs. Of course, this is not a good idea as UCT expands only single states while BDDs work well best when expanding sets of states. Nevertheless, those results might be seen as a kind of proof of concept, because for the games we have tested we have seen that more UCT runs can be performed in the same time.

In some special settings of model checking the approaches we have implemented in general game playing might be of use. For example, in the setting of μ-calculus parity games we have successfully applied the algorithm for solving two-player zero-sum games (Bakera et al., 2009). It is reasonable to assume that the newer algorithms can be applied in a similar manner.

Another important topic is to bring action planning and general game playing closer together. As we have seen, both are very similar under certain circumstances. Still, algorithms that are good in the one domain are only rarely used in the other. For example, it might help a lot to use the efficient classical planners in order to generate good results for single-player games. It is often assumed that UCT is bad in this setting, so that such a change of paradigm might actually improve the players. A first step to do so would be to translate GDL to PDDL, in order to use the planners as they are. Unfortunately, so far no such translation has been done and it seems rather difficult to achieve, though we believe that it is possible.

A relatively new trend in general game playing is the use of randomness and partial observability, introduced by Thielscher (2010) as GDL-II. In case of partial observability players often calculate so-called *belief sets*, i. e., sets of states that might be true in the current situation. Given that BDDs work well in case of large state sets, it might be that they bring some advantages in this setting.

When turning away from BDDs there is still lots of work to do in the GDL-II setting, especially as so far mainly some theoretical analyzes and only very few actual players have been published. We organized a first German Open in General Game Playing in 2011 where we ran a GDL-II track for the first time in an international competition. At that time only three teams participated at the GDl-II track, and only one player was really stable, while the others quite often produced errors. Thus, here research is just beginning.

Another topic further away from BDDs is the questions of when to use minimax and when to prefer UCT search. As we have seen, UCT works great in GO, both work similar in AMAZONS, and minimax is clearly better in CHESS. One criterion is that for CHESS very good evaluation functions are known, which is not the case in GO. But there are very likely further criteria that influence the effectiveness of the two approaches, which might give important insights into the algorithms.

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Appendix A

Planning and Game Specifications of the Running Example

A.1 Classical Planning Unit-Cost Actions

A.1.1 The Domain Description

```
(define (domain desertClassic)
 (:requirements :typing)
 (:types person location)
 (:constants
    gunslinger mib - person
 )
 (:predicates
    (position ?p - person ?l - location)
    (adjacent ?l1 ?l2 - location)
    (caughtMIB)
 )

 (:action move
    :parameters (?l1 ?l2 - location)
    :precondition
    (and
        (position gunslinger ?l1)
        (adjacent ?l1 ?l2)
    )
    :effect
    (and
        (not (position gunslinger ?l1))
        (position gunslinger ?l2)
    )
 )

 (:action catchMIB
    :parameters (?l - location)
    :precondition
    (and
        (position gunslinger ?l)
        (position mib ?l)
```

```
    )
    :effect
    (caughtMIB)
 )
)
```

A.1.2 The Problem Description

```
(define (problem desertClassicProb)
 (:domain desertClassic)
 (:objects
    p0 p1 p2 p3 p4 p5 p6 p7 p8 p9 - location
 )

 (:init
    (position gunslinger p0)
    (position mib p9)
    (adjacent p0 p1)   (adjacent p1 p0)
    (adjacent p0 p2)   (adjacent p2 p0)
    (adjacent p0 p3)   (adjacent p3 p0)
    (adjacent p1 p3)   (adjacent p3 p1)
    (adjacent p1 p4)   (adjacent p4 p1)
    (adjacent p2 p3)   (adjacent p3 p2)
    (adjacent p2 p5)   (adjacent p5 p2)
    (adjacent p3 p4)   (adjacent p4 p3)
    (adjacent p3 p5)   (adjacent p5 p3)
    (adjacent p3 p6)   (adjacent p6 p3)
    (adjacent p4 p6)   (adjacent p6 p4)
    (adjacent p4 p7)   (adjacent p7 p4)
    (adjacent p5 p6)   (adjacent p6 p5)
    (adjacent p5 p8)   (adjacent p8 p5)
    (adjacent p6 p7)   (adjacent p7 p6)
    (adjacent p6 p8)   (adjacent p8 p6)
    (adjacent p6 p9)   (adjacent p9 p6)
    (adjacent p7 p9)   (adjacent p9 p7)
    (adjacent p8 p9)   (adjacent p9 p8)
 )

 (:goal
    (caughtMIB)
 )
)
```

A.2 Classical Planning With General Action Costs

A.2.1 The Domain Description

```
(define (domain desertClassicCosts)
 (:requirements :typing :action-costs)
 (:types person location)
 (:constants
    gunslinger mib - person
 )
```

```
 (:predicates
    (position ?p - person ?l - location)
    (adjacent ?l1 ?l2 - location)
    (caughtMIB)
 )
 (:functions
    (total-cost) - number
    (moveCost ?l1 ?l2 - location) - number
 )

 (:action move
    :parameters (?l1 ?l2 - location)
    :precondition
    (and
        (position gunslinger ?l1)
        (adjacent ?l1 ?l2)
    )
    :effect
    (and
        (not (position gunslinger ?l1))
        (position gunslinger ?l2)
        (increase (total-cost) (moveCost ?l1 ?l2))
    )
 )

 (:action catchMIB
    :parameters (?l - location)
    :precondition
    (and
        (position gunslinger ?l)
        (position mib ?l)
    )
    :effect
    (caughtMIB)
 )
)
```

A.2.2 The Problem Description

```
(define (problem desertClassicCostsProb)
 (:domain desertClassiCosts)
 (:objects
    p0 p1 p2 p3 p4 p5 p6 p7 p8 p9 - location
 )

 (:init
    (position gunslinger p0)
    (position mib p9)
    (adjacent p0 p1)   (adjacent p1 p0)
    (adjacent p0 p2)   (adjacent p2 p0)
    (adjacent p0 p3)   (adjacent p3 p0)
    (adjacent p1 p3)   (adjacent p3 p1)
    (adjacent p1 p4)   (adjacent p4 p1)
    (adjacent p2 p3)   (adjacent p3 p2)
```

```
    (adjacent p2 p5)   (adjacent p5 p2)
    (adjacent p3 p4)   (adjacent p4 p3)
    (adjacent p3 p5)   (adjacent p5 p3)
    (adjacent p3 p6)   (adjacent p6 p3)
    (adjacent p4 p6)   (adjacent p6 p4)
    (adjacent p4 p7)   (adjacent p7 p4)
    (adjacent p5 p6)   (adjacent p6 p5)
    (adjacent p5 p8)   (adjacent p8 p5)
    (adjacent p6 p7)   (adjacent p7 p6)
    (adjacent p6 p8)   (adjacent p8 p6)
    (adjacent p6 p9)   (adjacent p9 p6)
    (adjacent p7 p9)   (adjacent p9 p7)
    (adjacent p8 p9)   (adjacent p9 p8)
    (= (moveCost p0 p1) 2)    (= (moveCost p1 p0) 2)
    (= (moveCost p0 p2) 1)    (= (moveCost p2 p0) 1)
    (= (moveCost p0 p3) 10)   (= (moveCost p3 p0) 10)
    (= (moveCost p1 p3) 2)    (= (moveCost p3 p1) 2)
    (= (moveCost p1 p4) 7)    (= (moveCost p4 p1) 7)
    (= (moveCost p2 p3) 4)    (= (moveCost p3 p2) 4)
    (= (moveCost p2 p5) 12)   (= (moveCost p5 p2) 12)
    (= (moveCost p3 p4) 3)    (= (moveCost p4 p3) 3)
    (= (moveCost p3 p5) 2)    (= (moveCost p5 p3) 2)
    (= (moveCost p3 p6) 7)    (= (moveCost p6 p3) 7)
    (= (moveCost p4 p6) 6)    (= (moveCost p6 p4) 6)
    (= (moveCost p4 p7) 5)    (= (moveCost p7 p4) 5)
    (= (moveCost p5 p6) 7)    (= (moveCost p6 p5) 7)
    (= (moveCost p5 p8) 3)    (= (moveCost p8 p5) 3)
    (= (moveCost p6 p7) 1)    (= (moveCost p7 p6) 1)
    (= (moveCost p6 p8) 1)    (= (moveCost p8 p6) 1)
    (= (moveCost p6 p9) 13)   (= (moveCost p9 p6) 13)
    (= (moveCost p7 p9) 3)    (= (moveCost p9 p7) 3)
    (= (moveCost p8 p9) 10)   (= (moveCost p9 p8) 10)
    (= (total-cost) 0)
 )

 (:goal
    (caughtMIB)
 )

 (:metric minimize (total-cost))
)
```

A.3 Net-Benefit Planning

A.3.1 The Domain Description

```
(define (domain desertNetben)
 (:requirements :typing :action-costs :numeric-fluents :goal-utilities)
 (:types person location thirstLevel)
 (:constants
    gunslinger mib - person
    t0 t1 t2 t3 - thirstLevel
 )
```

```
(:predicates
   (position ?p - person ?l - location)
   (adjacent ?l1 ?l2 - location)
   (caughtMIB)
   (waterAt ?l - location)
   (isDead ?p - person)
   (thirsty ?p - person ?t - thirstLevel)
)
(:functions
   (total-cost) - number
   (moveCost ?l1 ?l2 - location) - number
   (thirst) - number
)

(:action move
   :parameters (?l1 ?l2 - location)
   :precondition
   (and
       (< (thirst) 3)
       (position gunslinger ?l1)
       (adjacent ?l1 ?l2)
   )
   :effect
   (and
       (not (position gunslinger ?l1))
       (position gunslinger ?l2)
       (increase (total-cost) (moveCost ?l1 ?l2))
       (increase (thirst) 1)
   )
)

(:action catchMIB
   :parameters (?l - location)
   :precondition
   (and
       (< (thirst) 1)
       (position gunslinger ?l)
       (position mib ?l)
   )
   :effect
   (and
       (caughtMIB)
       (thirsty t0)
   )
)
(:action catchMIB
   :parameters (?l - location)
   :precondition
   (and
       (<= (thirst) 1)
       (>= (thirst) 1)
       (position gunslinger ?l)
       (position mib ?l)
```

```
    )
    :effect
    (and
        (caughtMIB)
        (thirsty t1)
    )
 )
 (:action catchMIB
    :parameters (?l - location)
    :precondition
    (and
        (<= (thirst) 2)
        (>= (thirst) 2)
        (position gunslinger ?l)
        (position mib ?l)
    )
    :effect
    (and
        (caughtMIB)
        (thirsty t2)
    )
 )

 (:action drink
    :parameters (?l - location)
    :precondition
    (and
        (< (thirst) 3)
        (position gunslinger ?l)
        (waterAt ?l)
    )
    :effect
    (assign (thirst) 0)
 )

 (:action hallucinate
    :parameters (?l - location)
    :precondition
    (>= (thirst) 3)
    :effect
    (and
        (caughtMIB)
        (isDead gunslinger)
    )
 )
)
```

A.3.2 The Problem Description

```
(define (problem desertNetbenProb)
 (:domain desertNetben)
 (:objects
    p0 p1 p2 p3 p4 p5 p6 p7 p8 p9 - location
 )
```

```
(:init
   (position gunslinger p0)
   (position mib p9)
   (adjacent p0 p1)   (adjacent p1 p0)
   (adjacent p0 p2)   (adjacent p2 p0)
   (adjacent p0 p3)   (adjacent p3 p0)
   (adjacent p1 p3)   (adjacent p3 p1)
   (adjacent p1 p4)   (adjacent p4 p1)
   (adjacent p2 p3)   (adjacent p3 p2)
   (adjacent p2 p5)   (adjacent p5 p2)
   (adjacent p3 p4)   (adjacent p4 p3)
   (adjacent p3 p5)   (adjacent p5 p3)
   (adjacent p3 p6)   (adjacent p6 p3)
   (adjacent p4 p6)   (adjacent p6 p4)
   (adjacent p4 p7)   (adjacent p7 p4)
   (adjacent p5 p6)   (adjacent p6 p5)
   (adjacent p5 p8)   (adjacent p8 p5)
   (adjacent p6 p7)   (adjacent p7 p6)
   (adjacent p6 p8)   (adjacent p8 p6)
   (adjacent p6 p9)   (adjacent p9 p6)
   (adjacent p7 p9)   (adjacent p9 p7)
   (adjacent p8 p9)   (adjacent p9 p8)
   (= (moveCost p0 p1) 2)    (= (moveCost p1 p0) 2)
   (= (moveCost p0 p2) 1)    (= (moveCost p2 p0) 1)
   (= (moveCost p0 p3) 10)   (= (moveCost p3 p0) 10)
   (= (moveCost p1 p3) 2)    (= (moveCost p3 p1) 2)
   (= (moveCost p1 p4) 7)    (= (moveCost p4 p1) 7)
   (= (moveCost p2 p3) 4)    (= (moveCost p3 p2) 4)
   (= (moveCost p2 p5) 12)   (= (moveCost p5 p2) 12)
   (= (moveCost p3 p4) 3)    (= (moveCost p4 p3) 3)
   (= (moveCost p3 p5) 2)    (= (moveCost p5 p3) 2)
   (= (moveCost p3 p6) 7)    (= (moveCost p6 p3) 7)
   (= (moveCost p4 p6) 6)    (= (moveCost p6 p4) 6)
   (= (moveCost p4 p7) 5)    (= (moveCost p7 p4) 5)
   (= (moveCost p5 p6) 7)    (= (moveCost p6 p5) 7)
   (= (moveCost p5 p8) 3)    (= (moveCost p8 p5) 3)
   (= (moveCost p6 p7) 1)    (= (moveCost p7 p6) 1)
   (= (moveCost p6 p8) 1)    (= (moveCost p8 p6) 1)
   (= (moveCost p6 p9) 13)   (= (moveCost p9 p6) 13)
   (= (moveCost p7 p9) 3)    (= (moveCost p9 p7) 3)
   (= (moveCost p8 p9) 10)   (= (moveCost p9 p8) 10)
   (waterAt p2)
   (waterAt p4)
   (waterAt p8)
   (= (thirst) 0)
   (= (total-cost) 0)
)

(:goal
   (and
       (caughtMIB)
       (preference pref_alive (not (isDead gunslinger)))
```

```
        (preference pref_notThirsty (thirsty t0))
        (preference pref_thirsty (thirsty t1))
        (preference pref_veryThirsty (thirsty t2))
    )
 )

 (:metric maximize
    (- 100
      (+
        (total-cost)
        (* (is-violated pref_alive) 50)
        (* (is-violated pref_notThirsty) 25)
        (* (is-violated pref_thirsty) 20)
        (* (is-violated pref_veryThirsty) 10)
      )
    )
 )
)
```

A.4 General Game Playing

```
(role gunslinger)
(role mib)

(init (position gunslinger p0))
(init (position mib p3))
(init (step 0))

(<= (legal gunslinger (move ?l1 ?l3))
    (true (position gunslinger ?l1))
    (adjacent ?l1 ?l2)
    (adjacent ?l2 ?l3)
    (distinct ?l1 ?l3)
)
(<= (legal mib (move ?l1 ?l2))
    (true (position mib ?l1))
    (adjacent ?l1 ?l2)
)

(<= (next (position ?p ?l2))
    (does ?p (move ?l1 ?l2))
)

(<= (next (step ?s2))
    (true (step ?s1))
    (succ ?s1 ?s2)
)

(succ 0 1)   (succ 1 2)   (succ 2 3)
(succ 3 4)   (succ 4 5)   (succ 5 6)
(succ 6 7)   (succ 7 8)   (succ 8 9)
(succ 9 10)
```

```
(adjacent p0 p1)   (adjacent p1 p0)
(adjacent p0 p2)   (adjacent p2 p0)
(adjacent p0 p3)   (adjacent p3 p0)
(adjacent p1 p3)   (adjacent p3 p1)
(adjacent p1 p4)   (adjacent p4 p1)
(adjacent p2 p3)   (adjacent p3 p2)
(adjacent p2 p5)   (adjacent p5 p2)
(adjacent p3 p4)   (adjacent p4 p3)
(adjacent p3 p5)   (adjacent p5 p3)
(adjacent p3 p6)   (adjacent p6 p3)
(adjacent p4 p6)   (adjacent p6 p4)
(adjacent p4 p7)   (adjacent p7 p4)
(adjacent p5 p6)   (adjacent p6 p5)
(adjacent p5 p8)   (adjacent p8 p5)
(adjacent p6 p7)   (adjacent p7 p6)
(adjacent p6 p8)   (adjacent p8 p6)
(adjacent p6 p9)   (adjacent p9 p6)
(adjacent p7 p9)   (adjacent p9 p7)
(adjacent p8 p9)   (adjacent p9 p8)

(<= caughtMIB
    (true (position gunslinger ?l))
    (true (position mib ?l))
    (not escapedMIB)
)

(<= escapedMIB
    (true (position mib p9))
)

(<= terminal
    caughtMIB
)
(<= terminal
    escapedMIB
)
(<= terminal
    (true (step 10))
)

(<= (goal gunslinger 100)
    caughtMIB
)
(<= (goal gunslinger 10)
    (true (step 10))
    (not caughtMIB)
    (not escapedMIB)
)
(<= (goal gunslinger 0)
    escapedMIB
)

(<= (goal mib 100)
```

```
    escapedMIB
)
(<= (goal mib 10)
    (true (step 10))
    (not caughtMIB)
    (not escapedMIB)
)
(<= (goal mib 0)
    caughtMIB
)
```

Appendix B

BNF of GDDL

B.1 BNF of GDDL's Problem File

⟨problem⟩::= (define (problem ⟨name⟩)
(:domain ⟨name⟩)
(:init ⟨atom⟩+)
(:goal ⟨expr⟩))
⟨expr⟩::= (and ⟨expr⟩+) |(or ⟨expr⟩+) |
(not ⟨expr⟩) | ⟨atom⟩
⟨atom⟩::= (⟨name⟩)
⟨name⟩::= ⟨letter⟩⟨symbol⟩*
⟨letter⟩::= a | . . . | z | A | . . . | Z
⟨symbol⟩::= ⟨letter⟩ | _ | - | ⟨number⟩
⟨number⟩::= 0 | . . . | 9

B.2 BNF of GDDL's Domain File

⟨domain⟩::= (define (domain ⟨name⟩)
(:predicates ⟨atom⟩+)
⟨multi-action⟩+
⟨reward⟩+)
⟨multi-action⟩::= (:multi-action
⟨player-action-pair⟩+
(:global-precondition ⟨expr⟩)
⟨precond-eff-pair⟩+)
⟨player-action-pair⟩::= (:player ⟨name⟩)
(:action⟨name⟩)
⟨precond-eff-pair⟩::= (:precondition ⟨expr⟩)
(:effect ⟨atom⟩)
⟨reward⟩::= (:reward
(:player ⟨name⟩)
(:value ⟨number⟩)
(:condition ⟨expr⟩))
⟨expr⟩::= (and ⟨expr⟩+) |(or ⟨expr⟩+) |
(not ⟨expr⟩) | ⟨atom⟩
⟨atom⟩::= (⟨name⟩)
⟨name⟩::= ⟨letter⟩⟨symbol⟩*
⟨letter⟩::= a | . . . | z | A | . . . | Z

⟨symbol⟩::= ⟨letter⟩ | _ | - | ⟨number⟩
⟨number⟩::= 0 | . . . | 9